Activity driven flows of dense bacteria suspensions in porous structures

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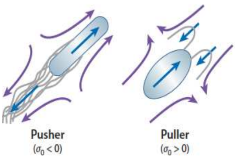
Motivation: Transport properties in bacterial flows

Continuous injection of energy into viscous medium ($Re \to 0$)

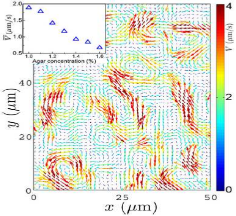


$|\alpha_0| n \sim \phi$ Active stress

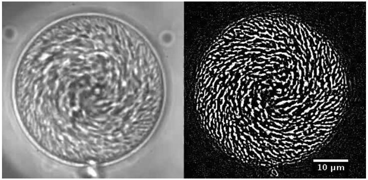
V_0 swimming velocity

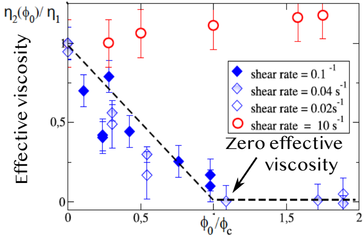
d_t translational diffusion

d_r rotational diffusion



Hydrodynamic interactions, flow alignment led to Collective motion.



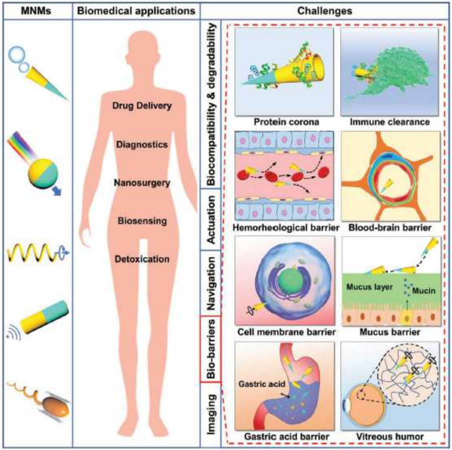


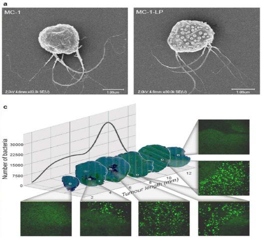
Confinement controls the Collective motion. Super fluid under shear rate condition.

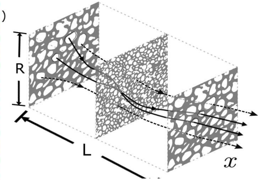
[H. P. Zhang et al Eur. Phys. Lett. (2009) , Wioland et al., Phys. Rev. Lett. (2013),Ganesh et al., Phys. Rev. Lett. (2025)]

Motivation: Active matter in crowded environment

Motile bacteria, catalytic Janus particles, nanobots



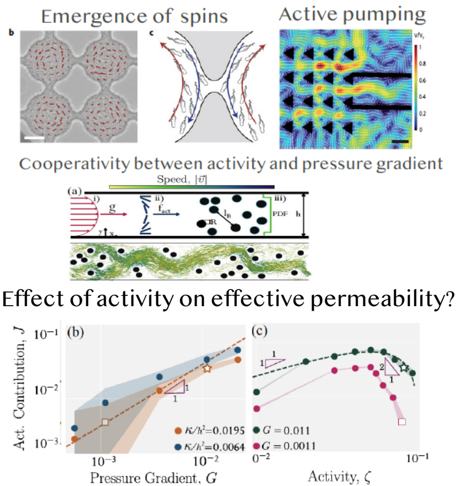




Controlling the navigation of active matter through confined and crowded environments under external forces

Gao et al Adv. Mater. (2021), Gao et al., Small (2013), Felfoul et al., Nat. Nanotech. (2016), Burada et al. Phys. Rev. Lett. (2008),Zhu et al. Phys. Rev. Lett. (2022)

Motivation: A Darcy's law for Active matter



Wioland, et al. Nat. Phys. (2016), Velez-Ceron, et al., Proc. Natl. Acad. Sci. (2025), Keogh et al Phs. Rev. Lett. (2024)

Study plan:

- Pressure driven flow of bacterial suspension in a Helle-Show cell
- Influence of periodic square obstacles

2D nematic model for semi-dilute suspensions

Model: Non-dimensional Macroscopic model

Density

$$\frac{D\rho}{Dt} = -\nabla \cdot \mathbf{J}_\rho$$

$$\mathbf{J}_\rho = Pe\nabla\mathbf{m} - D_t\nabla\rho$$

Polarization

$$\frac{D\mathbf{m}}{Dt} = -\nabla \cdot \mathbf{J}_\mathbf{m} + 0.5\mathbf{E}\cdot\mathbf{m} - \mathbf{W}\cdot\mathbf{m} - 2\mathbf{m}$$

$$\mathbf{J}_\mathbf{m} = Pe\mathbf{Q} + Pe\rho\mathbf{I}/2 - D_t\nabla\mathbf{m}$$

Nematic order tensor

$$\frac{DQ_{ij}}{Dt} = -\frac{\partial}{\partial x_k}J_{Q_{ij}} + 0.5\rho E_{ij} + 2/3E_{il}Q_{lj} - 4Q_{ij} - 1/3E_{kl}Q_{lk}\delta_{ij} + W_{ij}Q_{jl} - Q_{ij}W_{jl}$$

$$J_{Q_{ij}} = Pe\left(\frac{1}{4}\left(m_k\delta_{ij} + m_i\delta_{kj} + m_j\delta_{ki}\right) - \frac{1}{2}m_k\delta_{ij}\right) - D_t\frac{\partial}{\partial x_k}Q_{ij}$$

Stokes Equation

$$\nabla^2\mathbf{u} - \nabla \mathrm{P} = \alpha\nabla\cdot\mathbf{Q}, \quad \nabla\cdot\mathbf{u} = \mathbf{0}$$

$$\mathbf{V}_p = Pe\frac{\mathbf{m}}{\rho} + \mathbf{u}$$

Characteristic scale	Definition
τ	$1/d_r$
$\mathbf{x}^*$	H
$\mathbf{u}^*$	Hd_r
P^*	$\eta V_0/H$

[Theillard, et al., Soft Matter (2017)]

Adimensional variable	Definition
Pe	$\frac{V_0}{Hd_r}$
α	$\alpha_0 nH/(\eta V_0)$
D_t	$\frac{d_t\tau}{H^2} = Pe^2\frac{d_r d_t}{V_0^2}$

Periodic channel along the x-axis with $L_x = 8H$

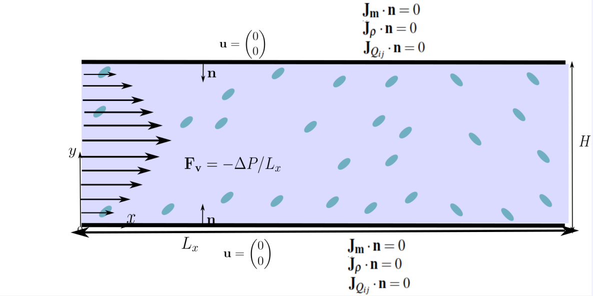


Figure: Poiseuille flow on a Hele-Shaw cell

$$\langle u_x \rangle_{x,y,t} = -\frac{k_{eff}(1,\alpha)}{\eta_0} \frac{\Delta P}{L_x}$$

Effective permeability $k_{eff}(1, \alpha)$

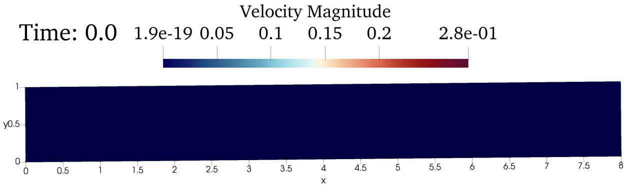
Average flow rate $\langle u_x \rangle_{x,y} = \int \int u_x(x,y) dA$

Average mass flow rate $\langle \dot{m}_x \rangle_{x,y} = \int \int u_x(x,y) \rho(x,y,t) dA$

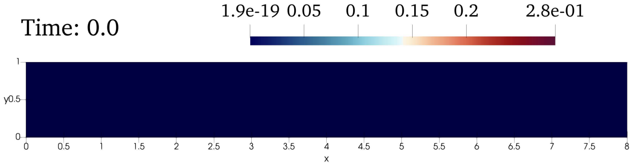
Results: Effect of the pressure gradient

$\alpha = -35.49,\ Pe = 0.044,\ D_t = 0.0016$

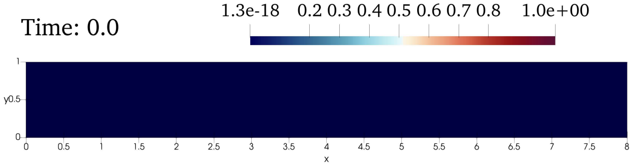
$\frac{\Delta P}{L_x} = 0.032$



$\frac{\Delta P}{L_x} = 0.4$



$\frac{\Delta P}{L_x} = 0.64$



Fixing the pressure gradient $-\frac{\Delta P}{L_x} = 0.3520$

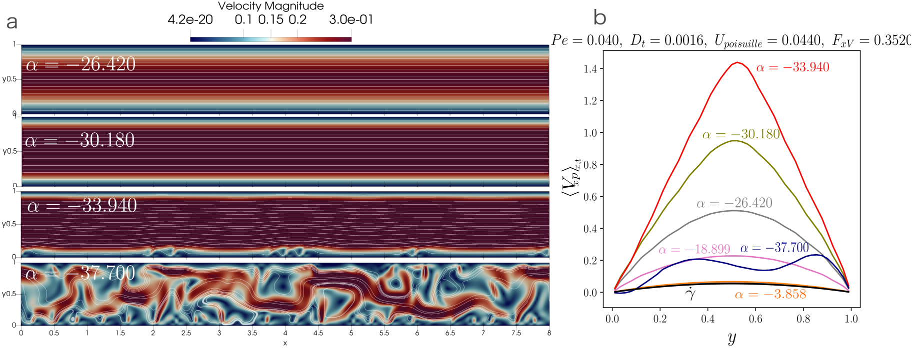


Figure: (a) fluid velocity (b) mean particle velocity profile $\mathbf{V}_p = Pe\frac{\mathbf{m}}{\rho} + \mathbf{u}$

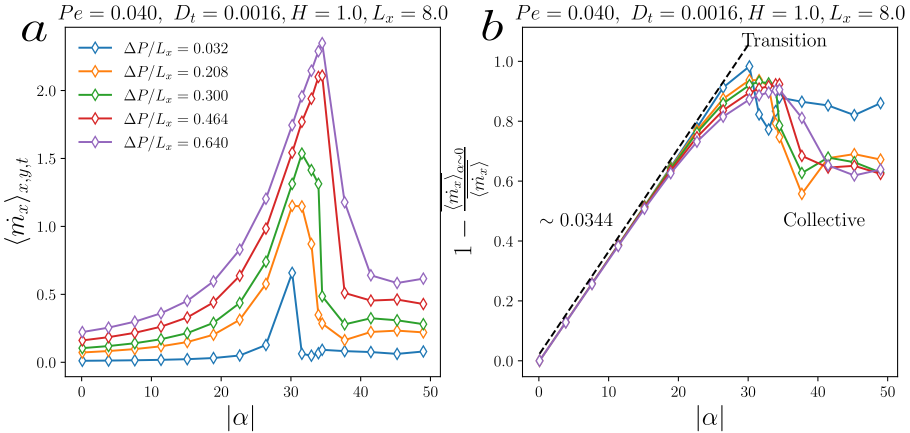


Figure: (a) Average mass flow rate as a function of α, (b) excess mass flow rate due activity.

$$\frac{\langle \dot{m}_x \rangle_{x,y,t}|_{\alpha\sim 0}}{\langle \dot{m}_x \rangle} \simeq \frac{\langle u_x \rangle_{x,y,t}|_{\alpha\sim 0}}{\langle u_x \rangle} \simeq \frac{\eta_{eff}}{\eta_0}$$

Results: Mass flow rate Darcy's law

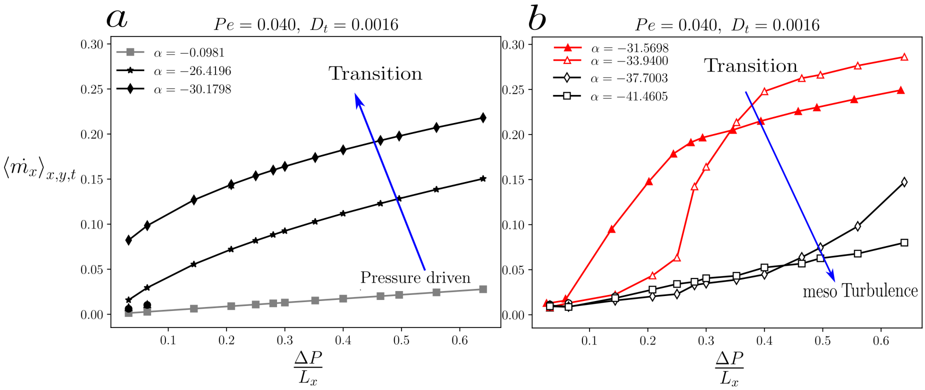


Figure: Average mass flow rate vs pressure gradient (a) from linear to transition (b) from transition to collective regime.

An array of 8 squares of length l_0

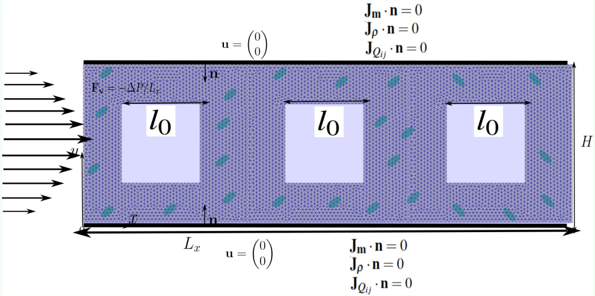


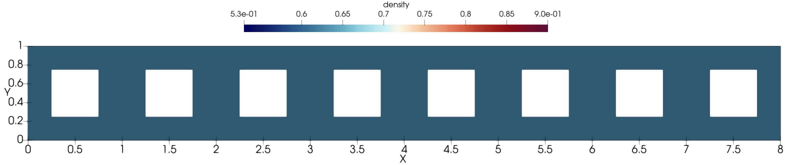
Figure: $l_0 = 0.5$

$$\langle u_x \rangle_{x,y,t} = -\frac{k_{eff}(\varepsilon,\alpha)}{\varepsilon \eta_0} \frac{\Delta P}{L_x}, \quad \varepsilon = 0.75$$

Fixing the pressure gradient $-\frac{\Delta P}{L_x} = 0.64$, $\alpha = -33.49$, $Pe = 0.044$, $D_t = 0.0016$

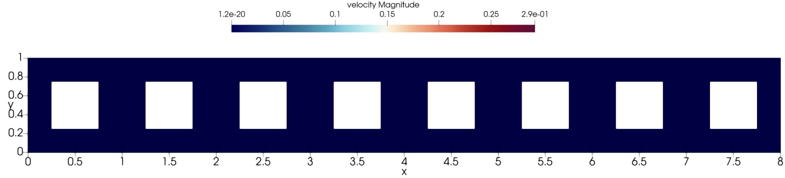
Bacterial density ρ

Time: 0.00



Fluid velocity $\mathbf{u}$

Time: 0.00



Results: Active matter through porous media

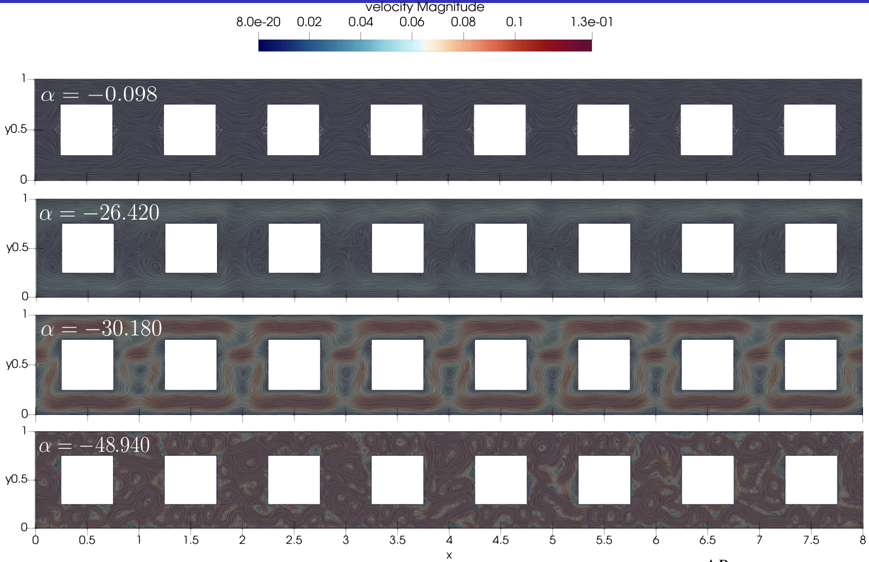


Figure: Array of squares modify the velocity profile at different α, $-\frac{\Delta P}{L_x} = 0.64$, $Pe = 0.044$, $D_t = 0.0016$.

Results: Active matter through porous media

$\langle \dot{m}_x \rangle_{x,y,t}$ for an HS cell (red) and a HS + porous media (black)

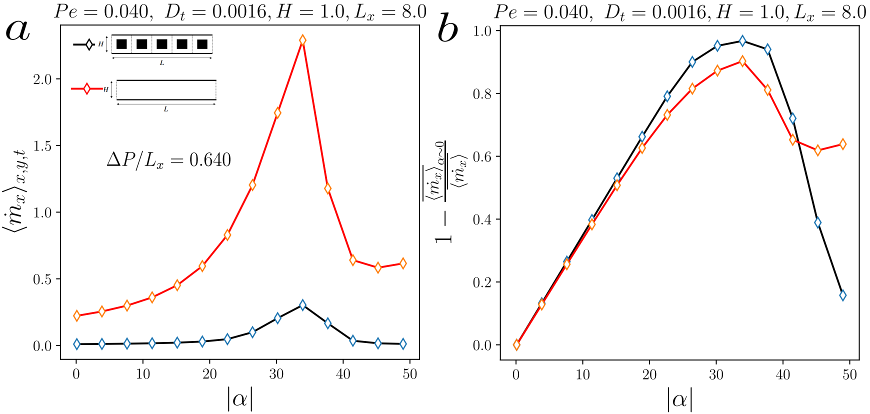


Figure: (a) Mass flow rate coefficient vs α, (b) excess mass flow rate due activity.

$$\langle \dot{m}_x \rangle_{x,y,t} \sim \langle u_x \rangle_{x,y,t}$$

Results: Active matter through Porous media

Relative mass flow rate compared to Helle-Show as function of α

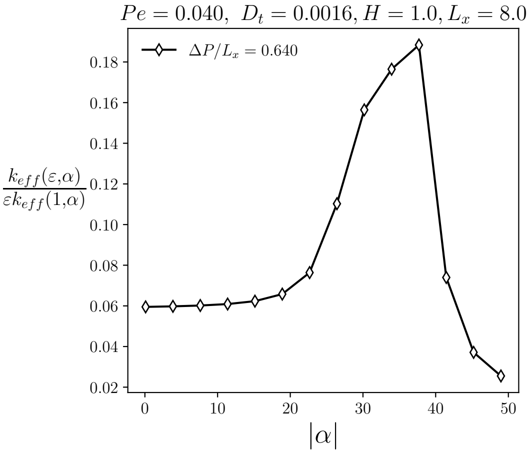


Figure: Relative permeability coefficient

$$\frac{\langle u_x\rangle_{x,y,t}(\varepsilon,\alpha)}{\langle u_x\rangle_{x,y,t}(1,\alpha)} = \frac{k_{eff}(\varepsilon,\alpha)}{\varepsilon k_{eff}(1,\alpha)}$$

Conclusions

- We found three flow regimes under pressure gradients: linear, transition, and collective or mesoturbulent.
- We found nonlinear Darcy's law in the transition regime.
- Although the average mass flow rate decreases, the transition regime occurs at the same critical conditions as in the empty channel.
- The relative permeability of the system presents a non-monotonic behavior with α.

Thank You