An incremental variational gradient damage model for saturated poroelastic media with THM coupling

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InterPore2026 in Nantes   May 21st, 2026

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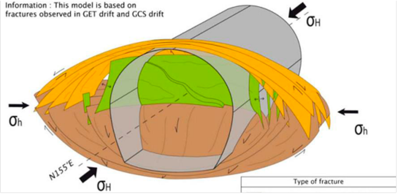
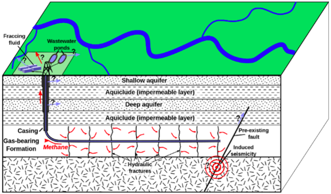
Outline

Introduction and motivation

Engineering problems:





Excavation damage zone (Armand et al., 2014) and hydraulic fracturing (Chen et al., 2021).

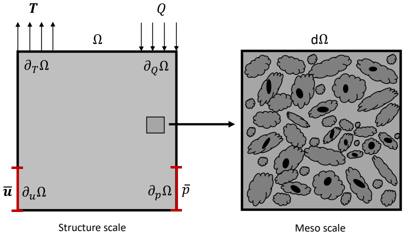
- Coupling between damage, thermo-poro-elasticity, fluid flow, and heat conduction.
- Non-local damage: transition from damage to fracture & less mesh dependency.

Objectives:

- A thermodynamics-based incremental variational formulation.
- Numerical implementation of a variational model.
- Applications to some engineering problems.

Hypothesis for porous media:

- Each material point contains a solid continuum and a fluid continuum.
- Solid skeleton = infinitesimally deformable solid matrix ∪ connected pore space.
- Fluid is incompressible and presents slow and laminar flow.



Saturated porous media.

Thermodynamic potentials:

- Clausius–Duhem inequality:

$$\mathcal{D} = \underbrace{\boldsymbol{\sigma} : \dot{\boldsymbol{\varepsilon}} - \dot{\psi}^s - \dot{p}\phi - \dot{T}s + Y_d \dot{d} + \boldsymbol{Y}_{\boldsymbol{\nabla} d} \cdot \boldsymbol{\nabla} \dot{d}}_{\mathcal{D}_{\text{int}}} \underbrace{- \boldsymbol{v} \cdot \boldsymbol{\nabla} p}_{\mathcal{D}_{\text{f}}} \underbrace{- \boldsymbol{q} \cdot \frac{\boldsymbol{\nabla} T}{T}}_{\mathcal{D}_{\text{th}}} \geq 0$$

- Thermodynamic potentials:
  - ☆ Reversible energy potential: $\psi^s := \psi^s(\boldsymbol{\varepsilon}, p, T, d, \boldsymbol{\nabla} d)$
  - ☆ Dissipation potential: $\varphi := \varphi^s(\dot{d}, \boldsymbol{\nabla}\dot{d}; d, \boldsymbol{\nabla} d) + \varphi^f(\boldsymbol{\nabla} p) + \varphi^{th}(\frac{\boldsymbol{\nabla} T}{T}, T)$

Incremental variational principle

✎ **Rate form of total energy functional** (Inspired by Ortiz and Stainier, JMPS, 1999; Zhang et al., JMPS, 2024):

$$\Pi(\dot{\boldsymbol{u}}, \dot{T}, \dot{p}, \dot{d}, \hat{s}, \hat{\phi}; p, T) = \int_\Omega \left(\underbrace{\dot{\psi}^s + \dot{p}\hat{\phi} + \dot{T}\hat{s}}_{\text{Reversible}} + \underbrace{\varphi^s + \varphi^f + \varphi^{th}}_{\text{Dissipative}} \right) \mathrm{d}\Omega - \underbrace{P_{ext}}_{\text{Power of external work}}$$

with $\hat{s}$ and $\hat{\phi}$ the entropy and porosity at non-equilibrium state, respectively.

✎ **Incremental form:**

$$I_n(\boldsymbol{u}_{n+1}, p_{n+1}, T_{n+1}, d_{n+1}, \hat{\phi}_{n+1}, \hat{s}_{n+1}) = \int_{t_n}^{t_{n+1}} \Pi \ \mathrm{d}t$$

✎ Incremental variational principle (6-field minimization):

$$\left(\boldsymbol{u}_{n+1}, p_{n+1}, T_{n+1}, d_{n+1}, \hat{\phi}_{n+1}, \hat{s}_{n+1}\right) =$$
$$\operatorname{Arg} \inf_{\boldsymbol{u}'_{n+1}, d'_{n+1}} \operatorname{stat}_{p'_{n+1}, T'_{n+1}, \hat{\phi}'_{n+1}, \hat{s}'_{n+1}} \left\{ I_n \left(\boldsymbol{u}'_{n+1}, p'_{n+1}, T'_{n+1}, d'_{n+1}, \hat{\phi}'_{n+1}, \hat{s}'_{n+1} \right) \right\}$$

Reduced scheme & a proposed variational model

Incremental variational principle (4-field minimization):

$$(\boldsymbol{u}_{n+1}, p_{n+1}, T_{n+1}, d_{n+1}) =$$
$$\text{Arg} \underset{\boldsymbol{u}'_{n+1},\, d'_{n+1}, p'_{n+1},\, T'_{n+1}}{\text{stat}} \left\{ I_n \left(\boldsymbol{u}'_{n+1}, p'_{n+1}, T'_{n+1}, d'_{n+1}; \hat{\phi} \simeq \phi_n, \hat{s} \simeq s_n \right) \right\}$$

- Reversible energy potential (Coussy 1995, 2004):

$$\psi^s = \frac{1}{2}\boldsymbol{\varepsilon} : g(d)\mathbb{C}_0 : \boldsymbol{\varepsilon} - p\phi_0 - (p - p_0)\boldsymbol{b}(d) : \boldsymbol{\varepsilon} - \frac{(p-p_0)^2}{2N(d)}$$
$$- Ts_0 - (T - T_0)\boldsymbol{\beta}(d) : \boldsymbol{\varepsilon} - \frac{c}{2T_0}(T - T_0)^2 + a_m(p - p_0)(T - T_0)$$

Note: Cohesive zone can be incorporated through appropriate choices of $g(d)$ (e.g., Lorentz and Godard, 2011 and Wu, 2017). See for instance You and Yoshioka, 2025; Xu et al., 2026, in case of HM coupling.

- Dissipation potential:

$$\varphi = \underbrace{Y_c(d)\dot{d} + 2l^2 w_1 \boldsymbol{\nabla} d \cdot \boldsymbol{\nabla} \dot{d}}_{\varphi^s} \underbrace{- \frac{1}{2}\boldsymbol{\nabla} p \cdot \frac{\boldsymbol{k}_f}{\mu_f} \cdot \boldsymbol{\nabla} p}_{\varphi^f} \underbrace{- \frac{1}{2T_0}\boldsymbol{\nabla} T \cdot \boldsymbol{k}_{th} \cdot \boldsymbol{\nabla} T}_{\varphi^{th}}$$

- Power of external work:

$$P_{ext} = \int_\Omega \boldsymbol{F} \cdot \dot{\boldsymbol{u}} \, \mathrm{d}\Omega + \int_{\partial_{\boldsymbol{T}}\Omega} \boldsymbol{T} \cdot \dot{\boldsymbol{u}} \, \mathrm{d}S + \int_{\partial_{Q^f}\Omega} Q^f p \, \mathrm{d}S + \int_{\partial_{Q^{th}}\Omega} Q^{th} log(\frac{T}{T_0}) \, \mathrm{d}S$$

Numerical implementation – staggered algorithm

I_n is not simultaneously convex with respect to $(\boldsymbol{u}, d) \Rightarrow$ staggered algorithm.

1. Saddle–point problem for thermo-poroelasticity (coupled with fluid flow and heat conduction):

$$(\boldsymbol{u}_{n+1}, p_{n+1}, T_{n+1}) = \text{Arg} \underset{\boldsymbol{u}'_{n+1}, p'_{n+1}, T'_{n+1}}{\text{stat}} \left\{ I_n^{\text{THM}} \left(\boldsymbol{u}'_{n+1}, p'_{n+1}, T'_{n+1}; d_n \right) \right\}$$

2. Minimizer for gradient damage:

$$d_{n+1} = \text{Arg} \min_{d'_{n+1}} \left\{ I_n^d \left(d'_{n+1} \right) \right\}$$

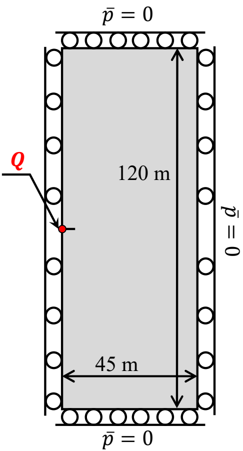
3. Iterating until the numerical computation converges.

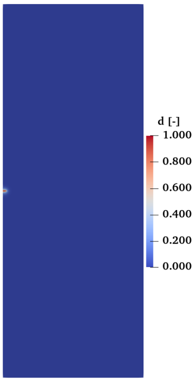
Remark:

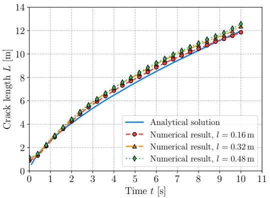
- Software: Finite element package FEniCS.
- AT2 model (Bourdin et al., JMPS 2000) (i.e. in gradient damage: $Y_c = 2w_1 d$; $w_1 = \frac{G_c}{2l}$).
- Isotropic permeability evolution (e.g. Mollaali et al., 2019, etc).

Numerical simulations

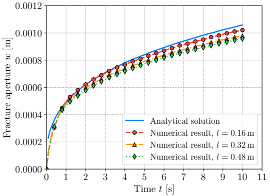
- Hydro-mechanical (HM) problems (Xu et al., 2026):
  - KGD fracture problem.
  - Hydraulic fracturing problems involving parallel or opposite cracks.
- Thermo-mechanical (TM) problems: thermal shock problems (Kamagate et al., 2025).
- Thermo-hydro-mechanical (THM) problem without damage: 3D thermal pressurization problem.
- Thermo-hydro-mechanical (THM) problem with damage: work in progress.







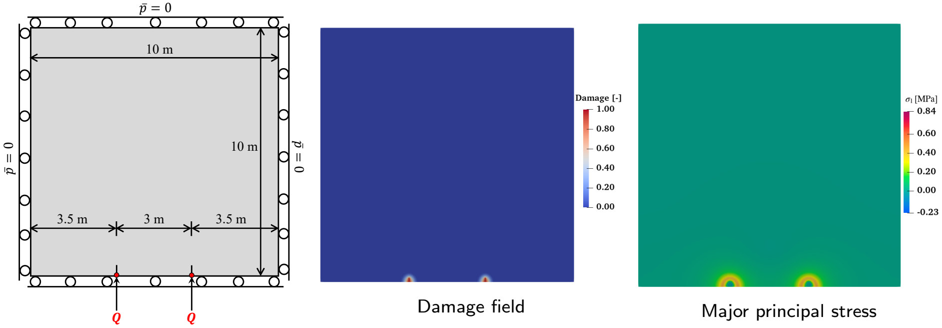
Crack length.



Fracture aperture.

- Horizontal fracture propagation driven by point fluid injection.
- The numerical results agree well with the analytical solutions (Garagash 2006, etc).
- As the internal length l decreases, the numerical results converge to the analytical solutions.

- $E = 5.8$ GPa; $\nu = 0.2$; $G_c = 100$ Pa $\cdot$ m; $Q = 3 \times 10^{-4}$ m^2/s.



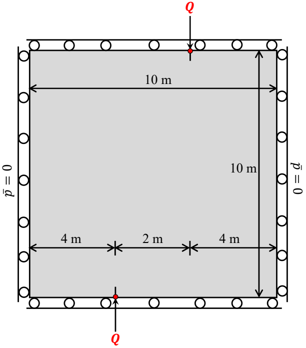
Damage field

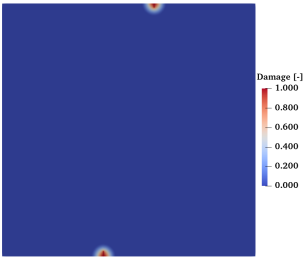
Major principal stress

- Fractures repel each other due to the modification of stress intensity at crack tips.
- Consistent with literature results (Paul et al., 2018; Wang, 2016).

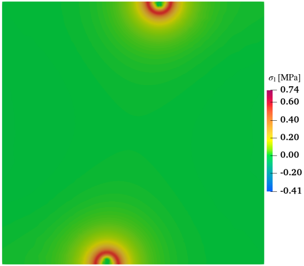
Simulation results – hydraulic fracturing problem with opposite cracks

- $E = 5.8$ GPa; $\nu = 0.2$; $G_c = 100$ Pa $\cdot$ m; $Q = 3 \times 10^{-4}$ m^2/s.





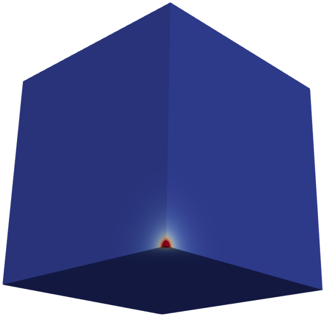
Damage field



Major principal stress

- Fractures attract each other due to the modification of stress intensity at crack tips.
- Consistent with literature results (Paul et al., 2018; Wang, 2016).

Simulation results – thermal pressurization problem



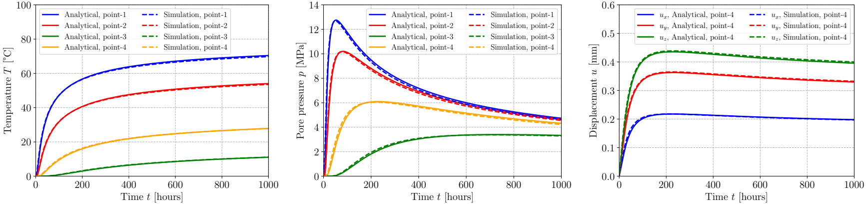
Geometry in the thermal pressurization problem.

Parameters in the thermal pressurization problem (Xu et al., 2020).

Parameters	Values
Young's modulus (E)	4500 MPa
Poisson's ratio (ν)	0.3
Porosity (ϕ)	0.15
Biot's coefficient (b)	1.0
Permeability (k)	4.5×10^{-20} m^2
Fluid dynamic viscosity (μ_f)	1×10^{-3} Pa s
Equivalent heat capacity (c_m)	1000 J kg^{-1} K^{-1}
Equivalent density (ρ_m)	2400 kg m^{-3}
Equivalent thermal conductivity (λ_m)	1.7 W m^{-1} K^{-1}
Volumetric thermal dilation coefficient of solid phase (α_s)	4.2×10^{-5} K^{-1}
Volumetric thermal dilation coefficient of fluid phase (α_f)	4×10^{-4} K^{-1}

- A constant heat flux imposed at a corner of the saturated porous medium.
- Analytical solution available (Booker and Savvidou (1985); Smith and Booker (1993)).

Simulation results – thermal pressurization problem



Evolution of temperature, pore pressure, and displacement in the thermal pressurization problem.

- The numerical results are in good agreement with the analytical solution.

Conclusions and Perspectives

Conclusions

- A thermodynamics based formulation of gradient damage coupled with thermo-poroelasticity.
- An incremental variational principle with a unified total energy functional, accounting for dissipative transport mechanisms (Darcy-type flow and heat conduction).
- A variational model was proposed and implemented using a staggered algorithm, and validated through several numerical applications.

Perspectives

- Further numerical validation and applications (2D and 3D) of the proposed THM-coupled gradient damage model.
- Application of the model to practical in situ THM-coupled gallery excavation problems.
- Extension to damage-induced anisotropy (e.g., permeability evolution), partially saturated conditions, and frictional crack modeling.

See corresponding works of our team for further details:

- Zhang, X. D., Cheng, L., Kondo, D., Giraud, A. (2024). Incremental variational approach to gradient damage coupled with poroelasticity of saturated media. Journal of the Mechanics and Physics of Solids, 187, 105614.
- Kamagaté, B., Cheng, L., Abdelmoula, R., Danho, E., Kondo, D. (2025). An incremental variational method to the coupling between gradient damage, thermoelasticity and heat conduction. Comptes Rendus. Mécanique, 353(G1), 1063-1084.
- Xu, Y., Cheng, L., Golfier, F., Zhang, Y. (2026). An incremental variational gradient damage model for saturated poroelastic media incorporating cohesive zone effects. Computer Methods in Applied Mechanics and Engineering, 458, 119045.

Thank you very much for listening!