On the flow of yield stress fluid in porous media : statistical properties, universality class and boundary conditions

Laurent Talon
Lab. FAST, CNRS, Univ. Paris-Saclay

Alberto Rosso (LPTMS, Orsay), Alex Hansen (NTNU, PoreLab, Norway)

Chen Liu, Federico Lanza, Andreas Hennig, Thibaud Chevalier

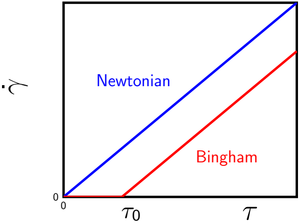
Fluides, Automatique et Systèmes Thermiques

Introduction : Yield Stress Fluids in porous media

Food, cosmetic, cements, mud, heavy oil...etc

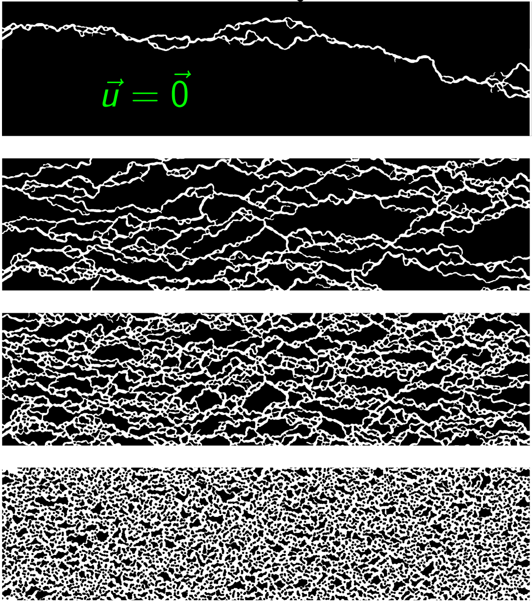
Applications in porous media :

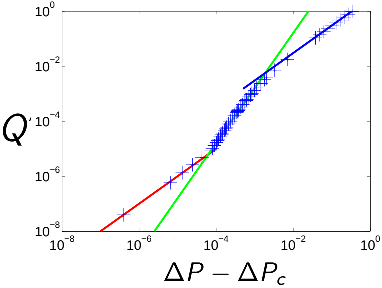
- Mud flow in fracture
- Proppent fluid for fracking
- Enhance Oil Recovery
- Cement injection in the ground



Flow paths

$\Delta P \sim \Delta P_c \quad \xrightarrow{Q}$

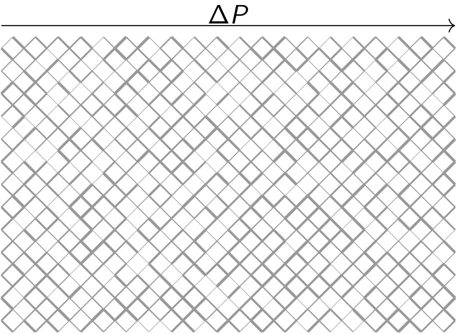


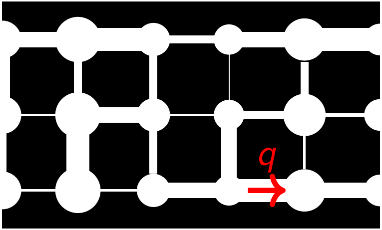


3 scaling regimes:-

- $Q \propto (\Delta P - \Delta P_c)^1$
- $Q \propto (\Delta P - \Delta P_c)^2$
- $Q \propto (\Delta P - \Delta P_c)^1$

Network of bounds with random radius b_{ij}





Each link i (throat) :

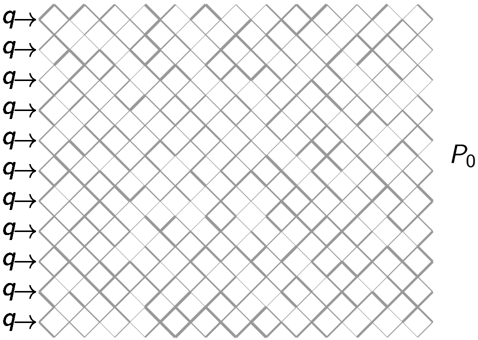
$q = \frac{k_i}{\eta}(\delta p - \delta p_{ci})$, with $\delta p > \delta p_{ci}$

$\delta p_{ci} = \frac{l_i}{b_i}\tau_0$

$k_i = \frac{\pi b_i^4}{4}$

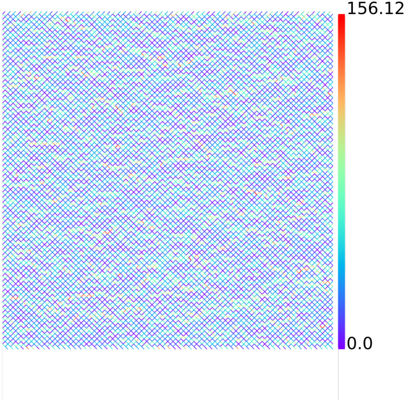
- Solve Kirchhoffs' equations

Influence of the boundary condition?

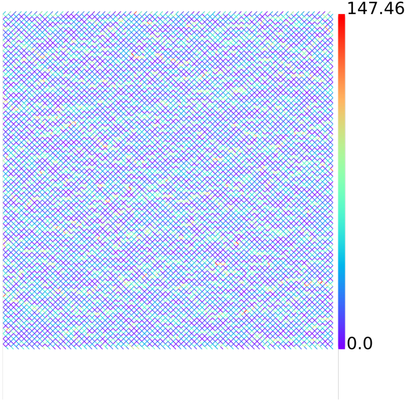


Newtonian fluid

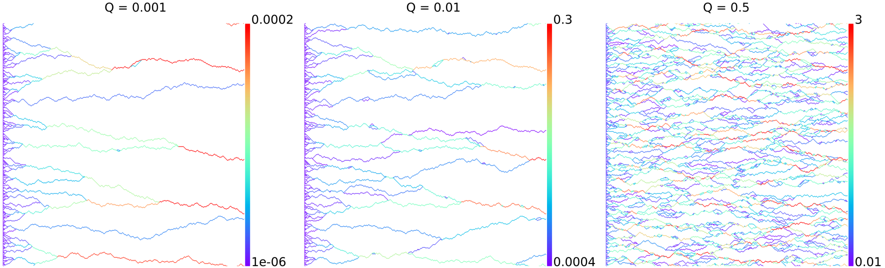
Q imposed

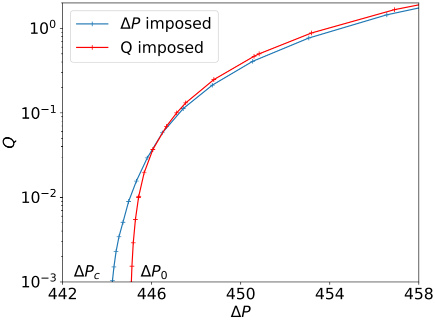


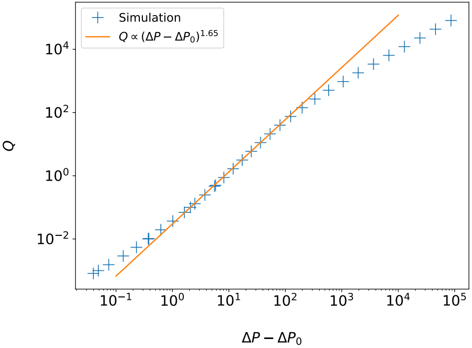
ΔP imposed



Bingham:

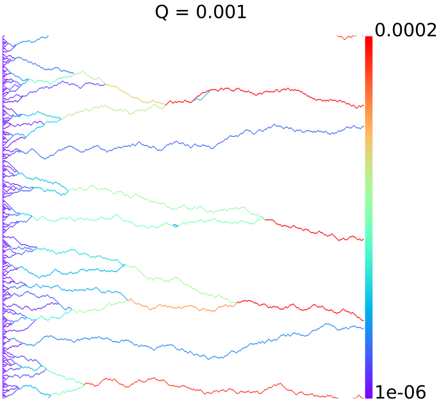


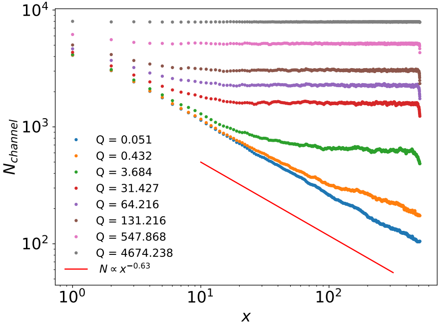




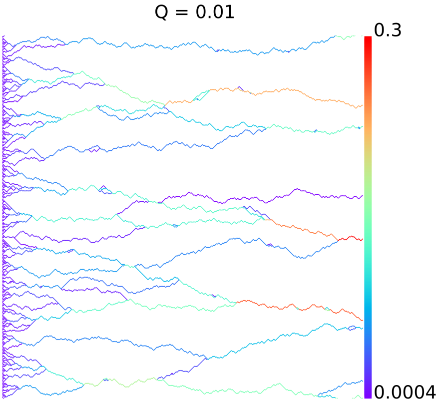
$$Q \propto (\Delta P - \Delta P_0)^{\beta}, \quad (1)$$

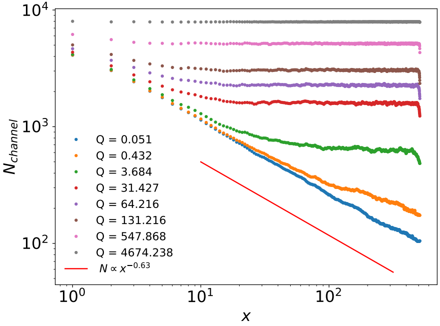
$$\beta \simeq 1.6$$



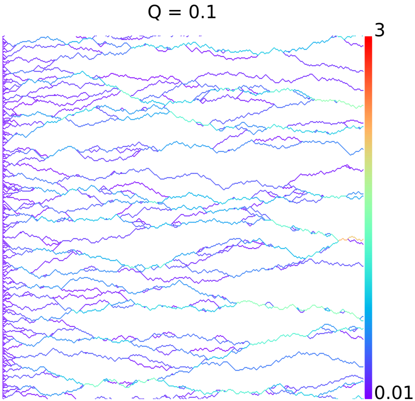


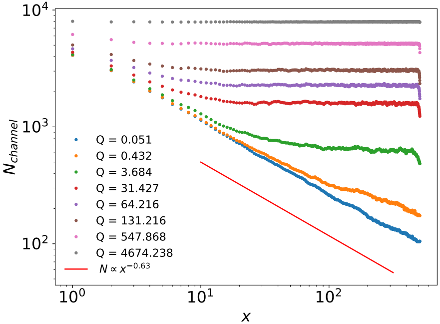
$$N_{\text{channel}}(x \geq 1, Q \to 0) \simeq \frac{W}{2} x^{-\delta}.$$



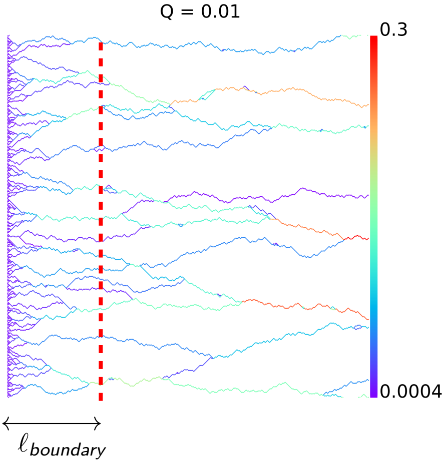


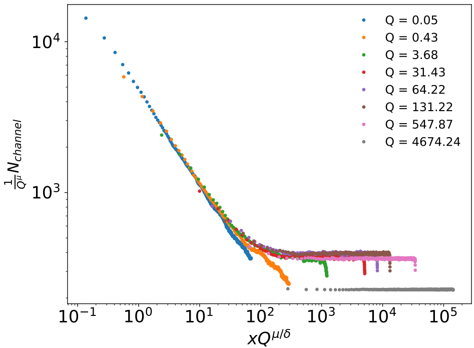
$$N_{\text{channel}}(x \geq 1, Q \to 0) \simeq \frac{W}{2} x^{-\delta}.$$





$$N_{\text{channel}}(x \geq 1, Q \to 0) \simeq \frac{W}{2} x^{-\delta}.$$





$$N_{\text{channel}}(x \geq 1, Q \to 0) \simeq \frac{W}{2} x^{-\delta}.$$

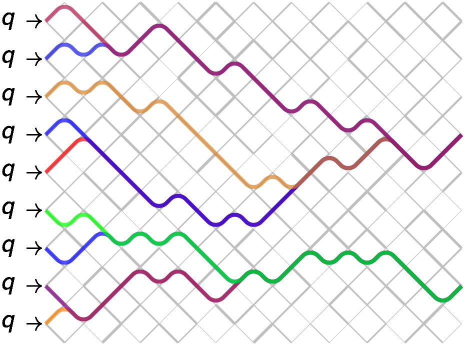
$$N_{\text{channel}}(x, Q) = Q^{\mu} f(xQ^{\mu/\delta}) \ ,$$

$$\mu = 0.42 \pm 0.02 \text{ and } \delta \simeq 0.63 \pm 0.05.$$

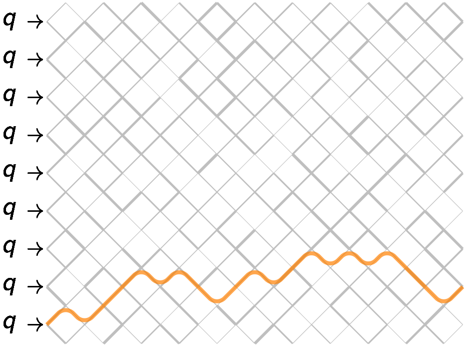
$$\ell_{boundary} \propto Q^{-\mu/\delta}$$

KPZ avalanches

$$\Delta P_0 = \lim_{Q \to 0} \frac{1}{Q} \min_{\{q\} \in \Omega} \sum_{(ij)} \delta p_{ci} |q_{ij}|$$



$$\Delta P_0 = \lim_{Q\to 0} \frac{1}{Q} \min_{\{q\}\in\Omega} \sum_{(ij)} \delta p_{ci} |q_{ij}|$$



$$\Delta P_0 = \frac{1}{N_{in}} \sum_{in} \min_{\mathcal{C}_{in}} \left[\sum_{\mathcal{C}_{in}} \delta p_{ci}\, dx \right]$$

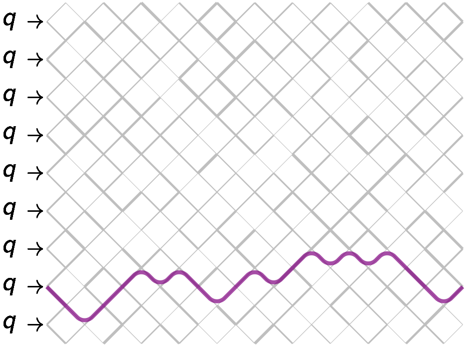
Kardar, Parisi & Zhang (1986)

Directed polymer : $\min_{\mathcal{C}_{in}} \left[\sum_{\mathcal{C}_{in}} \delta p_{ci}\, dx \right]$

$W \propto L^{\zeta}$, $\zeta = 2/3$.

KPZ avalanches

$$\Delta P_0 = \lim_{Q \to 0} \frac{1}{Q} \min_{\{q\} \in \Omega} \sum_{(ij)} \delta p_{ci} |q_{ij}|$$



$$\Delta P_0 = \frac{1}{N_{in}} \sum_{in} \min_{\mathcal{C}_{in}} \left[\sum_{\mathcal{C}_{in}} \delta p_{ci} dx \right]$$

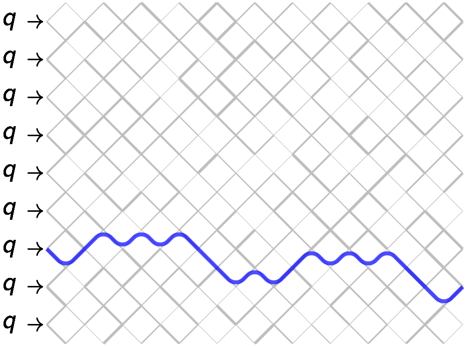
Kardar, Parisi & Zhang (1986)

Directed polymer : $\min_{\mathcal{C}_{in}} \left[\sum_{\mathcal{C}_{in}} \delta p_{ci} dx \right]$

$W \propto L^{\zeta}$, $\zeta = 2/3$.

KPZ avalanches

$$\Delta P_0 = \lim_{Q \to 0} \frac{1}{Q} \min_{\{q\} \in \Omega} \sum_{(ij)} \delta p_{ci} |q_{ij}|$$



$$\Delta P_0 = \frac{1}{N_{in}} \sum_{in} \min_{\mathcal{C}_{in}} \left[\sum_{\mathcal{C}_{in}} \delta p_{ci} dx \right]$$

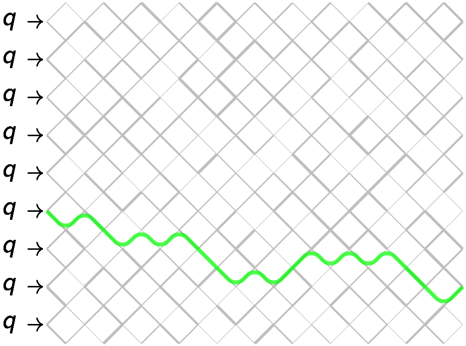
Kardar, Parisi & Zhang (1986)

Directed polymer : $\min_{\mathcal{C}_{in}} \left[\sum_{\mathcal{C}_{in}} \delta p_{ci} dx \right]$

$W \propto L^{\zeta}$, $\zeta = 2/3$.

KPZ avalanches

$$\Delta P_0 = \lim_{Q \to 0} \frac{1}{Q} \min_{\{q\} \in \Omega} \sum_{(ij)} \delta p_{ci} |q_{ij}|$$



$$\Delta P_0 = \frac{1}{N_{in}} \sum_{in} \min_{\mathcal{C}_{in}} \left[\sum_{\mathcal{C}_{in}} \delta p_{ci} dx \right]$$

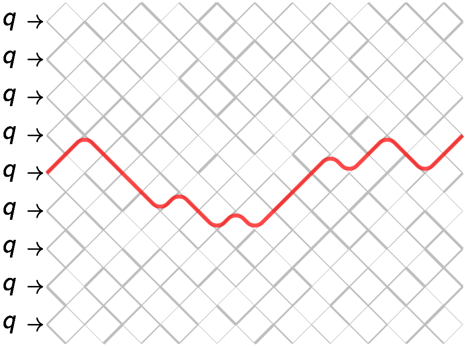
Kardar, Parisi & Zhang (1986)

Directed polymer : $\min_{\mathcal{C}_{in}} \left[\sum_{\mathcal{C}_{in}} \delta p_{ci} dx \right]$

$W \propto L^{\zeta}$, $\zeta = 2/3$.

KPZ avalanches

$$\Delta P_0 = \lim_{Q \to 0} \frac{1}{Q} \min_{\{q\} \in \Omega} \sum_{(ij)} \delta p_{ci} |q_{ij}|$$



$$\Delta P_0 = \frac{1}{N_{in}} \sum_{in} \min_{\mathcal{C}_{in}} \left[\sum_{\mathcal{C}_{in}} \delta p_{ci} dx \right]$$

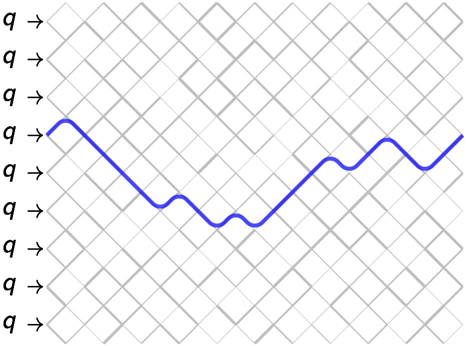
Kardar, Parisi & Zhang (1986)

Directed polymer : $\min_{\mathcal{C}_{in}} \left[\sum_{\mathcal{C}_{in}} \delta p_{ci} dx \right]$

$W \propto L^{\zeta}$, $\zeta = 2/3$.

KPZ avalanches

$$\Delta P_0 = \lim_{Q \to 0} \frac{1}{Q} \min_{\{q\} \in \Omega} \sum_{(ij)} \delta p_{ci} |q_{ij}|$$



$$\Delta P_0 = \frac{1}{N_{in}} \sum_{in} \min_{\mathcal{C}_{in}} \left[\sum_{\mathcal{C}_{in}} \delta p_{ci} dx \right]$$

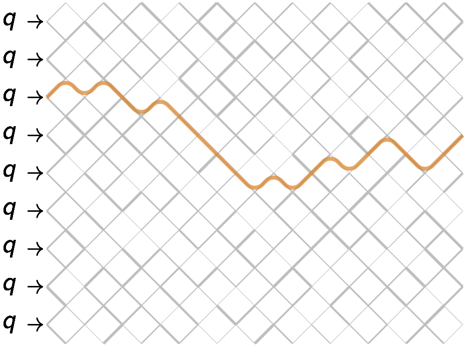
Kardar, Parisi & Zhang (1986)

Directed polymer : $\min_{\mathcal{C}_{in}} \left[\sum_{\mathcal{C}_{in}} \delta p_{ci} dx \right]$

$W \propto L^{\zeta}$, $\zeta = 2/3$.

KPZ avalanches

$$\Delta P_0 = \lim_{Q\to 0} \frac{1}{Q} \min_{\{q\}\in\Omega} \sum_{(ij)} \delta p_{ci} |q_{ij}|$$



$$\Delta P_0 = \frac{1}{N_{in}} \sum_{in} \min_{\mathcal{C}_{in}} \left[\sum_{\mathcal{C}_{in}} \delta p_{ci} dx \right]$$

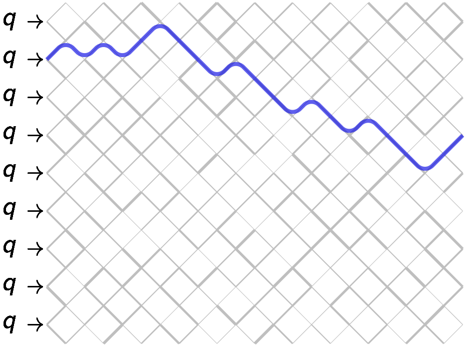
Kardar, Parisi & Zhang (1986)

Directed polymer : $\min_{\mathcal{C}_{in}} \left[\sum_{\mathcal{C}_{in}} \delta p_{ci} dx \right]$

$W \propto L^{\zeta}$, $\zeta = 2/3$.

KPZ avalanches

$$\Delta P_0 = \lim_{Q \to 0} \frac{1}{Q} \min_{\{q\} \in \Omega} \sum_{(ij)} \delta p_{ci} |q_{ij}|$$



$$\Delta P_0 = \frac{1}{N_{in}} \sum_{in} \min_{\mathcal{C}_{in}} \left[\sum_{\mathcal{C}_{in}} \delta p_{ci} dx \right]$$

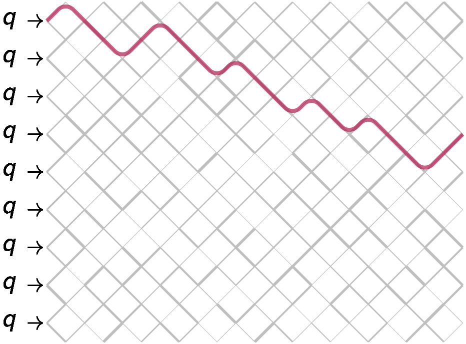
Kardar, Parisi & Zhang (1986)

Directed polymer : $\min_{\mathcal{C}_{in}} \left[\sum_{\mathcal{C}_{in}} \delta p_{ci} dx \right]$

$W \propto L^{\zeta}$, $\zeta = 2/3$.

KPZ avalanches

$$\Delta P_0 = \lim_{Q \to 0} \frac{1}{Q} \min_{\{q\} \in \Omega} \sum_{(ij)} \delta p_{ci} |q_{ij}|$$



$$\Delta P_0 = \frac{1}{N_{in}} \sum_{in} \min_{\mathcal{C}_{in}} \left[\sum_{\mathcal{C}_{in}} \delta p_{ci} dx \right]$$

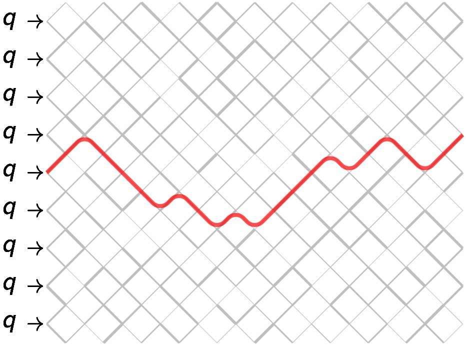
Kardar, Parisi & Zhang (1986)

Directed polymer : $\min_{\mathcal{C}_{in}} \left[\sum_{\mathcal{C}_{in}} \delta p_{ci} dx \right]$

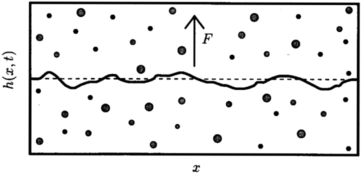
$W \propto L^{\zeta}$, $\zeta = 2/3$.

KPZ avalanches

$$\Delta P_0 = \lim_{Q \to 0} \frac{1}{Q} \min_{\{q\} \in \Omega} \sum_{(ij)} \delta p_{ci} |q_{ij}|$$



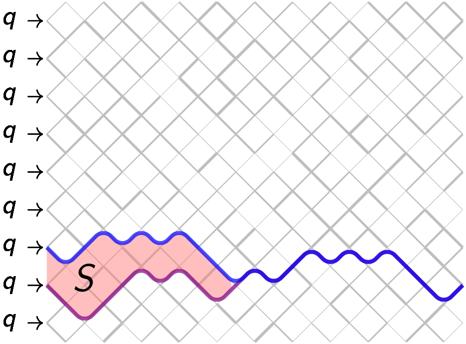
$$\Delta P_0 = \frac{1}{N_{in}} \sum_{in} \min_{\mathcal{C}_{in}} \left[\sum_{\mathcal{C}_{in}} \delta p_{ci} dx \right]$$



Avalanche dynamic (Elastic line in random disorder)

KPZ avalanches

$$\Delta P_0 = \lim_{Q \to 0} \frac{1}{Q} \min_{\{q\} \in \Omega} \sum_{(ij)} \delta p_{ci} |q_{ij}|$$



$$\Delta P_0 = \frac{1}{N_{in}} \sum_{in} \min_{\mathcal{C}_{in}} \left[\sum_{\mathcal{C}_{in}} \delta p_{ci} dx \right]$$

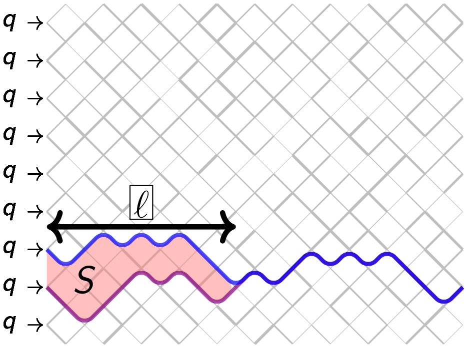
S : Size of avalanches

ℓ : length

Avalanche dynamic (Elastic line in random disorder)

KPZ avalanches

$$\Delta P_0 = \lim_{Q \to 0} \frac{1}{Q} \min_{\{q\} \in \Omega} \sum_{(ij)} \delta p_{ci} |q_{ij}|$$

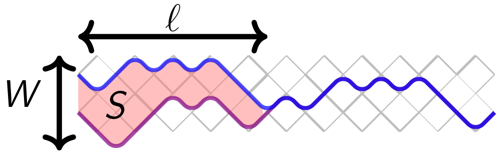


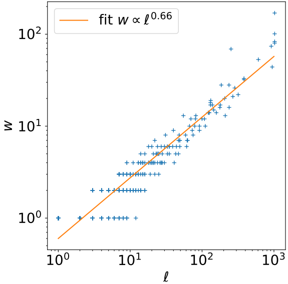
$$\Delta P_0 = \frac{1}{N_{in}} \sum_{in} \min_{\mathcal{C}_{in}} \left[\sum_{\mathcal{C}_{in}} \delta p_{ci} dx \right]$$

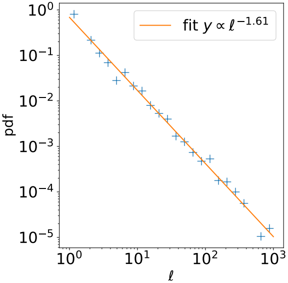
S : Size of avalanches

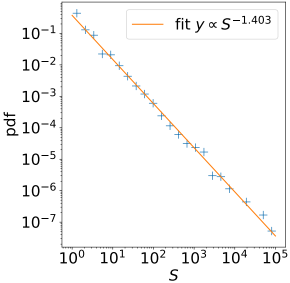
ℓ : length

Avalanche dynamic (Elastic line in random disorder)









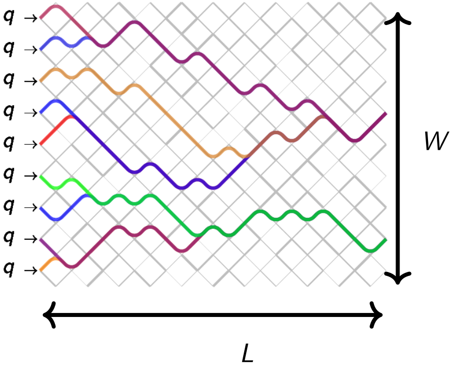
Roughness:
$W \propto \ell^{\zeta}$
$\zeta = 2/3.$

Avalanche length:
$P(\ell) \propto \ell^{-\tau_\ell}$
$\tau_\ell \simeq 1.6$

Avalanche size:
$P(S) \propto S^{-\tau_S}$
$\tau_S \simeq 1.4$

$$\tau_S \neq 2 - \frac{2}{1+\zeta} \qquad (2)$$

Narayan, O. & Fisher, D. S. *Threshold critical dynamics of driven interfaces in random media Phys. Rev. B, , 1993, 48, 7030-7042*



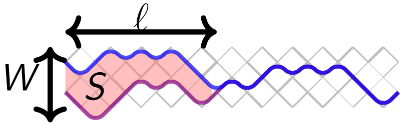
$$\tau_S = 2 - \frac{1}{1+\zeta}$$

After W avalanches, surface covered: $W\langle S\rangle = WL$.

$$W\langle S\rangle \approx WL \Longrightarrow L^{(1+\zeta)(2-\tau_S)} \approx L\ , \qquad (3)$$

Conclusion

Exponent ζ governed many scaling laws



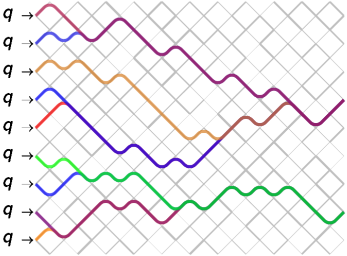
$$\tau_S = 2 - \frac{1}{1+\zeta} \quad (4)$$

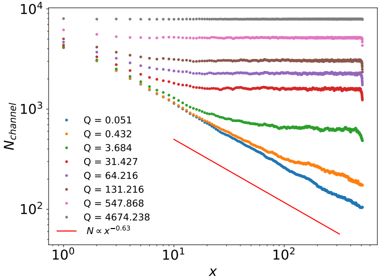
Roughness:
$W \propto \ell^{\zeta}$
$\zeta = 2/3.$

Avalanche length:
$P(\ell) \propto \ell^{-\tau_\ell}$
$\tau_\ell \simeq 1.6$
$\tau_\ell = 1 + \zeta = \frac{5}{3}$

Avalanche size:
$P(S) \propto S^{-\tau_S}$
$\tau_S \simeq 1.4$
$\tau_S = 2 - \frac{1}{1+\zeta} = \frac{7}{5}$

Talon et al. *Influence of the imposed flow rate boundary condition on the flow of Bingham fluid in porous media* PRFluids, 2024





$$N_{\text{channel}}(x \geq 1, Q \to 0) \simeq \frac{W}{2} x^{-\delta}.$$

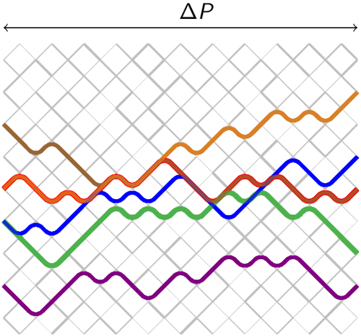
$$N_{\text{channel}}(x, Q) = Q^{\mu} f(xQ^{\mu/\delta}) \; ,$$

$$\mu = 0.42 \pm 0.02 \text{ and } \delta \simeq 0.63 \pm 0.05.$$

$N_{\text{channel}}(x) =$ number of avalanches larger than $x \Rightarrow \delta = \xi$

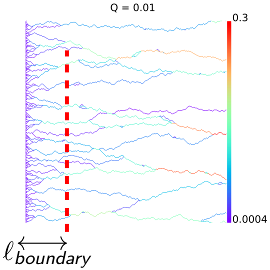
We are still missing an interpretation for μ and β.

$$Q \propto (\Delta P - \Delta P_0)^{\beta} \tag{5}$$



Conclusion

- Critical system (fractal flow field, etc.)
- KPZ controls the boundary length
  $P(\ell) \propto \ell^{-\tau_\ell}, \quad \tau_\ell = 1 + \zeta$
  $P(S) \propto S^{-\tau_S}, \quad \tau_S = 2 - \frac{1}{1+\zeta}$
  $N_{\text{channel}}(x \geq 1, Q \to 0) \simeq \frac{W}{2} x^{-\xi}$



- Homogenization
- Effect of pressure imposed boundary condition?
- Explicit determination of the flow curve in the Cayley tree, for low Q.

  Schimmenti, V. M. et al. *Darcy's law of yield stress fluids on a treelike network* PRE, 2023
- 3D PNM, Nathan Abitbol (PhD).