Comparative analysis of methods for solving conduction-radiation coupling in heterogeneous materials

RAD-25 - 11th International Symposium on Radiative Transfer

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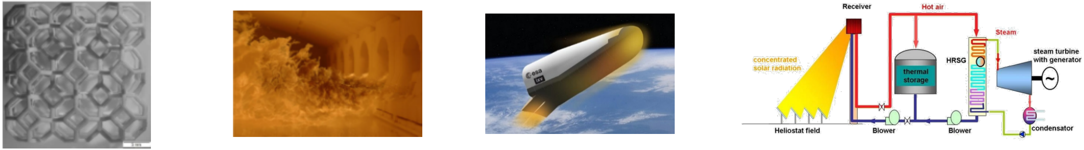
Monday, June 16th 2025

Outline

CNRS 2047 TAMARYS[1] thematic network

- Bringing together scientific and industrial players to address **multi-scale** and **multi-physics** difficulties in the sustainable of radiative-dominated **industrial processes and systems**.
- Research on resolving conduction-radiation coupling in **heterogeneous semi-transparent materials**.



Illustrations from B. Rousseau's presentation at CNRS Paris in April 2024, shared by CEA Le Ripault, Saint-Gobain, ArianeGroup, and PROMES...

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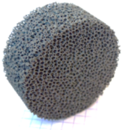
[1] Thematic network "**T**ransferts r**A**diatifs, **MA**tériaux, p**R**océdés et s**Y**stèmes a**S**sociés"

Key players and their mission

- Since 2019, the TAMARYS thematic network has set up the **Industrial Partners Club**:

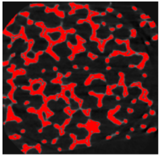
cnes ENGIE MERSEN SYENSQO ArcelorMittal arianeGROUP
cea 3DCERAM Air Liquide TotalEnergies SAINT-GOBAIN

- The **need**: quantify **apparent thermal properties** for dimensioning purposes.

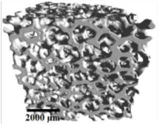


Thermal conductivity?

Thermal diffusivity?



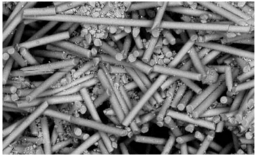
X-Ray µ-tomography

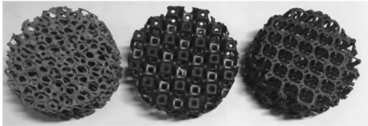


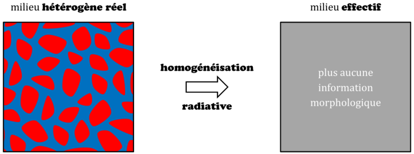
3D reconstruction
Spatial resolution : 24µm/voxel

Excerpts from B. Rousseau's presentation on 3D textural characterization at the 13th European Ceramics Society Conference (4/07/2023).

Motivation


Y. Maanane, PhD thesis, 2020


M. Sans, PhD thesis, 2019



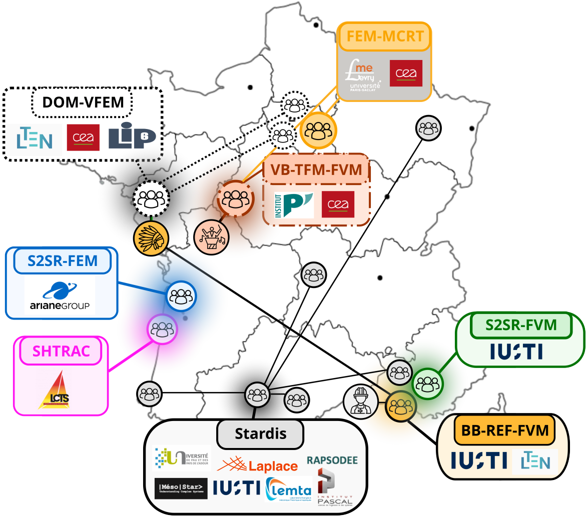
F. Enguehard, MATTER 2022

- **Conduction-radiation coupling** is crucial in porous, fibrous, or semi-transparent material.
- Solving the Radiative Transfer Equation (RTE) in **heterogeneous and non-homogenizable domains** remains a challenge.

Objective

Compare 8 state-of-the-art **numerical approaches** from French research teams (GDR TAMARYS) on a common configuration.

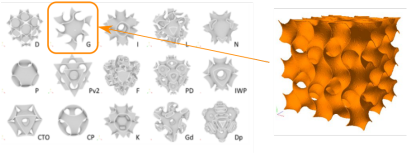
Collaboration among academic and industrial teams



A benchmark initiative

- First collective milestone of a **long-term comparative project** (*since 2017*)
- **Common case study**:

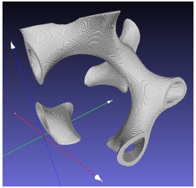
TPMS : “Triply Periodic Minimal Surface”



B. Rousseau, 13th European Ceramics Society Conference, 2023) -

Gyroid generated by genMat for a shape parameter (τ) equal to 0.

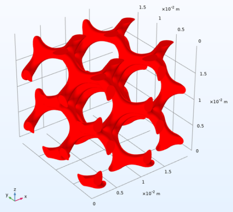
$$G_\tau = \sin x \cos y + \sin z \cos x + \sin y \cos z = \tau$$

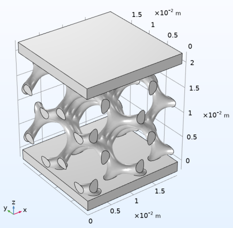


Gyroid generated by genMat for a shape parameter (τ) equal to 1.2.

Physical problem definition

- **2x2x2 gyroid** geometry (non-homogenizable)
- Opaque conductive solid + transparent non-conductive fluid
- Guarded hot plate configuration: 2000 K / 1000 K
- Opaque grey surfaces with specular reflection
- **Shared parameters**:
  - Same geometry and mesh (3 types provided)
  - Same material properties (SiC, $k = 13$ W/mK, emissivity $= 0.85$)
  - Same boundary conditions (Dirichlet + adiabatic)





Overview of the methodologies

Deterministic methods

- DOM[a]-VFEM[b]
- VB[c]-TFM[d]-FVM[e]
- BB[f]-REF[g]-FVM
- *Commercial solvers*:
    - S2SR[h]-FEM
    - S2SR-FVM

Hybrid methods

- FEM-MCRT[i]

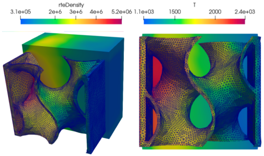
Stochastic methods

- SHTRAC[j]
- Stardis

[a] Discrete Ordinates Method | [b] Vectorial Finite Element Method | [c] Voxel-Based | [d] Two-Flux Method | [e] Finite Volume Method | [f] Block-Based | [g] Radiative Exchange Factors | [h] Surface-to-Surface Radiation | [i] Monte Carlo Ray Tracing | [j] Simulation of Heat Transfer by Radiation and Conduction

Overview of the methodologies: Deterministic methods 1/2

DOM-VFEM

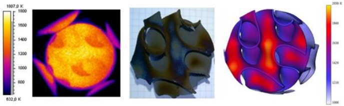


Y. Favennec, 2024

- All finite elements
- **Vectorial** and stabilized finite elements
- Discrete Ordinates Method
- Highly parallelizable

P. Jolivet et al., JCP, 2021 | S. Ouchtout et al., UHMT, 2022

VB-TFM-FVM

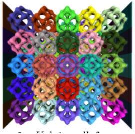


C. Heisel et al., 2021

- THERMIVOX software
- Conduction solved by finite volumes
- Radiation solved by a simplified approach with **homogeneous** voxels and **isotropic luminances** by half-spaces

M. Niezgoda et al., JPCS, 2012 | C. Heisel et al.,

BB-REF-FVM



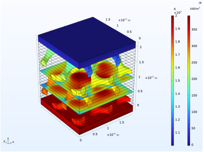
A. Kumar et al., 2021

- genMat software
- Conduction solved by finite volumes **for each sub-volume** (block)
- Radiation solved by a Discrete Ordinate Method **from voxel to voxel**

S. Guévelou, PhD thesis, 2015 | A. Kumar et al.,

Overview of the methodologies: Deterministic methods 2/2

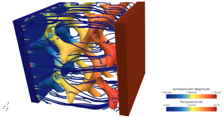
S2SR-FEM



D. Jehl, 2025

- **COMSOL Multiphysics®** FEM solver using Surface-to-Surface Radiation (S2SR).
- Heat fluxes computed via **Hemicube method** (conduction) and ray tracing (radiation).
- Radiative fluxes extracted on predefined planar surfaces using transparency tricks.

S2SR-FVM



Y. Jobic and F. Topin, 2025

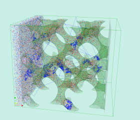
- **Star-CCM®** commercial FVM/FEM solver using S2SR model.
- Surface-to-surface radiation via view factors; non-participating medium assumed.
- Radiative fluxes indirectly obtained from temperature gradients in the solid.

Overview of the methodologies: Hybrid and Stochastic methods

FEM-MCRT

- **In-house hybrid solver**: FEM for conduction, Monte Carlo Ray Tracing for radiation.
- Surface-to-surface exchanges on unstructured tetrahedral mesh.
- Future developments: stochastic RTE and model order reduction for conduction.

SHTRAC

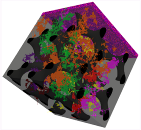


G. Vignoles, 2025

- **Hybrid random walkers**: Brownian (conduction) and ballistic (radiation).
- Simplified Marching Cubes technique to handle solid/void interfaces.
- Computes effective thermal conductivities from steady-state walker distributions.

G. Vignoles et al., IJHMT, 2016

Stardis



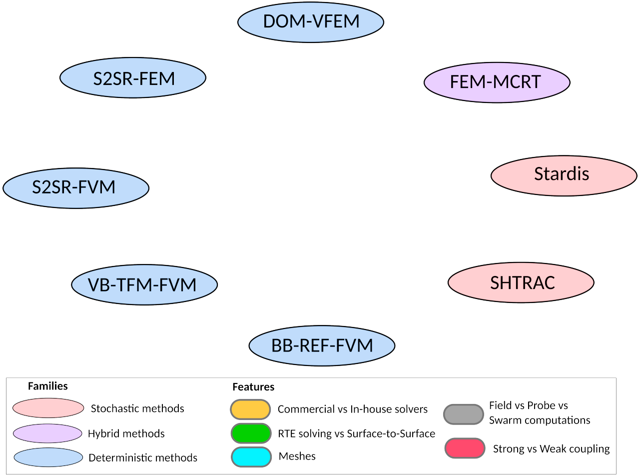
A. A. Fortunat, 2024

- Monte Carlo method solving conduction–radiation coupling via **path sampling**.
- No volume mesh required
- Non-linear radiative transfer handled through null-collision and modified Picard schemes.

J.-M. Tregan et al., PLOS One, 2023

L. Penazzi et al., Comphy Journal, 2024

Shared features between methods



DOM-VFEM

S2SR-FEM

FEM-MCRT

Stardis

S2SR-FVM

VB-TFM-FVM

SHTRAC

BB-REF-FVM

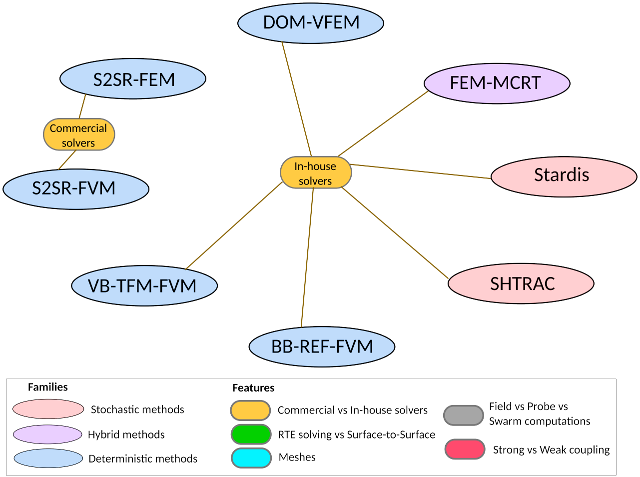
Families

- 🟥 Stochastic methods
- 🟪 Hybrid methods
- 🟦 Deterministic methods

Features

- 🟨 Commercial vs In-house solvers
- 🟩 RTE solving vs Surface-to-Surface
- 🟦 Meshes
- ⬛ Field vs Probe vs Swarm computations
- 🟥 Strong vs Weak coupling

Shared features between methods



DOM-VFEM

S2SR-FEM

Commercial solvers

S2SR-FVM

FEM-MCRT

In-house solvers

Stardis

VB-TFM-FVM

SHTRAC

BB-REF-FVM

Families

- Stochastic methods
- Hybrid methods
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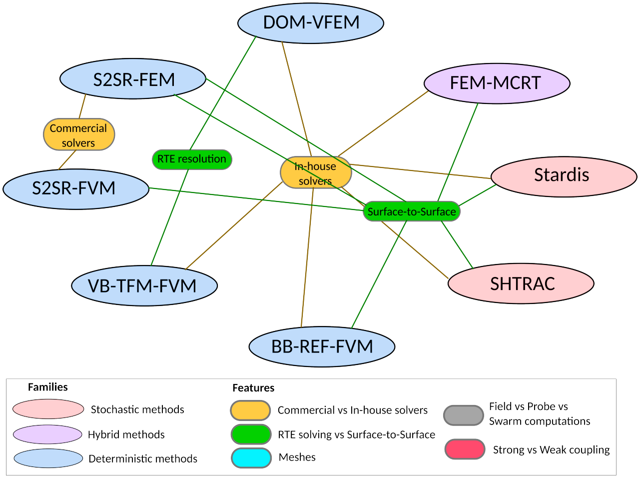
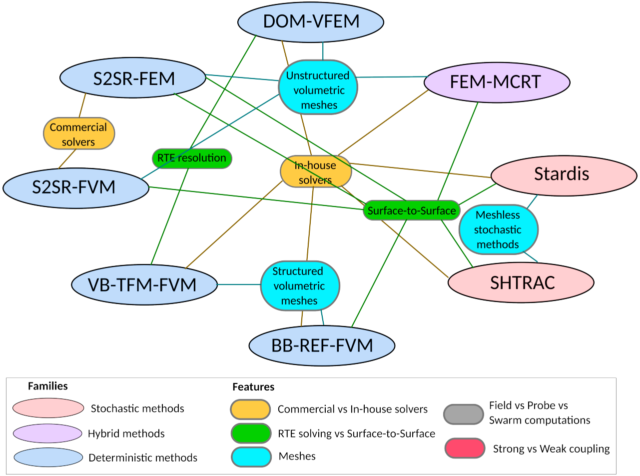
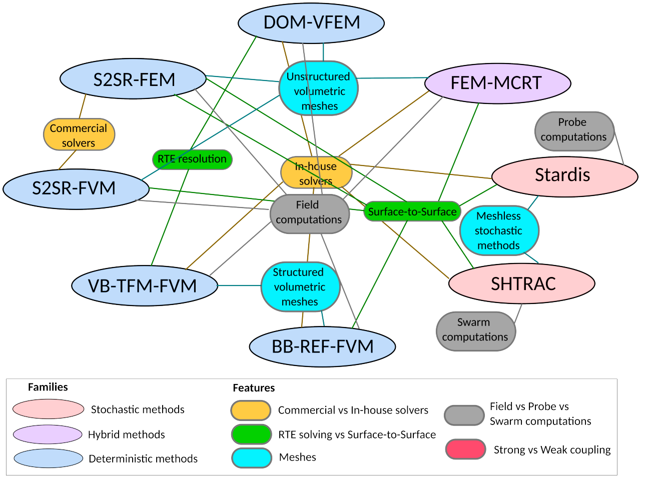
Shared features between methods



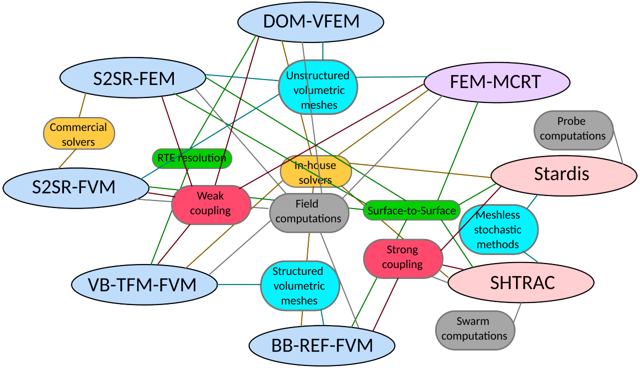
Shared features between methods



Shared features between methods



Shared features between methods



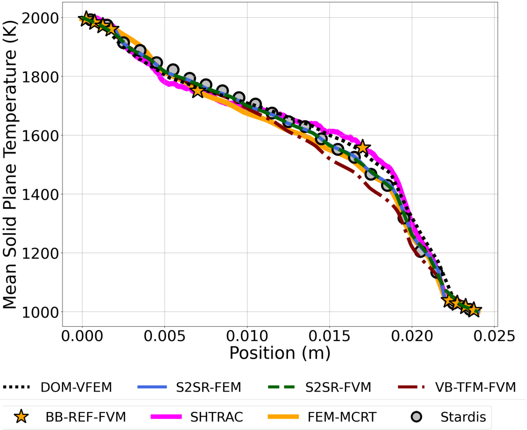
Families

- Stochastic methods
- Hybrid methods
- Deterministic methods

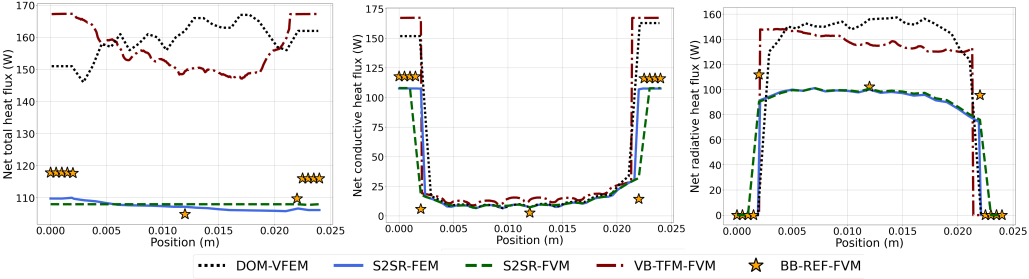
Features

- Commercial vs In-house solvers
- RTE solving vs Surface-to-Surface
- Meshes
- Field vs Probe vs Swarm computations
- Strong vs Weak coupling

Temperature Profiles



Heat Flux Profiles



Discussion: Open Questions

- **Two groups clearly emerge**:
  - Methods solving the RTE (DOM-VFEM, VB-TFM-FVM) yield consistent high fluxes.
  - Commercial solvers (S2SR-FEM/FVM) and BB-REF-FVM yield lower, consistent values.
- **Yet, no definitive explanation** can be drawn at this stage.

Elements currently under consideration:

- Radiative flux discrepancies likely stem from modeling assumptions (e.g. isotropic intensity in VB-TFM-FVM).
- Solver robustness is affected by parameters like angular discretization, interface treatments, etc.
- Energy conservation is not ensured in all approaches (FVM: conservative by design; FEM: not always).
- Confidence bounds are difficult to define in the absence of a reference solution.
- Commercial solvers offer stable results, but cannot yet be assumed as a reference by default.

Key take-aways

- **A collective ambition years in the making:** The idea of this comparative benchmark emerged in 2017. It materialized concretely through my postdoc starting in early 2024.
- **Time is a critical factor:** It took nearly a year to agree on a common geometry (gyroid) and mesh. The first usable results have been available for only a few months — highlighting the effort behind such coordination.
- **No "best" method – and that's the point:** Each approach is shaped by specific needs and modeling philosophies. The goal is not to rank them, but to learn from their differences and complementarities.
- **Shared configuration, diverse outcomes:** Benchmarking methods on a common case reveals strengths for some, and challenges for others — providing precious insight into modeling choices.
- **This is only the beginning:** We now have a foundation, a first achievement. The comparative analysis can truly begin, and will evolve in future publications and collaborations.

Perspectives

- **Step 1 – Toward validation:** Use a simplified test case with analytical solution: the classical **Viskanta configuration**. → A common benchmark for all methods, to assess convergence and accuracy.
- **Step 2 – Preparing the extended paper (JQSRT):**
  - Explicitly state assumptions for each method (angular and spatial discretization, convergence criteria).
  - Quantify numerical uncertainties and sensitivity to key parameters.
  - Compare **CPU time**, **memory usage** and convergence behaviors.
- **Step 3 – Extending the benchmark scope:**
  - Semi-transparent **participating solids**.
  - **Non-grey media**, more realistic optical properties.
  - More complex geometries and boundary conditions.

Thank you for your attention!
Questions?

This benchmark is a collective effort within the GDR CNRS TAMARYS.
Let's keep the discussion going – your feedback is welcome.

Contact: lea.penazzi@univ-amu.fr
More info: `https://gdr-tamarys.cnrs.fr/`

Methodology

(1) Definition of **morphological and geometrical parameters** → Share common geometry files (STL, RAW) and 3D images

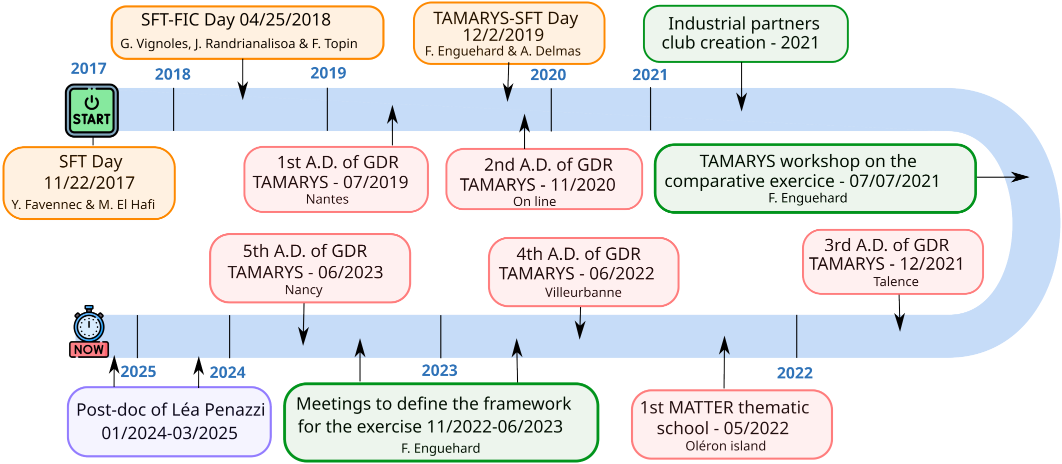
↓

(2) Definition of a set of **thermophysical parameters** and **boundary conditions** in conduction and radiation for collective applications

↓

(3) **Production** of initial **results** (temperature and heat flux profiles)

Origins and history of this exercise



SFT Day
11/22/2017
Y. Favennec & M. El Hafi

SFT-FIC Day 04/25/2018
G. Vignoles, J. Randrianalisoa & F. Topin

1st A.D. of GDR
TAMARYS - 07/2019
Nantes

TAMARYS-SFT Day
12/2/2019
F. Enguehard & A. Delmas

2nd A.D. of GDR
TAMARYS - 11/2020
On line

Industrial partners
club creation - 2021

TAMARYS workshop on the
comparative exercice - 07/07/2021
F. Enguehard

3rd A.D. of GDR
TAMARYS - 12/2021
Talence

1st MATTER thematic
school - 05/2022
Oléron island

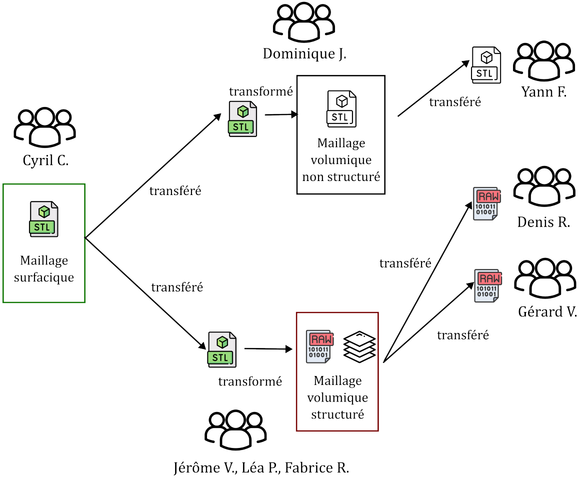
4th A.D. of GDR
TAMARYS - 06/2022
Villeurbanne

Meetings to define the framework
for the exercise 11/2022-06/2023
F. Enguehard

5th A.D. of GDR
TAMARYS - 06/2023
Nancy

Post-doc of Léa Penazzi
01/2024-03/2025

Mesh workflow



Registration is now open! International CNRS Thematic School on Radiative Transfer in Semi-Transparent Media

Vacation Village Azureva, Hendaye, France (3-9 October 2026)

Supported by

This autumn Thematic School will offer general lectures on radiation transfer in semi-transparent media, covering gases, plasmas, particles suspensions as well as dense and porous materials. Numerical methodologies to solve the radiative transfer equation will be tackled.

The program aims to offer basic and advanced courses that provide doctoral students, researchers and engineers from academic institutions and private companies with robust methodologies enabling them to meet major contemporary industrial challenges where radiative transfers are inevitable.

This includes both the reliable decarbonisation of high-temperature industrial processes as well as the reusability of spacecraft subjected to hypersonic flight regimes. The school will be held at the Vacation Village Azureva in Hendaye, France. All courses will be taught in English.

Registration fees*

Researchers, engineers, Ph-D students from CNRS	PhD students	Ass. professors, professors, engineers	Participants from private institutions, members of the IPC	Participants from private institutions, outside the IPC
0 €	300 €	500 €	700 €	1000 €

*Training + Full accommodation on a double room basis including catering for 5 or 7 days. IPC : TAMARYS Industrial Partners Club

Website for registration : https://rtstm.sciencesconf.org/ ; there are 65 places available: register now! Registration closes on **30 June 2026.**

Lectures

Basics of thermal radiation
Prof. F. Enguehard,
*Université de Poitiers, **France***

Methods for solving the RTE
Prof. P. Coelho,
Instituto Superior Técnico,
Portugal

Physics of gas radiation
Dr A. Soufiani,
*CNRS, **France***

Approximate gas radiation models
Prof. F. Liu,
*NRCC, **Canada***

Radiation scattering in particulate media
Prof. R. Carminati,
ESPCI Paris - PSL University,
France

Near-field radiative transfer
Prof. M. Francoeur,
*University of Utah, **USA***

Radiation physics of solids
Dr B. Rousseau,
*CNRS, **France***

Identification of the rad. prop. of semi-transaparent media
Prof. K. Daun,
*University of Waterloo, **Canada***

A history of radiative transfer
Prof. P. Menguc,
*Ozyeğin University, **Turkey***

Practical works

Advanced Monte Carlo methods
*Prof. R. Fournier, Univ. de Toulouse & Prof. M. El Hafi, IMT Mines Albi, **France***

Infrared thermography and spectroscopy
Prof. G. Parent,
*Université de Lorraine, **France***

Modelling gases radiative properties
Dr F. André,
*CNRS, **France***