Beyond Spectral Bias in Geothermal Heat Transport

A Comparative Analysis of Fourier Neural Operators and DeepONet Architectures in Heterogeneous Media

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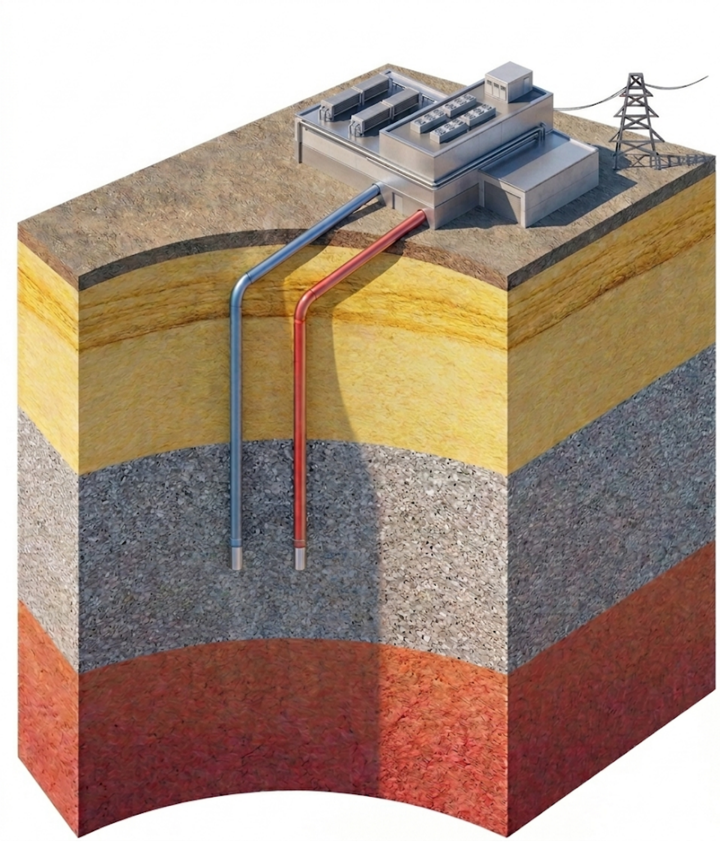
Geothermal energy: challenges

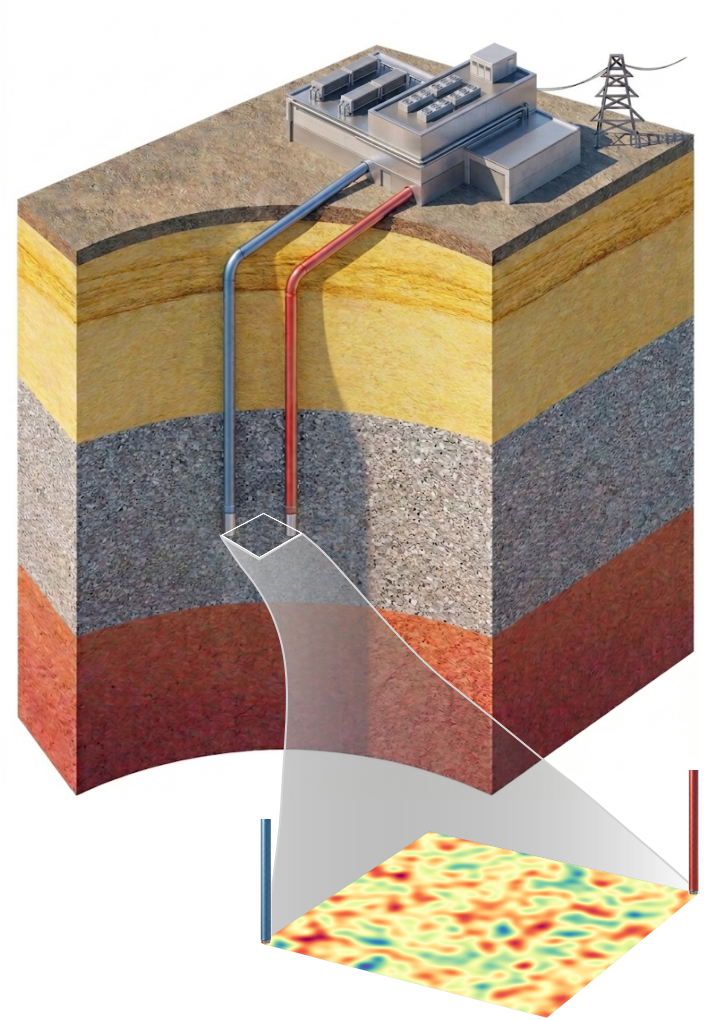
- **The Potential:** Stable, continuous, and resilient renewable energy source.
- **The Geothermal Doublet:** Cold water injection and hot fluid extraction.
- **Scalability Barriers:** High spatial variability and subsurface heterogeneity.



Geothermal energy: the shift towards digital twins

- **The Computational Bottleneck:** Traditional methods (FEM/FVM) are too slow for many-query analyses.
- AI-based surrogate model can replace costly numerical simulations for real-time optimization.

Research Objective: Use neural operators to predict geothermal temperature behaviour.



1. Governing Equations:

$$\nabla \cdot \mathbf{q} = 0, \quad \mathbf{q} = -\frac{k(\mathbf{x})}{\mu}\nabla p(\mathbf{x})$$

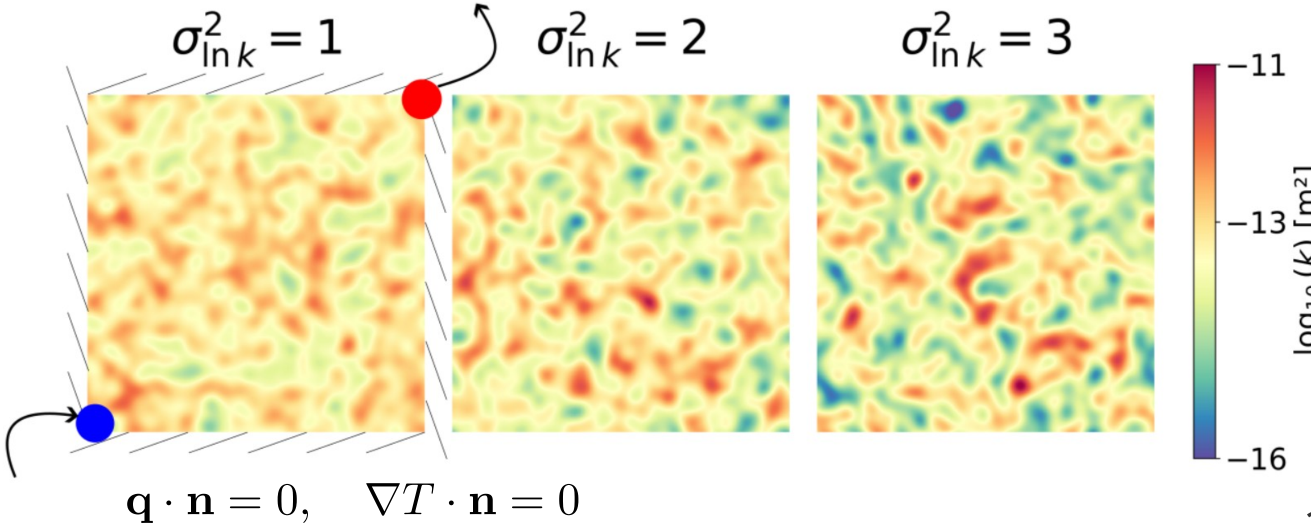
$$\frac{\partial T}{\partial t} + \mathbf{q}(\mathbf{x}) \cdot \nabla T = \nabla \cdot \left(\frac{U \cdot L}{Pe}\nabla T\right)$$

2. Characteristic Parameters:

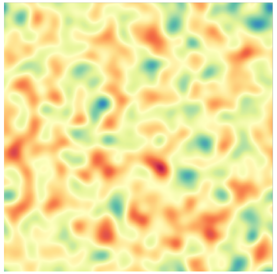
$$Pe = \frac{\rho_f c_f L}{\kappa_{\text{eff}}} U, \quad t_c = \frac{L^2(\rho c)_{\text{eff}}}{\kappa_{\text{eff}}}\left(\frac{1}{Pe}\right)$$

Permeability fields

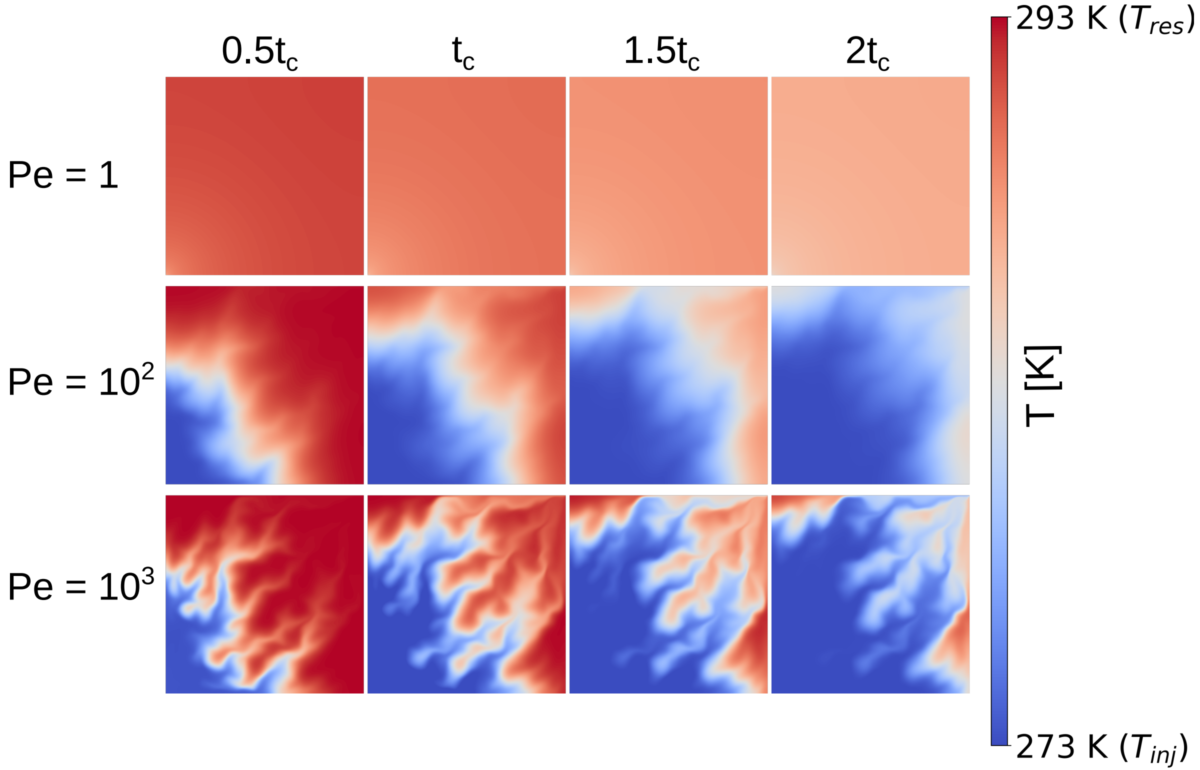
- Log-normal spectral generation
- Log-variance: σ^2 = 1, 2, 3
- Correlation length: 0.03



Methodology: physical problem and governing equations



Temperature fields



Data generation for training

- Three FEM simulations per permeability field
- Conducted at Péclet numbers 1, 100, and 1000
- Each resolved over four time steps

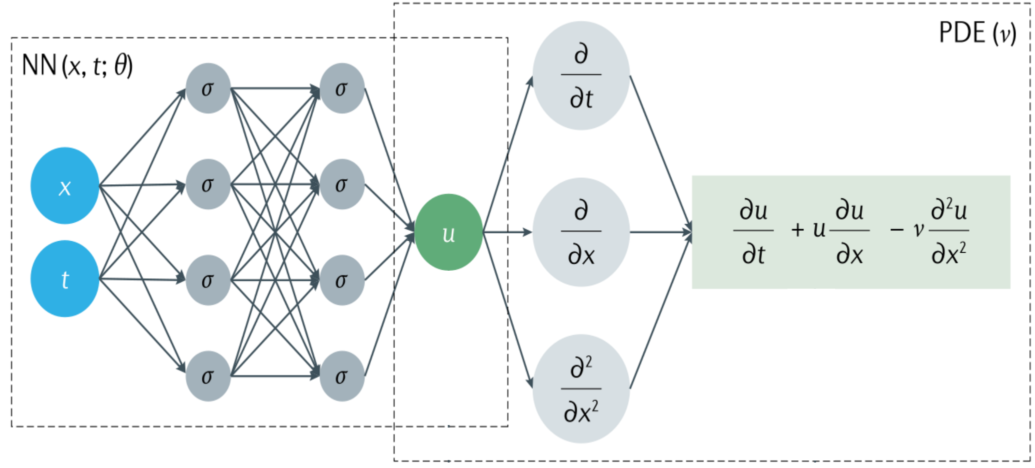
Methodology: operator learning

PINNs

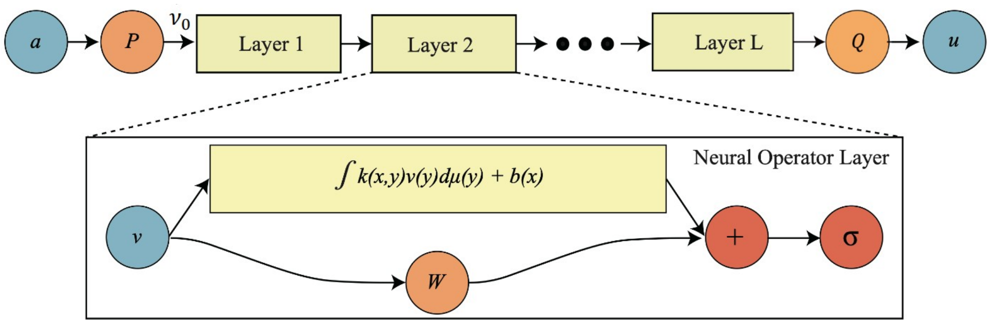
- Learns one solution for one specific problem instance
- New parameters → retrain from scratch

Neural operators

- Learns the map between function spaces: any K in → corresponding T out
- Train once on {K_i, T_i} pairs, generalise instantly to unseen K



(Karniakis et al. 2021)

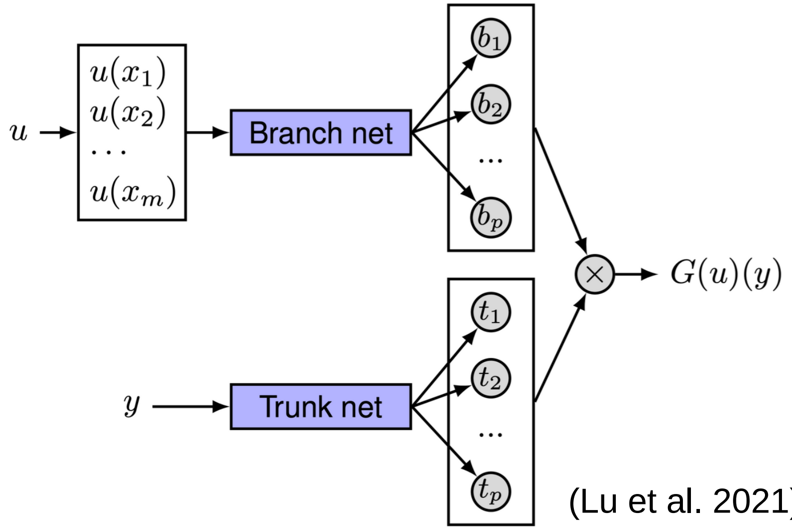


(Li et al. 2020)

Methodology: architectures

DeepONet

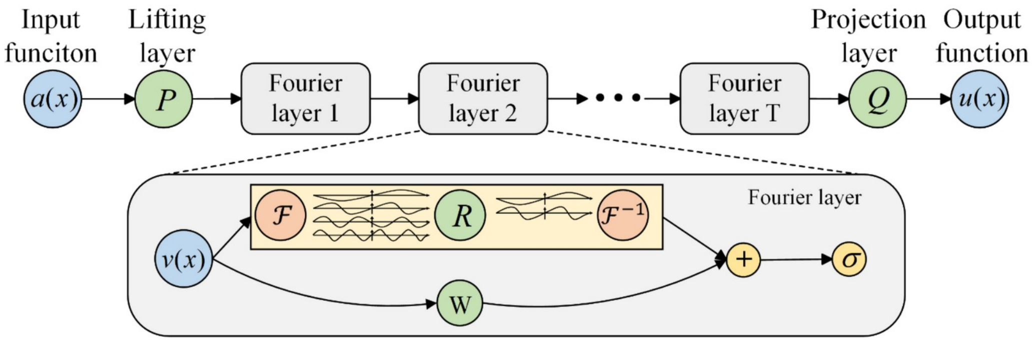
- Two parallel networks: one encodes the input field, one encodes spatial coordinates
- Combines them via dot product to predict the output at any point in space



(Lu et al. 2021)

Fourier Neural Operator (FNO)

- Transforms the input field into the frequency domain via Fourier transform
- Learns global spatial patterns directly from spectral coefficients, then maps back to physical space



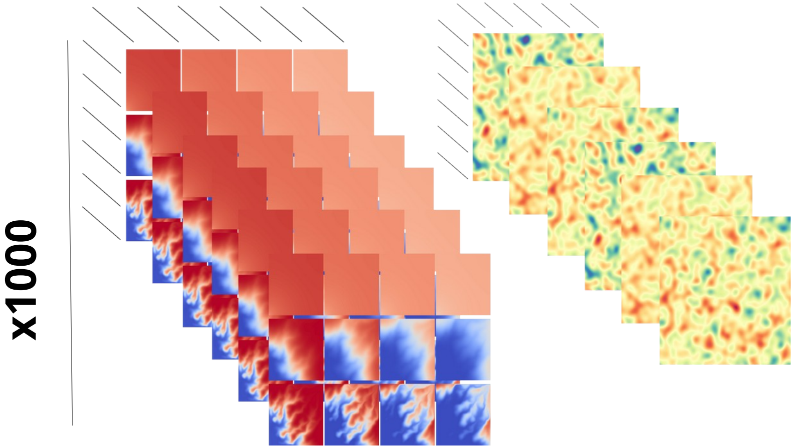
(Li et al. 2023)

Both minimize the distance between predictions and FEM simulations:

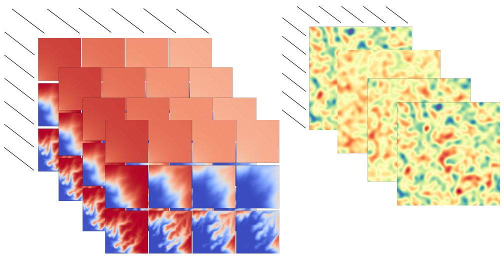
$$\mathcal{L}_{L^2} = \frac{1}{N}\sum_{i=1}^{N}\frac{\|\hat{T}_i - T_i\|_2}{\|T_i\|_2}$$

Methodology: experiment setup

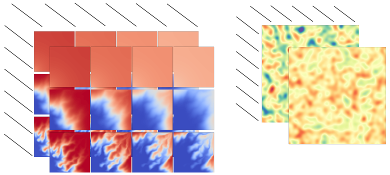
- 1,000 FEM simulations per Péclet number
- 700 train / 150 validation / 150 test
- One FNO and DeepONet per step: K, $T_i \rightarrow T_{i+1}$



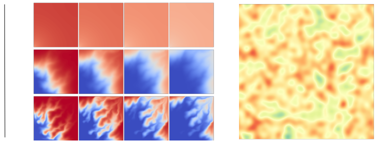
Train set x700



Valid and Test: x150 per set

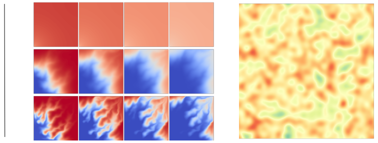


Baseline



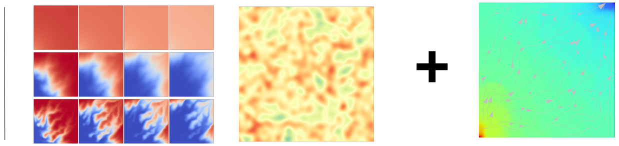
$$\mathcal{L}_{L^2} = \frac{1}{N}\sum_{i=1}^{N}\frac{\|\hat{T}_i - T_i\|_2}{\|T_i\|_2}$$

Experiment A: Sobolev norm



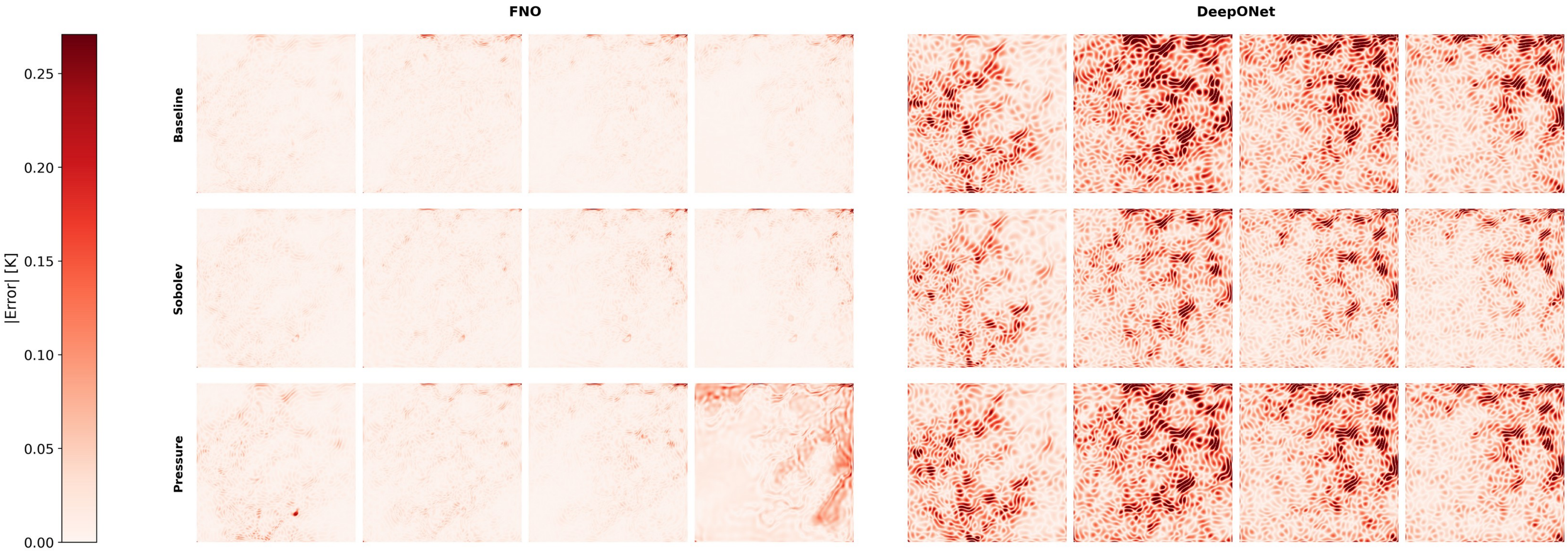
$$\mathcal{L}_{H^1} = \mathcal{L}_{L^2} + \lambda\frac{1}{N}\sum_{i=1}^{N}\frac{\|\nabla\hat{T}_i - \nabla T_i\|_2}{\|\nabla T_i\|_2}$$

Experiment B: Pressure field

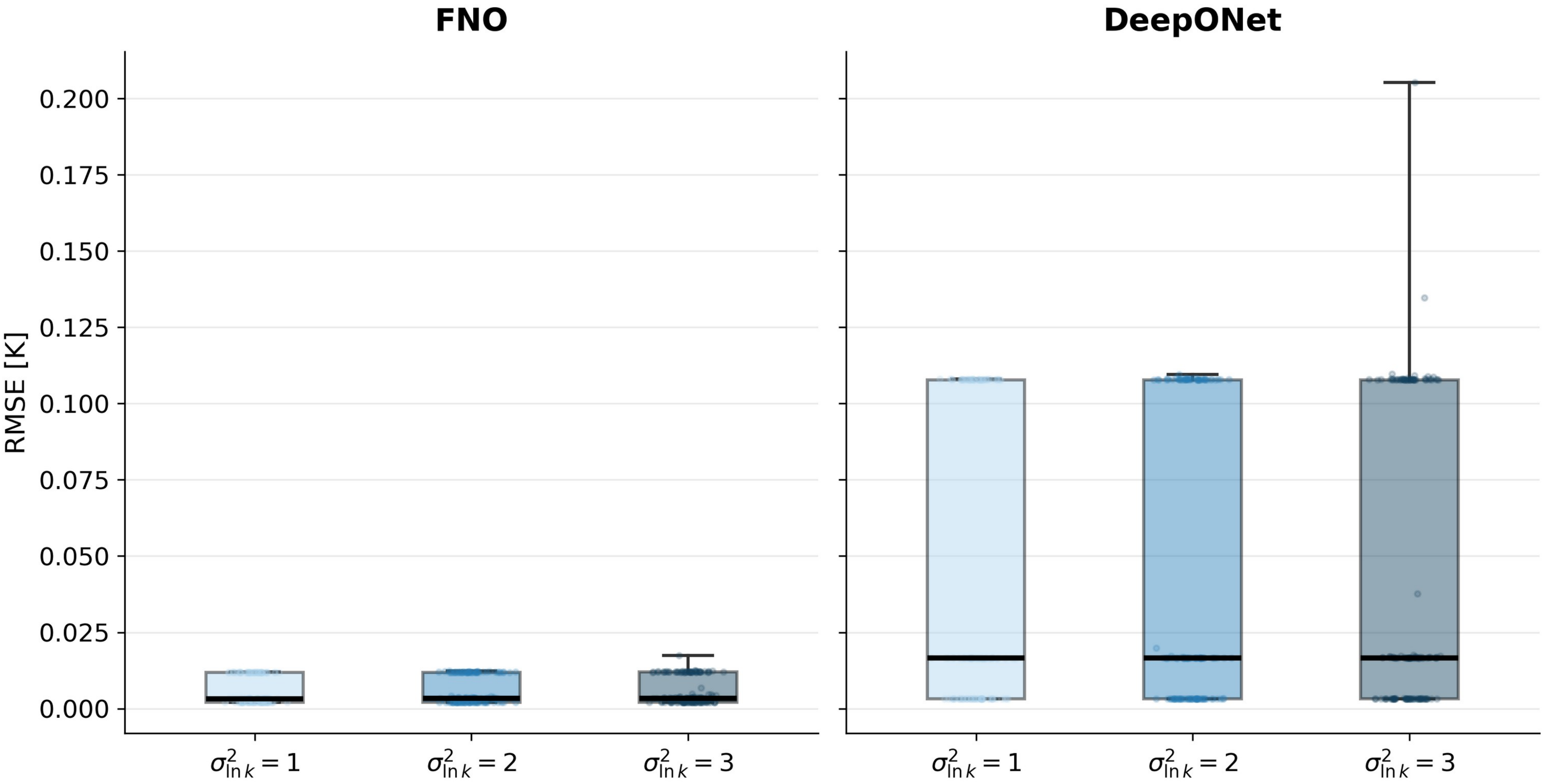


$$\mathcal{L}_{L^2} = \frac{1}{N}\sum_{i=1}^{N}\frac{\|\hat{T}_i - T_i\|_2}{\|T_i\|_2}$$

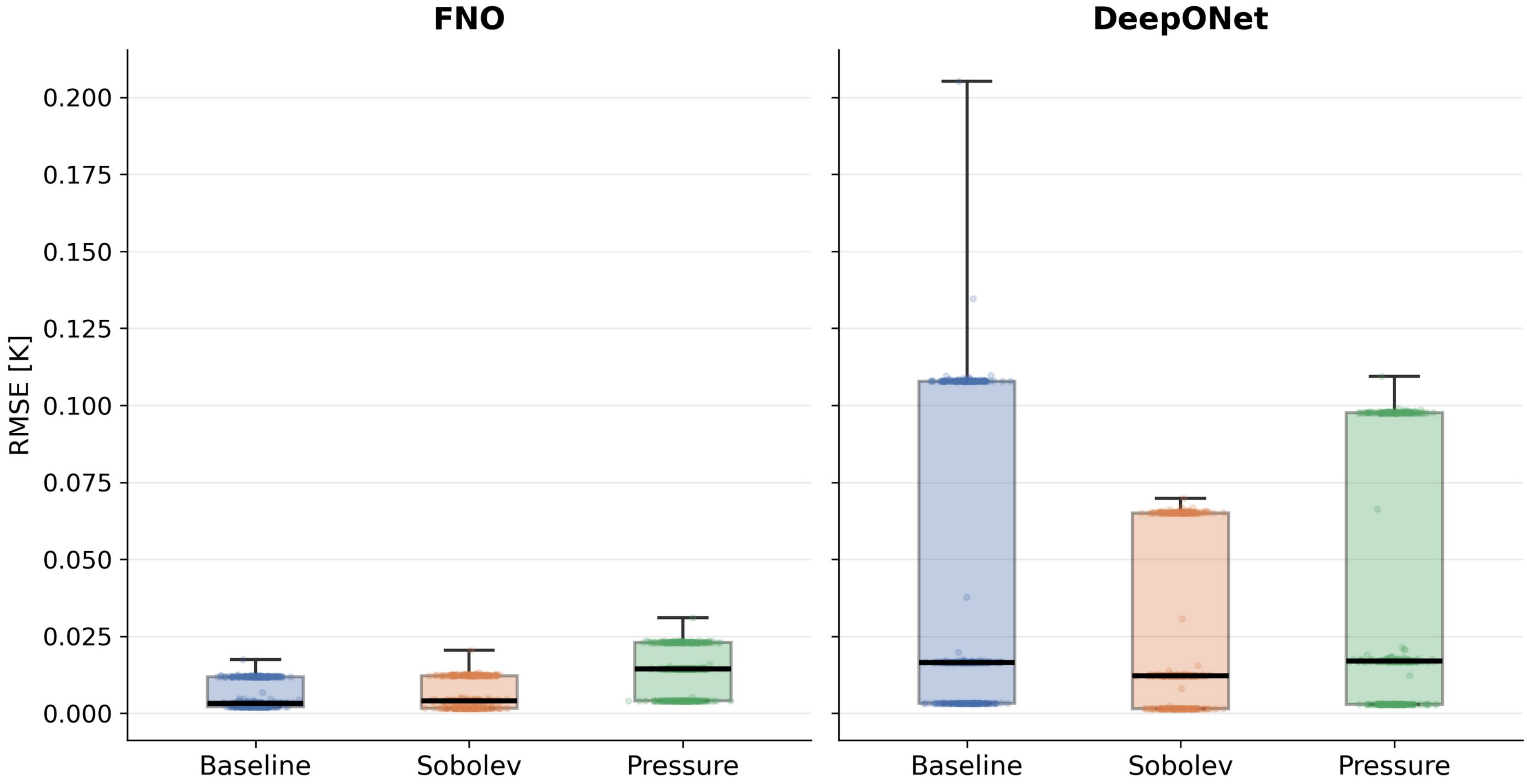
Results: permeability field with $\sigma^2 = 3$ at Pe = 1000



Results: baseline comparison across variances



Results: model comparison across experiments



Summary & Outlook

Conclusions

- FNOs outperform DeepONets across all experiments
- High resolution (400×400) likely facilitates L2 loss in resolving thermal fronts that may not hold at coarser resolutions
- Low test errors reflect in-distribution evaluation: train and test share the same log-variance distribution

Future directions

- OOD evaluation: higher log-variances, channelized/fractured fields, unseen Pe values
- Assess Sobolev loss impact at lower resolutions
- Extend rollout beyond 4 steps to study error accumulation

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Thank you for your attention

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