Model for Flow in Karstified Carbonate Reservoirs using Two-Level Upscaling and Surrogate-Based Transmissibilities under Geomechanical Effects

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Computational Modelling of Porous Materials

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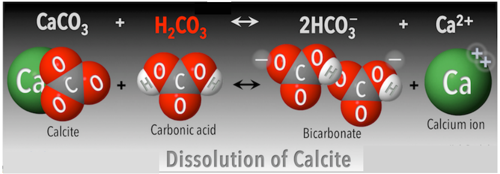
Outline

1. Motivation
2. Literature Review
3. MsEDKM
4. MsEDKM coupled with Geomechanics
5. Computational Results
6. Wrap-up

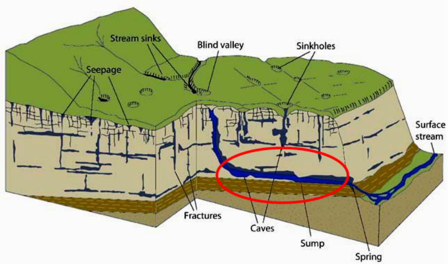
Karstified Carbonate Reservoir

- Sedimentary carbonate rocks are highly reactive to fluids



- Dissolution processes $\Longrightarrow$ karst:
  vugs, enlarged fractures, caves, conduit networks
- Karst in carbonate reservoirs:
  - impact reservoir storage capacity and subsurface fluid flow
  - collapse of wells
  - fluid loss during drilling
- Models need to incorporate these complexities

Localized flow - Conduits



Runkel, A. C. et al, (2003).

Literature Review

- $2\phi 2K$ with enhanced permeabilities (Machado, 2020)
- Coupled 3D-1D Discrete Model (Murad; Pereira, 2020)
- Triple Porosity Models (Kang, 2004)
- EDKM (Correia at al, 2018)
  - Extension of the well established EDFM (Li and Lee, 2008; Moinfar et al. 2014)
  - Single porosity model with two grid domains (matrix and karst properties)
  - Communication between matrix/karst via special connections (NNC)
  - **Karst representation: same block size as refined geological grid**
  - **Transmissibility matrix/karst computed analitycally.**

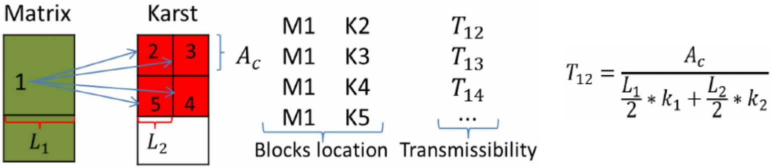
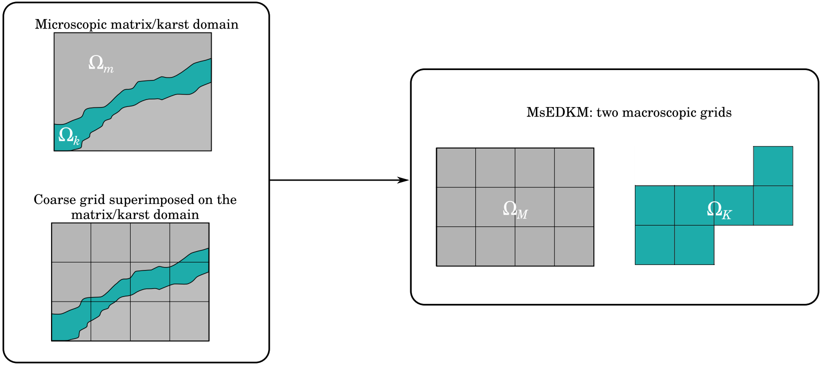


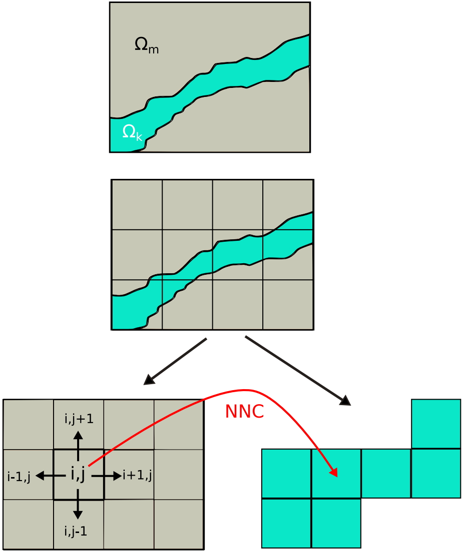
Figure 3—Application of special connections.

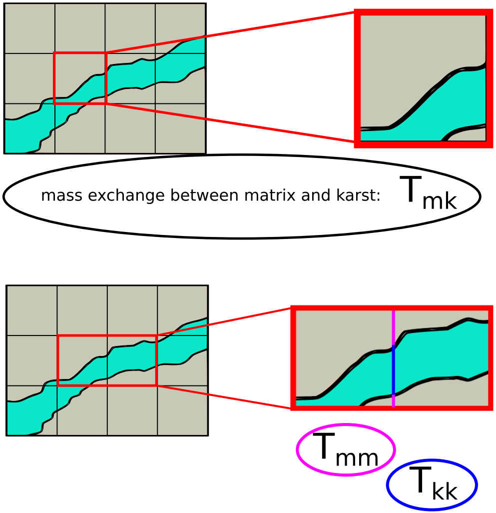
Literature Review

- MsEDKM: multiscale embedded discrete karst model
  - **Karst representation: same block size as the matrix grid**
  - **Matrix-karst transmissibilities computed via numerical simulations**
  - **Surrogate model trained on high-fidelity numerical simulations to reduce computational cost and enable scalable use of the methodology**



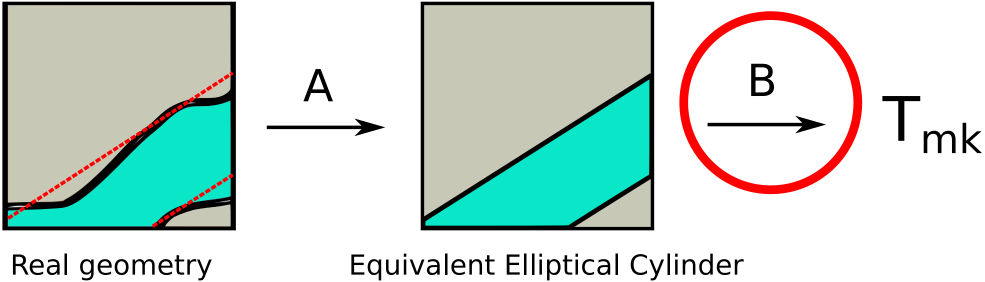
MsEDKM


Ωm
Ωk
i,j+1
i-1,j
i,j
i+1,j
i,j-1
NNC


mass exchange between matrix and karst: Tmk
Tmm
Tkk

Two-Level Upscaling for T_{mk} Computation

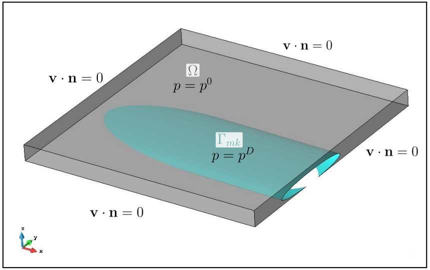
- **A**. Moment of inertia based upscaling (micro-meso)
  Goal: regularization of the conduit surface roughness
- **B**. Flow based upscaling (meso-macro)
  Goal: computation of hydraulic transmissibility between karst and matrix (T_{mk})



Numerical Computation of T_{mk} ($K_k >> K_m$): No Geomechanical Effects

Solve the local microscopic transient problem:

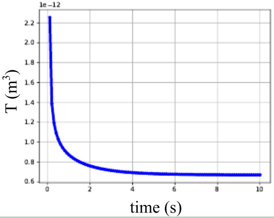
$$\begin{cases} \beta \dfrac{\partial p}{\partial t} + \nabla \cdot \ \mathbf{v} = 0 & \text{in} \quad \Omega_m \\ \mathbf{v} = -\dfrac{\mathbf{K}}{\mu} \nabla p & \text{in} \quad \Omega_m \\ \\ p(\mathbf{x}, 0) = p^0(\mathbf{x}) & \text{in} \quad \Omega_m \\ p(\mathbf{x}, t) = p^D(\mathbf{x}) & \text{on} \quad \Gamma_{mk} \\ \mathbf{v} \cdot \mathbf{n} = 0 & \text{on} \quad \Gamma_m \end{cases}$$



Compute T_{mk}:

$$T_{mk}(t) = \frac{\mu q}{\overline{p} - p^D}$$

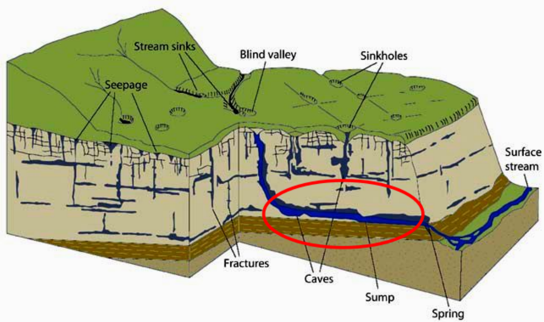
$$T_{mk} = \lim_{t \to \infty} T_{mk}(t)$$



Our Goal

Extend the MsEDKM framework to incorporate the impact of geomechanical effects on flow through stress-dependent hydraulic properties.

Localized flow - Conduits

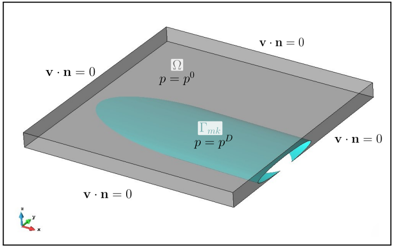


Runkel, A. C. et al, (2003).

Numerical Computation of T_{mk} $(K_k >> K_m)$

Solve the **non-linear** local microscopic transient problem:

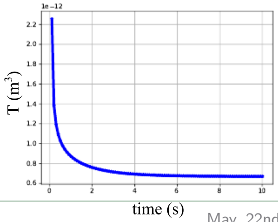
$$\begin{cases} \beta_{\text{eff}} \dfrac{\partial p}{\partial t} + \nabla \cdot \mathbf{v} = 0 & \text{in} \quad \Omega_m \\ \mathbf{v} = -\dfrac{\mathbf{K}}{\mu} \nabla p & \text{in} \quad \Omega_m \\ \boxed{\mathbf{K} = \mathbf{K}(\phi(p))} & \\ p(\mathbf{x}, 0) = p^0(\mathbf{x}) & \in \Omega_m \\ p(\mathbf{x}, t) = p^D(\mathbf{x}) & \text{on} \quad \Gamma_{mk} \\ \mathbf{v} \cdot \mathbf{n} = 0 & \text{on} \quad \Gamma_m \end{cases}$$



Compute T_{mk}:

$$T_{mk}(t) = \frac{\mu q}{\overline{p} - p^D}$$

$$T_{mk} = \lim_{t \to \infty} T_{mk}(t)$$



Effective compressibility:

$$\beta_{\text{eff}} = \underbrace{\frac{1}{M}}_{\text{Fluid compressibility}} + \underbrace{\frac{\alpha^2}{\mathbb{K}}\left(\frac{1+\nu}{3(1-\nu)}\right)}_{\text{Geomechanical contribution}}$$

$$\frac{1}{M} = \frac{\phi^{\text{ref}}}{\mathbb{K}_{\text{f}}} + \frac{\alpha - \phi^{\text{ref}}}{\mathbb{K}_{\text{s}}}, \qquad \alpha = 1 - \frac{\mathbb{K}}{\mathbb{K}_{\text{s}}}, \qquad \mathbb{K} = \frac{E}{3(1-2\nu)}$$

$\mathbb{K}_{\text{f}}$: fluid bulk modulus,

$\mathbb{K}_{\text{s}}$: solid grain bulk modulus,

$\mathbb{K}$: porous matrix bulk modulus.

α : Biot coefficient,

E : Young's modulus,

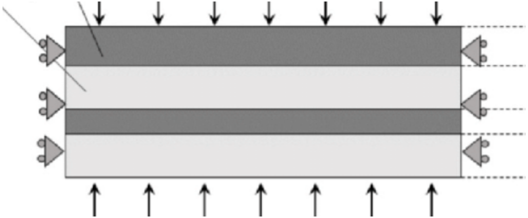
ν : Poisson's ratio.

Analytical Geomechanics Approach: Oedometric Stress Path

Assumptions:

- *In-situ* state (input data): $\{p^{\text{ref}}, \sigma^{\text{ref}}, \mathbf{K}^{\text{ref}}, \phi^{\text{ref}}\}$.
- Total stress main directions are aligned to the coordinate system $\{x, y, z\}$
- **Prescribed total stress path (oedometric)**

- Lateral strains vanish on outer interface $\Delta\epsilon_x = 0$
- Constant vertical total stress $\Delta\sigma_V = 0$
- Effective stress changes with pressure
  - $\Delta\sigma'_V = \alpha\Delta p$
  - $\Delta\sigma'_H = \Delta\sigma'_h = \dfrac{\nu}{1-\nu}\Delta\sigma'_V$



Geomechanical Steps

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$$\boxed{\mathbf{K} = \mathbf{K}(p)}$$

$$p \longrightarrow \Delta\boldsymbol{\sigma}' \longrightarrow \varepsilon_v \longrightarrow \phi \longrightarrow \mathbf{K}$$

Mechanical response

Effective stress:

$$\sigma' = \sigma'^{\text{ref}} - \Delta\sigma',$$

with

$$\Delta\boldsymbol{\sigma}' = \alpha\left(p - p^{\text{ref}}\right)\begin{pmatrix} \frac{\nu}{1-\nu} & 0 & 0 \\ 0 & \frac{\nu}{1-\nu} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Volumetric Strain:

$$\varepsilon_v = \frac{\text{tr}\left(\Delta\boldsymbol{\sigma}'\right)}{3\mathbb{K}}$$

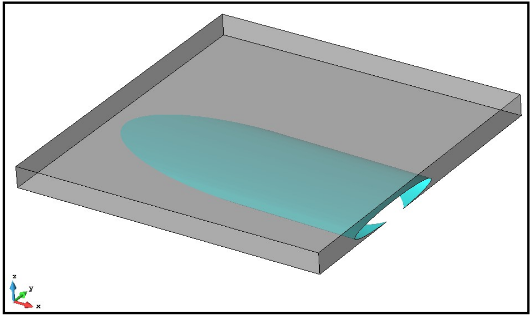
Porosity and permeability

$$\phi = 1 - \left(1 - \phi^{\text{ref}}\right)\exp\left(-\varepsilon_v\right)$$

$$\mathbf{K} = \mathbf{K}^{\text{ref}}\left(\frac{\phi}{\phi^{\text{ref}}}\right)^{A}$$

A : permeability-porosity sensitivity exponent.

Computational Simulations



- cell dimension: 150x150x10 m
- conduit volume fraction: $\approx 10\%$

In-situ state:

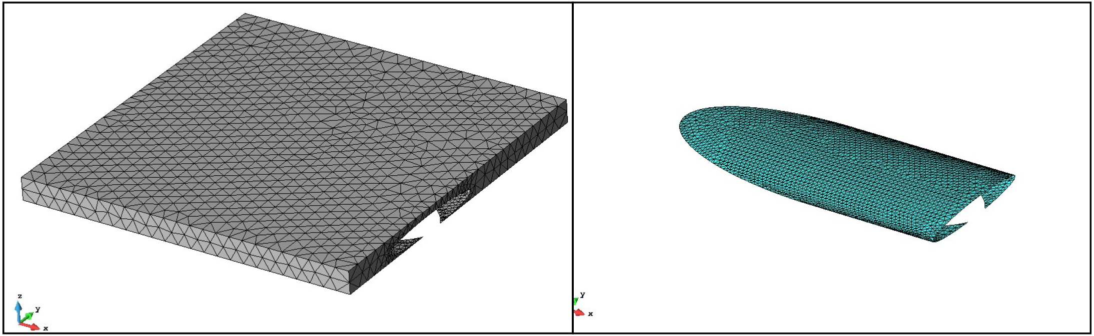
Parameter	Value	Unit (SI)
p^{ref}	60	MPa
$\boldsymbol{\sigma}^{\text{ref}}$	(80, 80, 100)	MPa
$\mathbf{K}^{\text{ref}}$	$(100, 100, 100)$	mD
ϕ_{ref}	0.2	–

Input Parameters:

Parameter	Value	Unit (SI)
E	2.2222×10^9	Pa
ν	0.3	–
$\mathbb{K}_s$	1.0×10^{32}	Pa
$\mathbb{K}_f$	2.0×10^9	Pa
α	1.0	–
μ_f	1.0×10^{-3}	Pa s
Δt	1.0×10^2	s

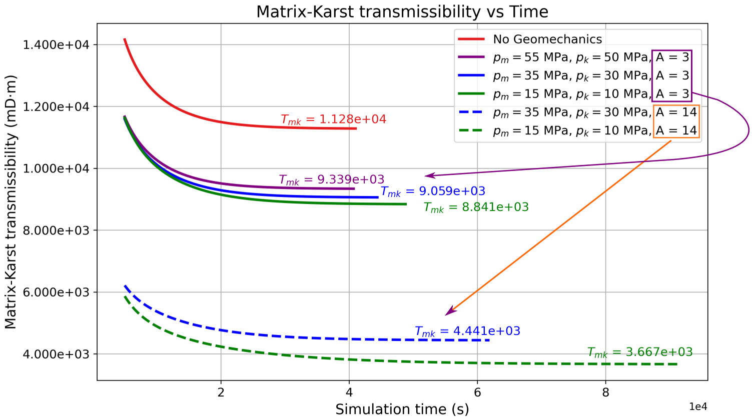
Computational Simulations

Mesh refinement



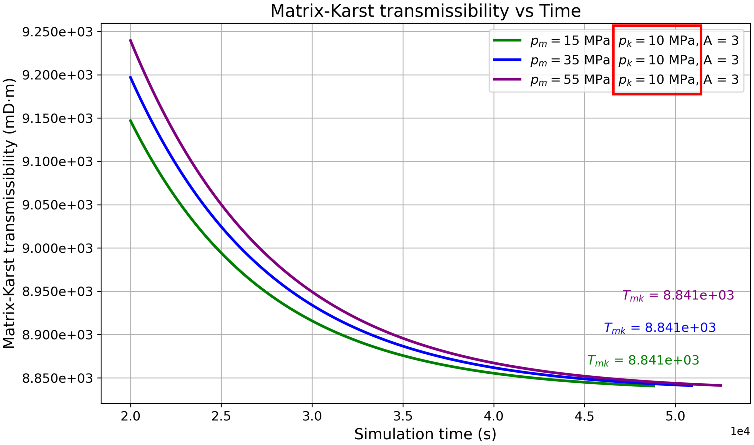
- Finer mesh along the conduit surface, where the flow rate is computed

Computational Simulations



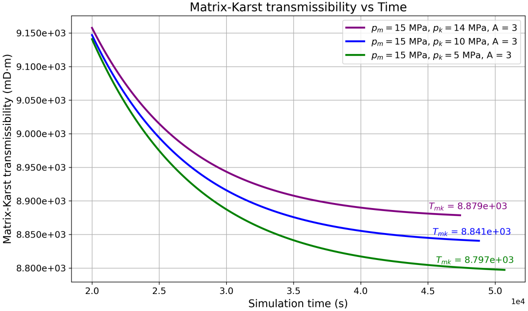
- $\Delta p = (p_m - p_k) = 5$ MPa
- A: permeability-porosity sensitivity exponent
- Depletion scenario
- Geomechanical effects reduce transmissibility between matrix/karst transmissibility
- $A \uparrow,\ \phi \downarrow,\ \mathbf{K} \downarrow \Rightarrow T_{mk} \downarrow$
- Lower range of pressure: higher geomechanical impact

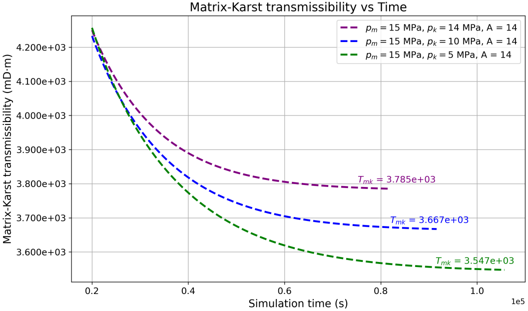
Computational Simulations



- Depletion scenario
- p_k fixed
- Geomechanical effects: $T_{mk} \downarrow$
- T_{mk} are the same for distinct p_m

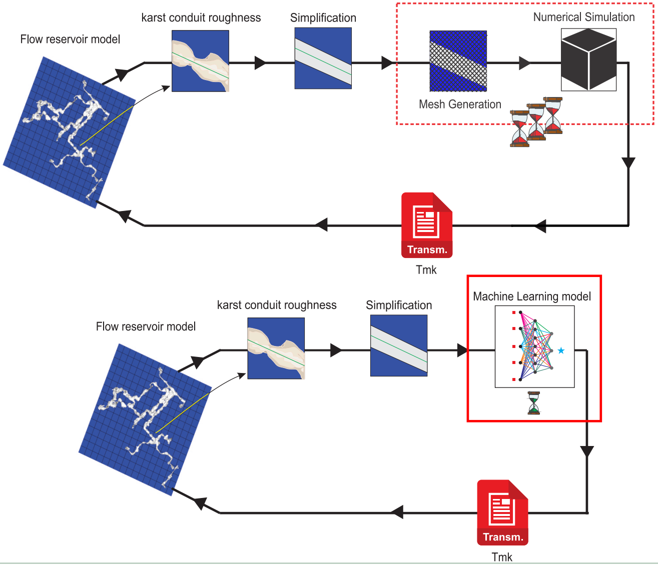
Computational Simulations





- p_m fixed
- T_{mk} depends on p_k

MsEDKM Workflow



Surrogate Model Attributes

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Descriptor vector for the surrogate model

- Geometric and hydraulic attributes

$$\mathbf{X} = \{L_x, L_y, L_z, a_k, b_k, \mathbf{x}_k^{\text{in}}, \mathbf{x}_k^{\text{out}}, L_k, \eta_k, A_{mk}, V_k, \mathbf{K}\}$$

- Geomechanical attributes

$$\mathbf{X}_{\text{geomechanics}} = \{E, \nu, \mathbb{K}_{\text{s}}, \mathbb{K}_{\text{f}}, p^{\text{ref}}, \boldsymbol{\sigma}^{\text{ref}}, \mathbf{K}^{\text{ref}}, A, p_m, p_k\}$$

Steps Toward the Complete Methodology

- Build a new database from hydromechanical numerical simulations
- Construct and train a surrogate model or predicting matrix-karst NNC transmissibilities
- Compute local transmissibilities accounting for geomechanical effects through permeability updates
- Implement the macroscopic flow model incorporating the surrogate model and the local transmissibilities

Wrap-up

- MsEDKM: a systematic methodology for integrating karst conduits into reservoir simulators
- Ongoing work extends the MsEDKM framework to account for geomechanical effects through stress-dependent hydraulic properties
- Geomechanical effects are incorporated analytically, under a prescribed stress path, as an alternative to fully coupled hydromechanical numerical simulations
- Matrix–karst mass transfer is computed through numerical simulations, using flow-based upscaling
- ML-based surrogate model will enable large-scale simulations