Coupled Hyperbolic Approach to Solve Buoyant Two-Phase Flow and Transport in Heterogeneous Porous Media

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[3] Chevron Corporation

Motivation

- Fully implicit methods: expensive
- Sequential methods: require small time steps

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Goal

- Tightly coupled, explicit flow and transport scheme
- Two phases
- Suited for ACTI and GPUs

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- Time integration with adapted Riemann solver

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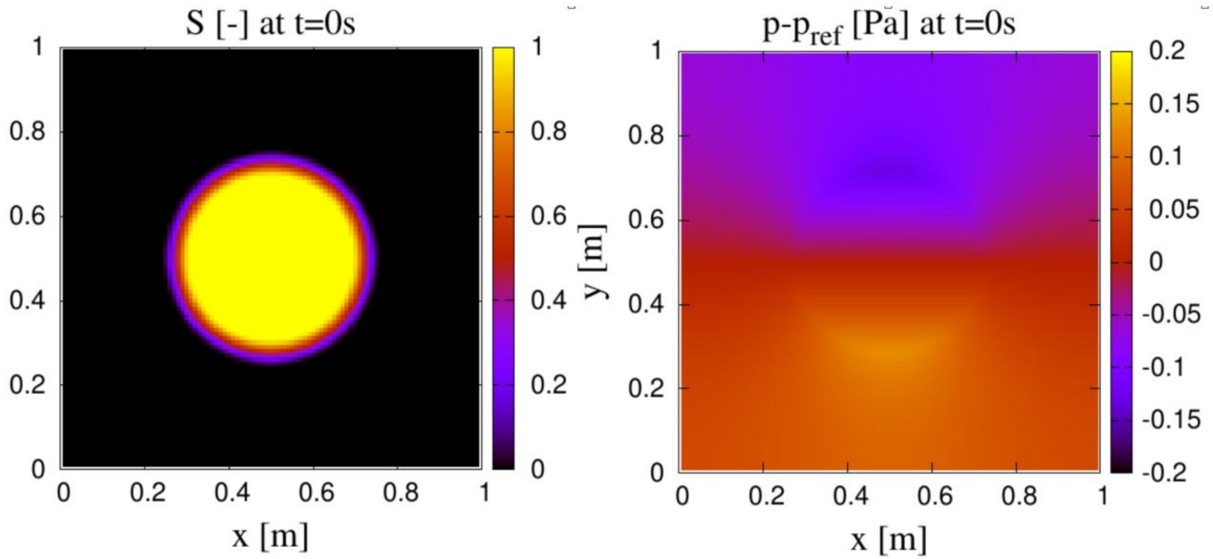
Vision

- Adaptive in space and time
- For complex problems in 3D
- Suited for GPUs

Motivation



S [-] at t=0s
1
0.8
0.6
0.4
0.2
0
0
0.2
0.4
0.6
0.8
1
x [m]
p-p$_{ref}$ [Pa] at t=0s
y [m]
0.2
0.15
0.1
0.05
0
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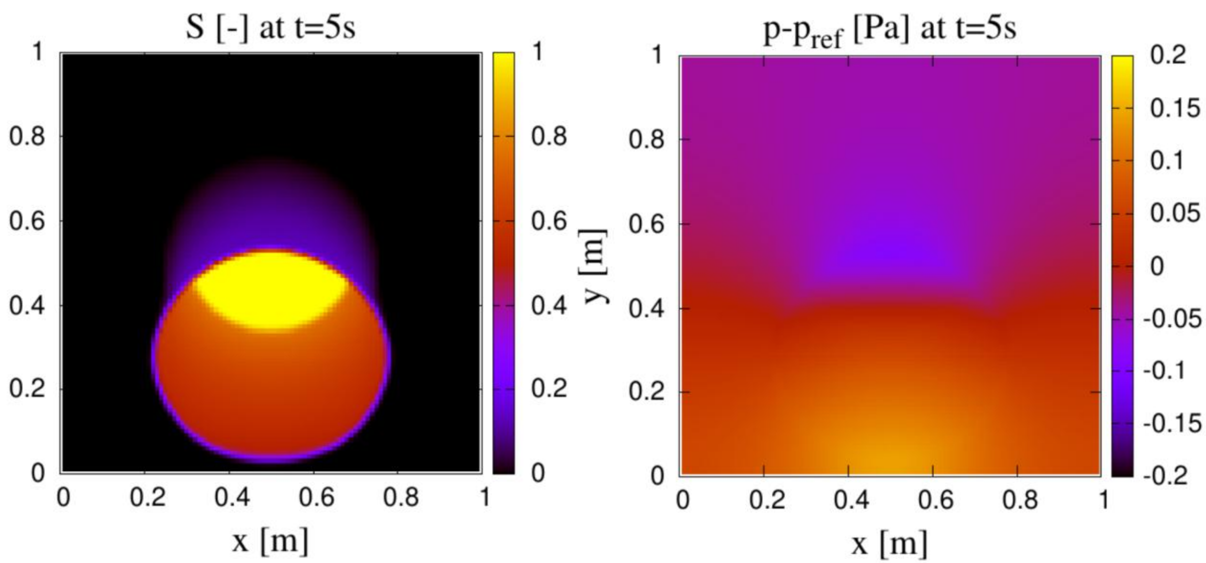

S [-] at t=5s
p-p$_{ref}$ [Pa] at t=5s
x [m]
y [m]
0
0.2
0.4
0.6
0.8
1
0.2
0.15
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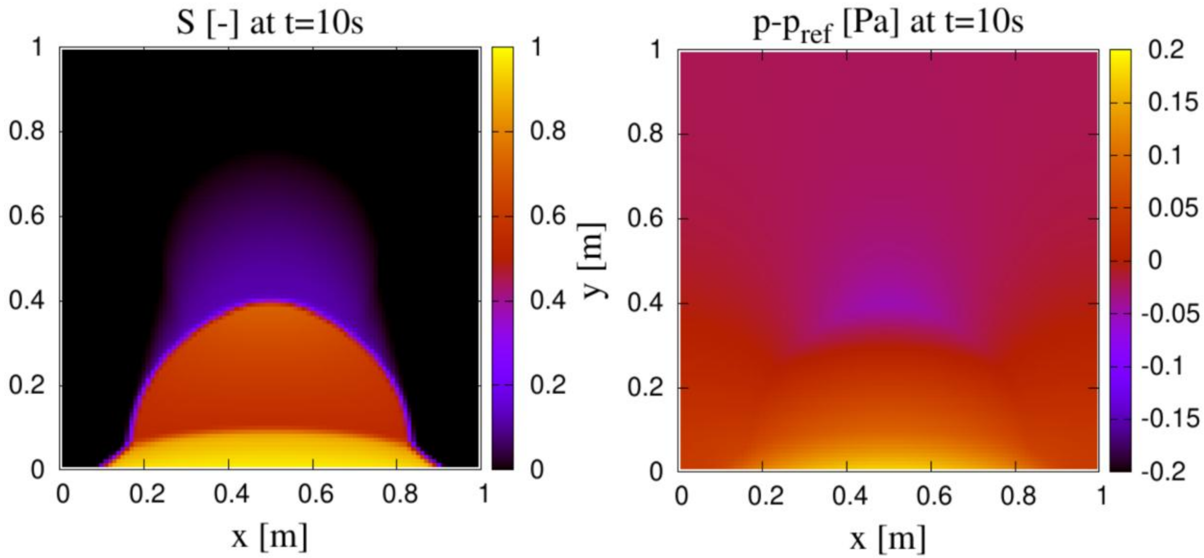
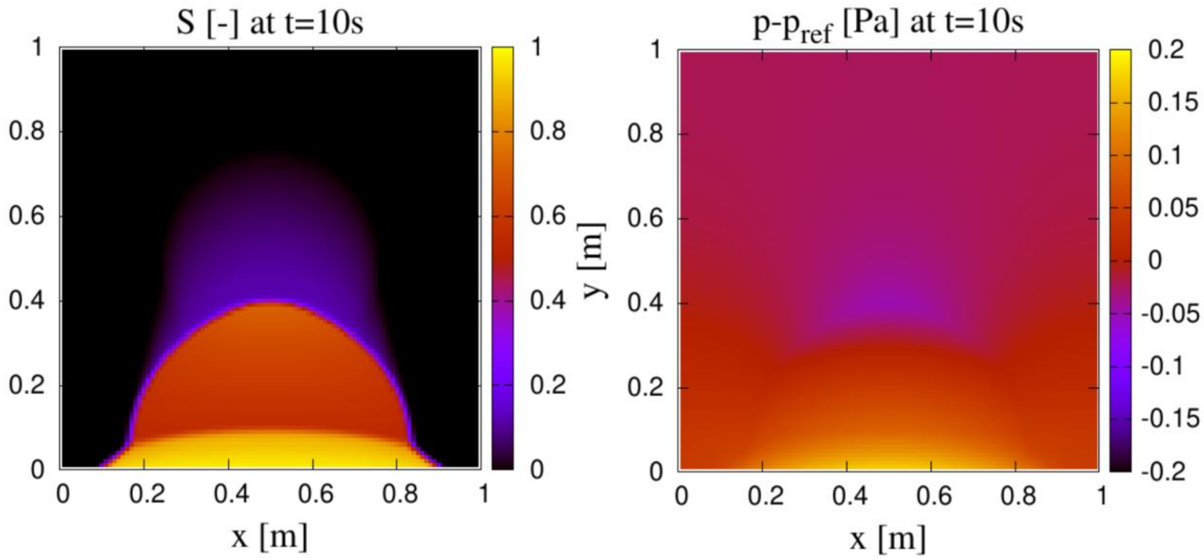


Classical System for Two-Phase Flow and Transport



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$\uparrow$

$$\boldsymbol{u} = -\lambda k \nabla p + k\gamma \boldsymbol{g}$$

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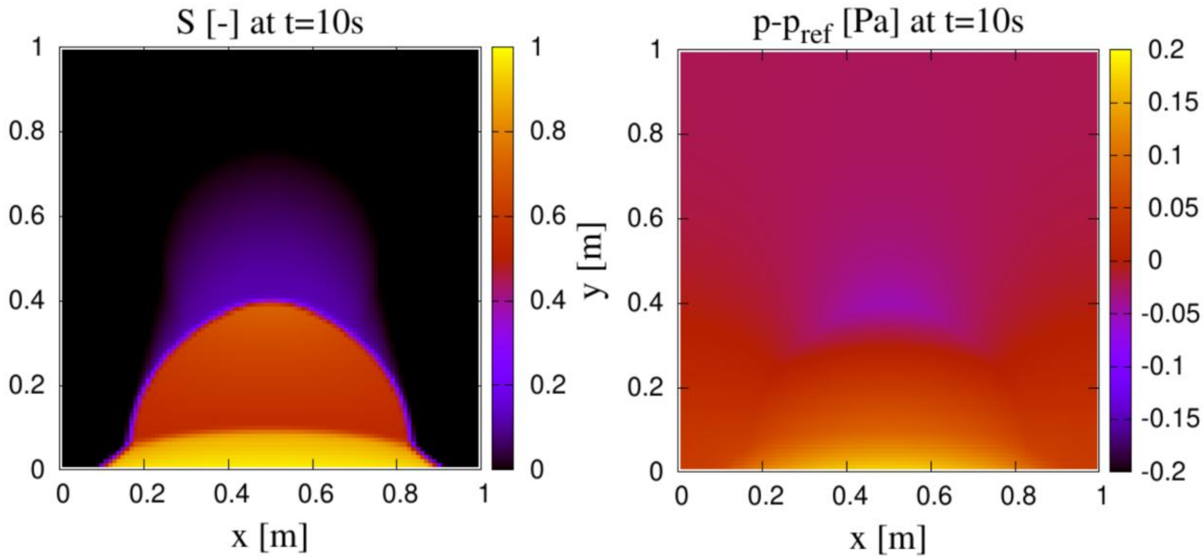

$$\frac{\partial \phi S}{\partial t} + \nabla \cdot \boldsymbol{u}_{p1} = 0$$

$$\boldsymbol{u}_{p1} = f_1 \boldsymbol{u} + f_{12} \boldsymbol{g}$$

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New: Hyperbolic System for Flow and Transport

$$\underbrace{\frac{\partial}{\partial t}\begin{pmatrix}\rho\\ \rho u\\ \rho v\end{pmatrix}}_{\boldsymbol{U}} + \frac{\partial}{\partial x}\underbrace{\begin{pmatrix}\rho u\\ \rho u^2 + p\\ \rho u v\end{pmatrix}}_{\boldsymbol{F}_x} + \frac{\partial}{\partial y}\underbrace{\begin{pmatrix}\rho v\\ \rho v u\\ \rho v^2 + p\end{pmatrix}}_{\boldsymbol{F}_y} = \underbrace{\begin{pmatrix}0\\ 0\\ 0\end{pmatrix}}_{\boldsymbol{Q}}$$

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$$\boldsymbol{R} = -\frac{1}{\lambda k}\boldsymbol{u} + \frac{\gamma}{\lambda}\boldsymbol{g}$$

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Good approximation if:

- Low Mach number
- Small relative density differences
- Source Terms dominate

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Numerical Scheme: Approximate Riemann Solver

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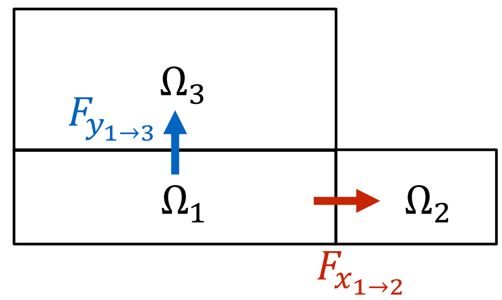

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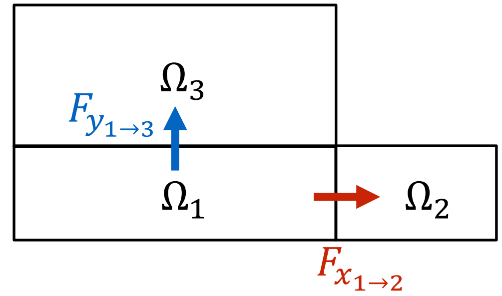

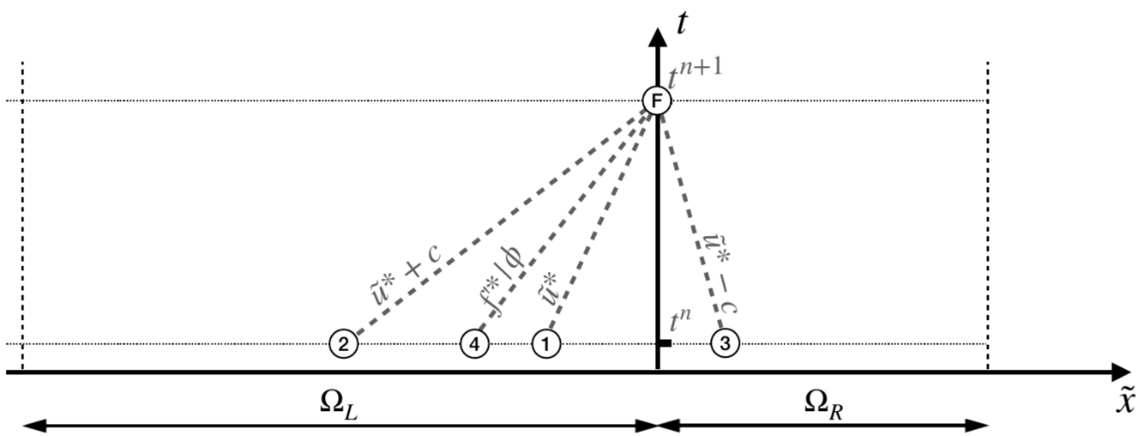

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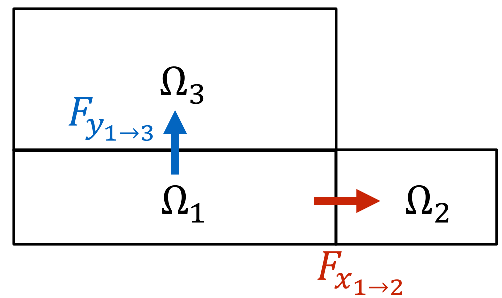
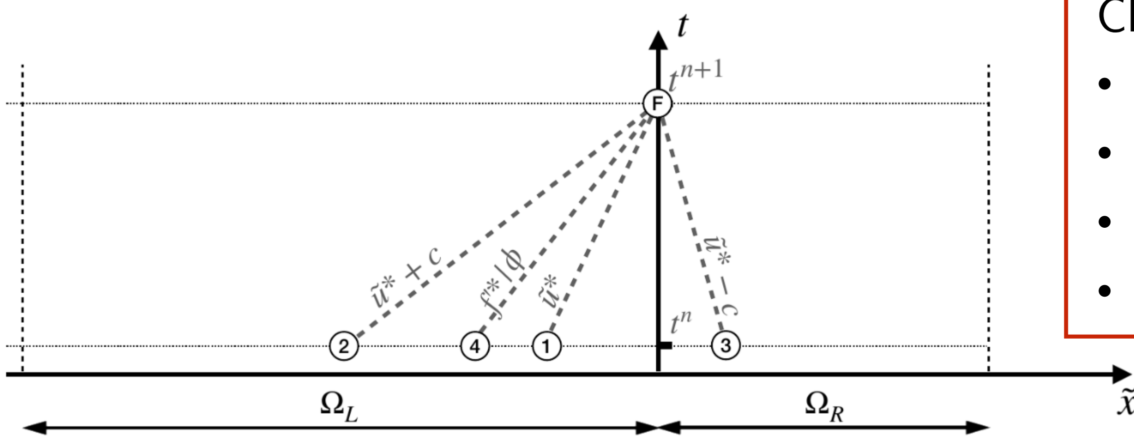
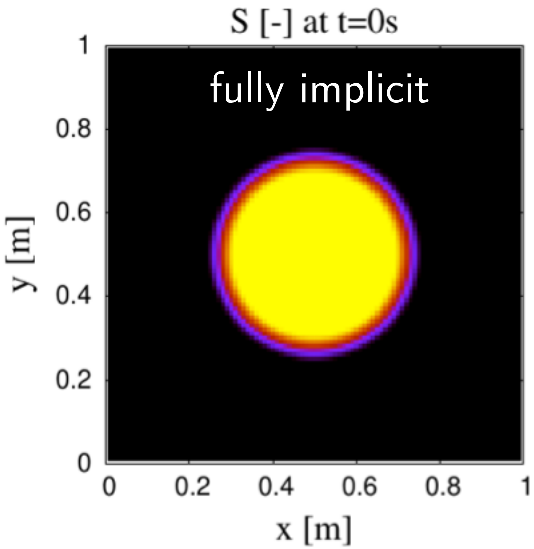




Characteristics-based Riemann solver:

- Cheap
- Robust
- Accurate
- Suited for GPUs

Case 1: 2D Plume



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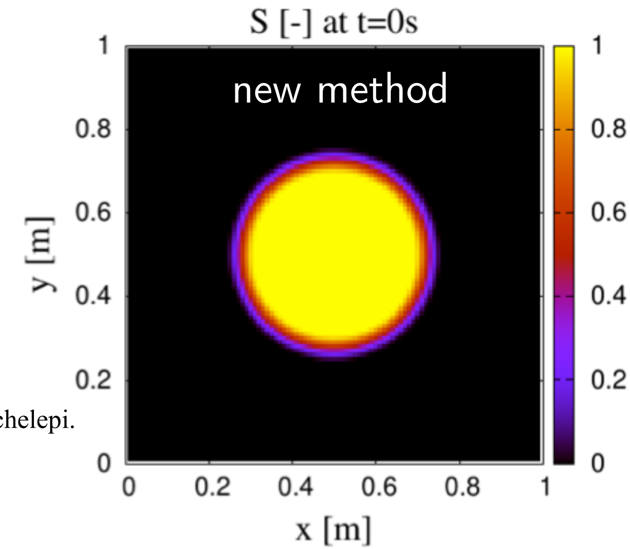

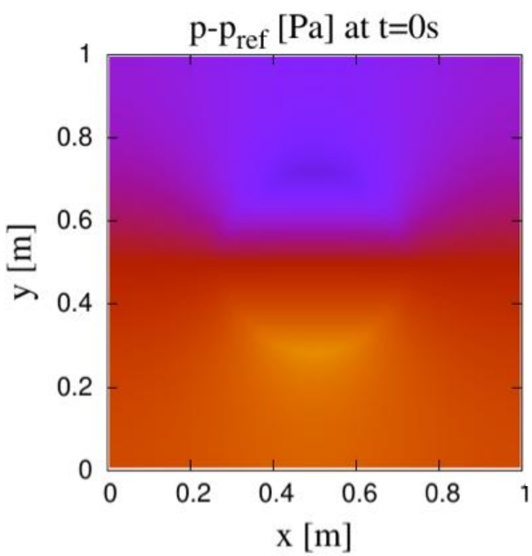

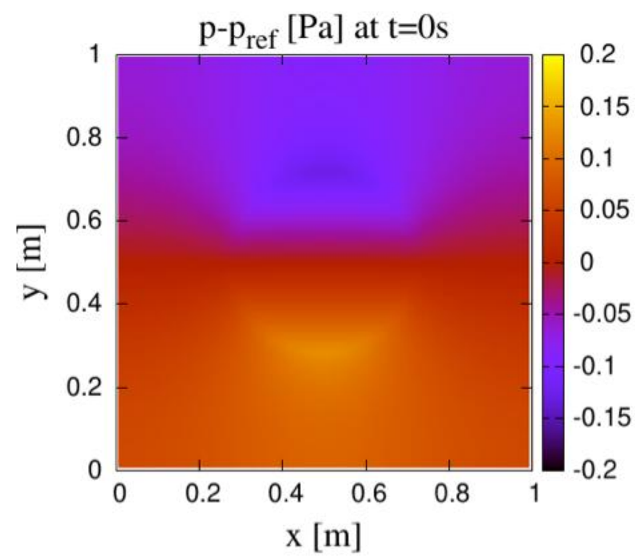

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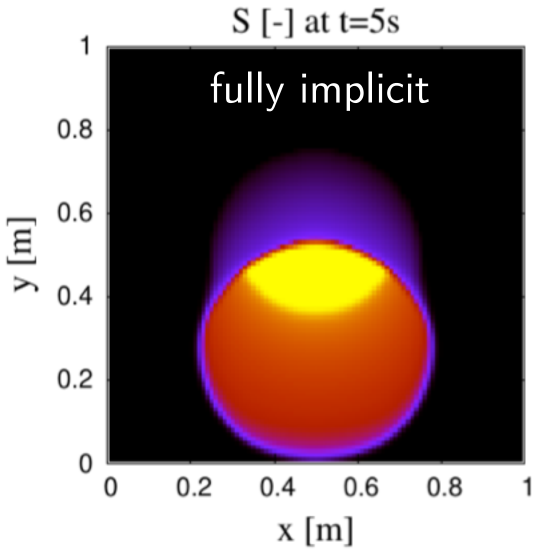

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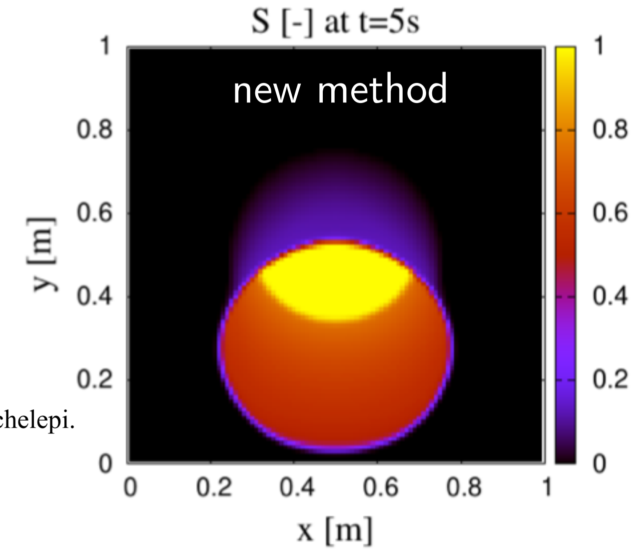

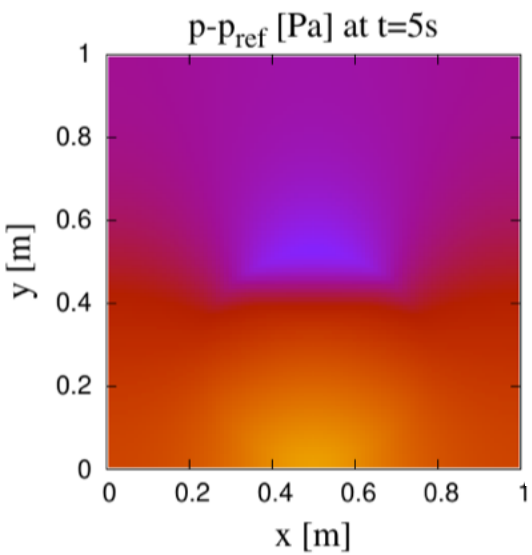

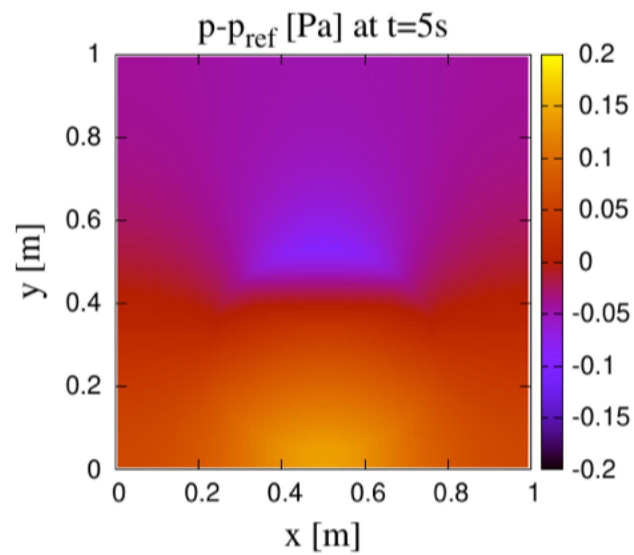

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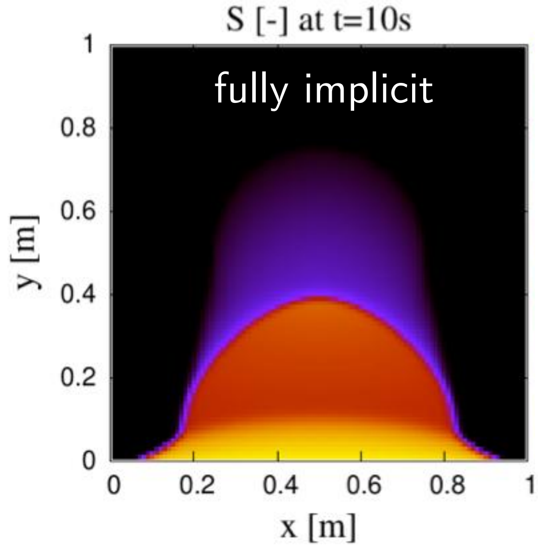

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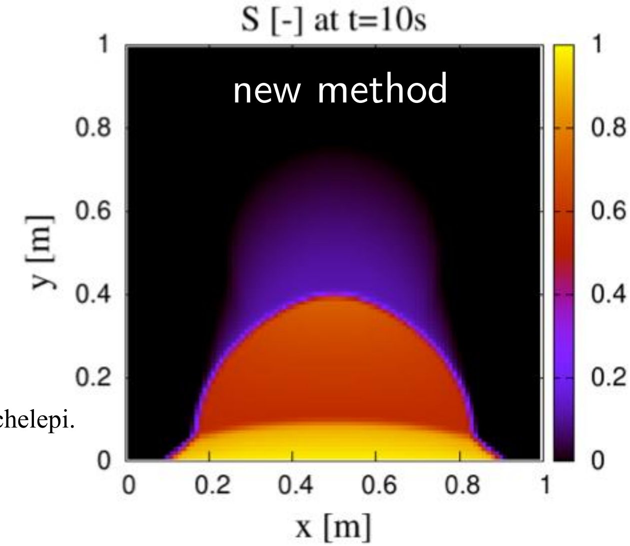

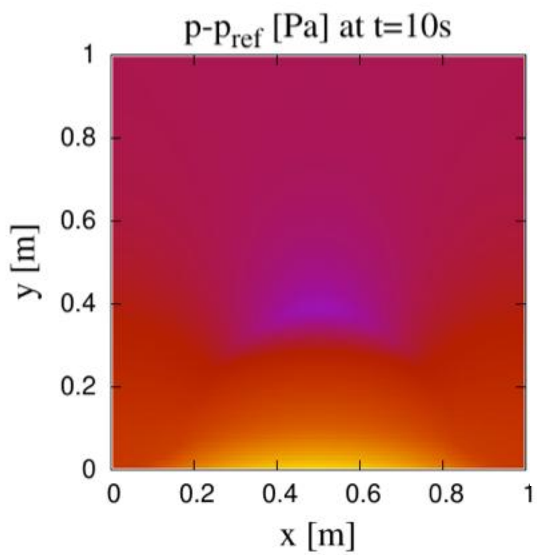

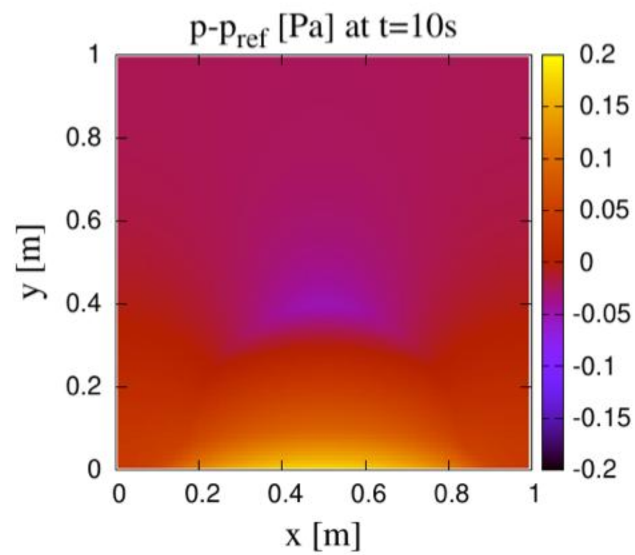

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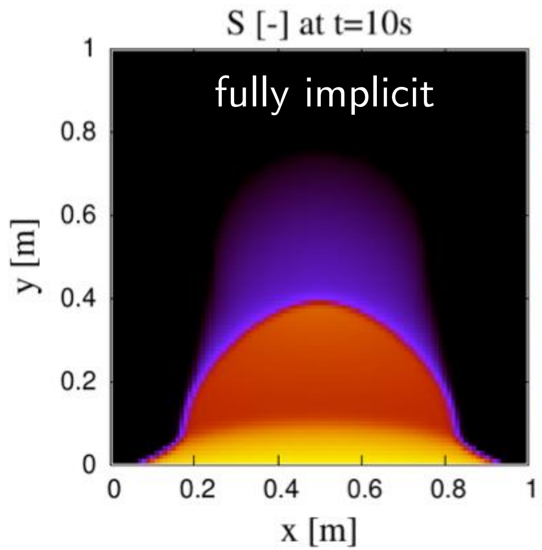

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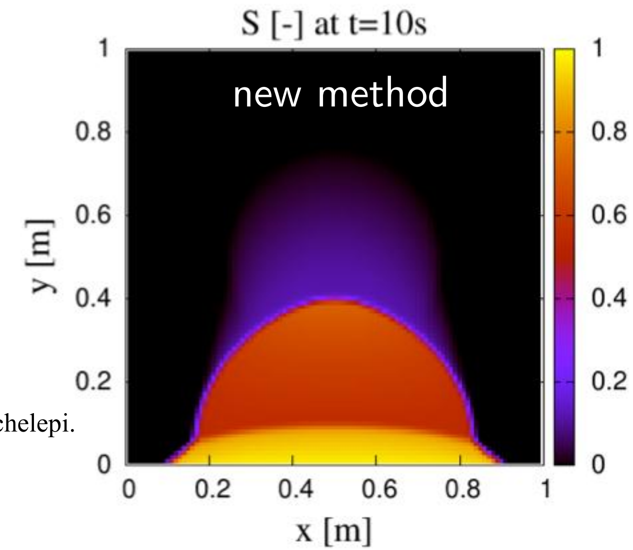
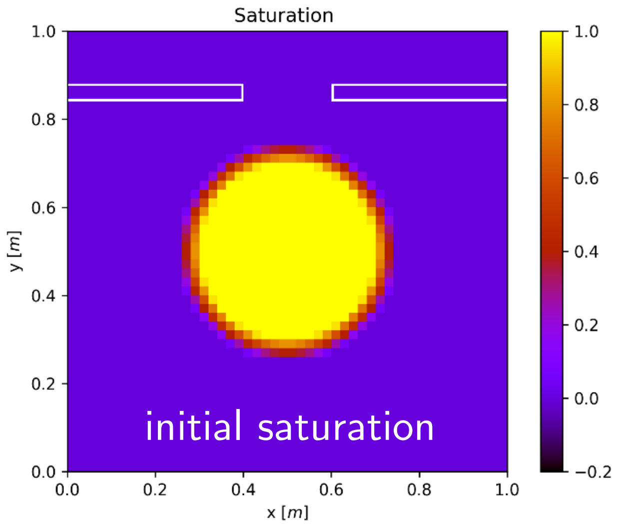


Hyperbolic vs Standard Simulator

- Very good agreement
- Sharper shocks with hyperbolic simulator

Case 2: 2D Plume with Shale



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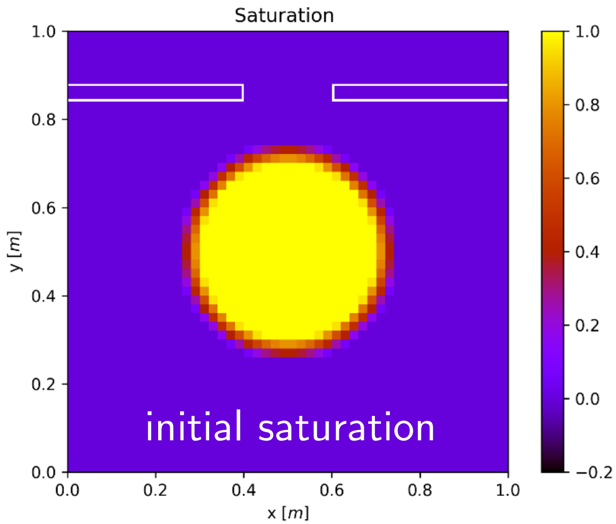

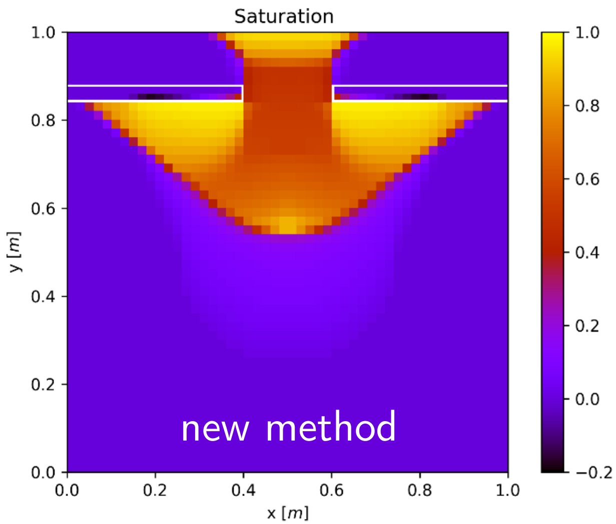

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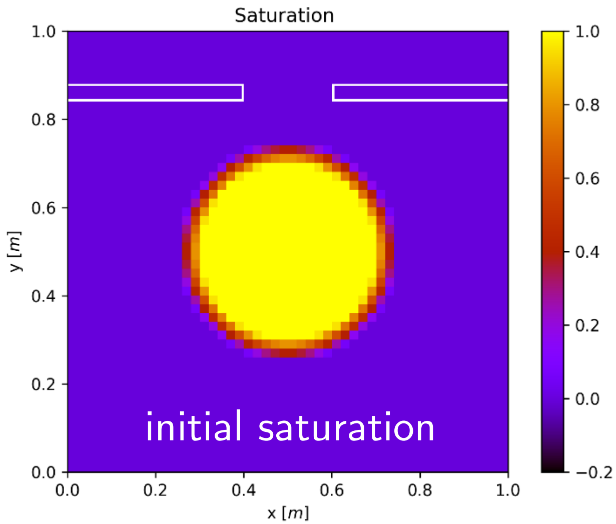

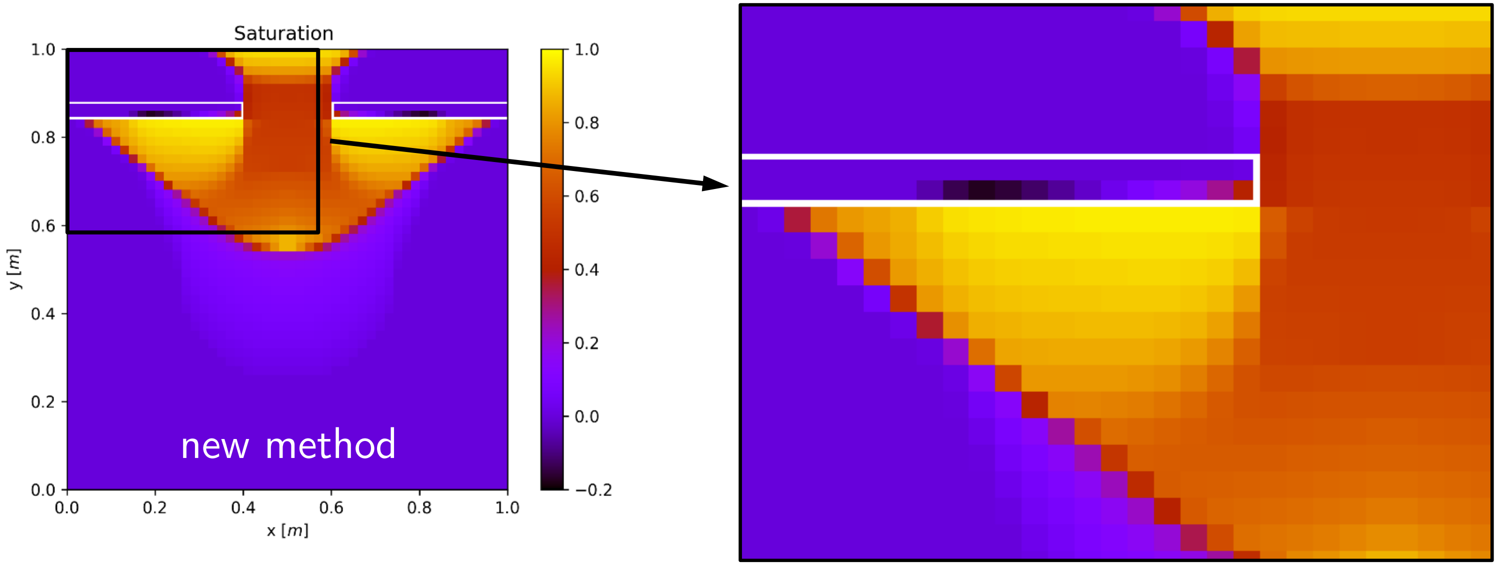

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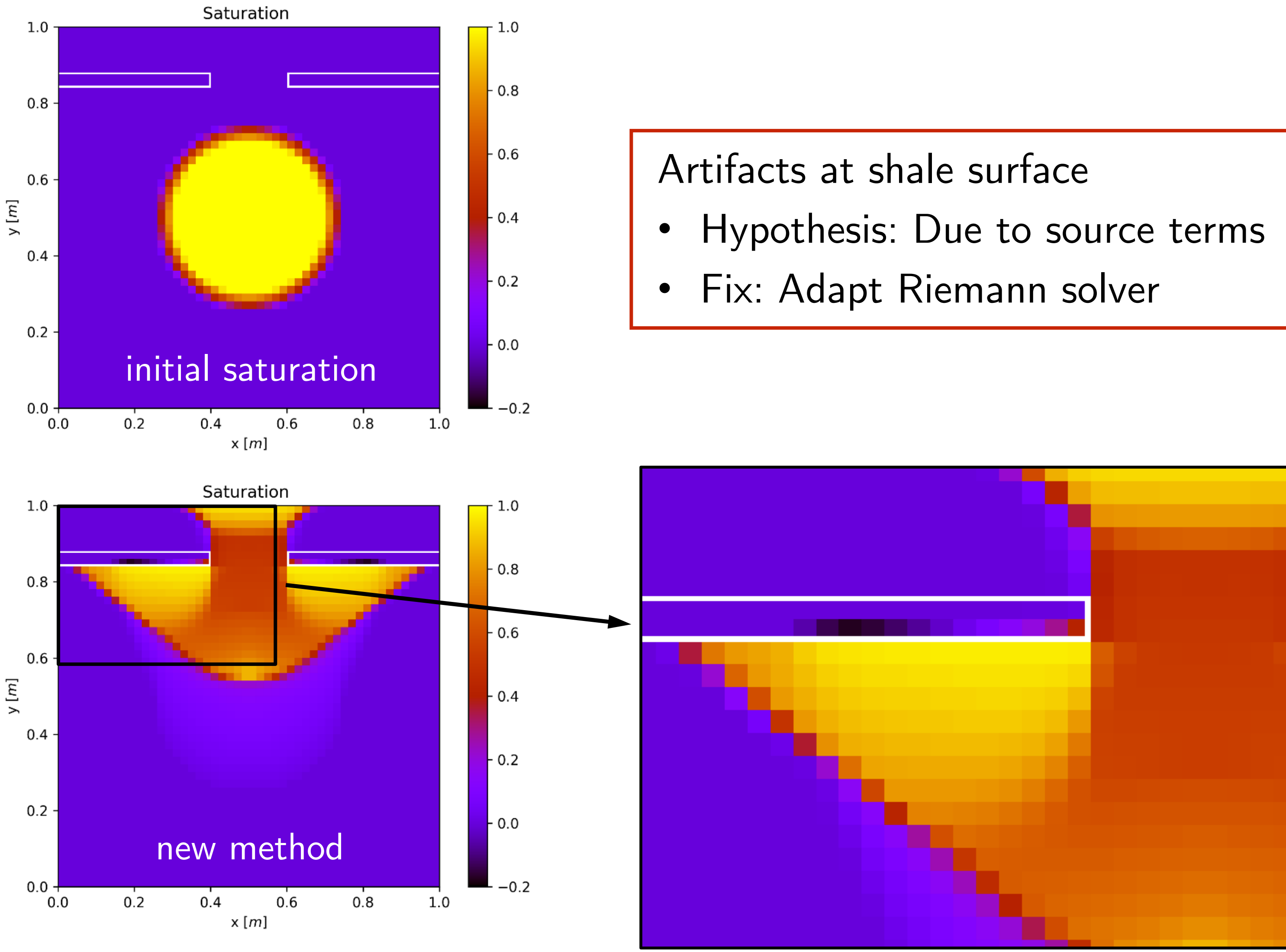
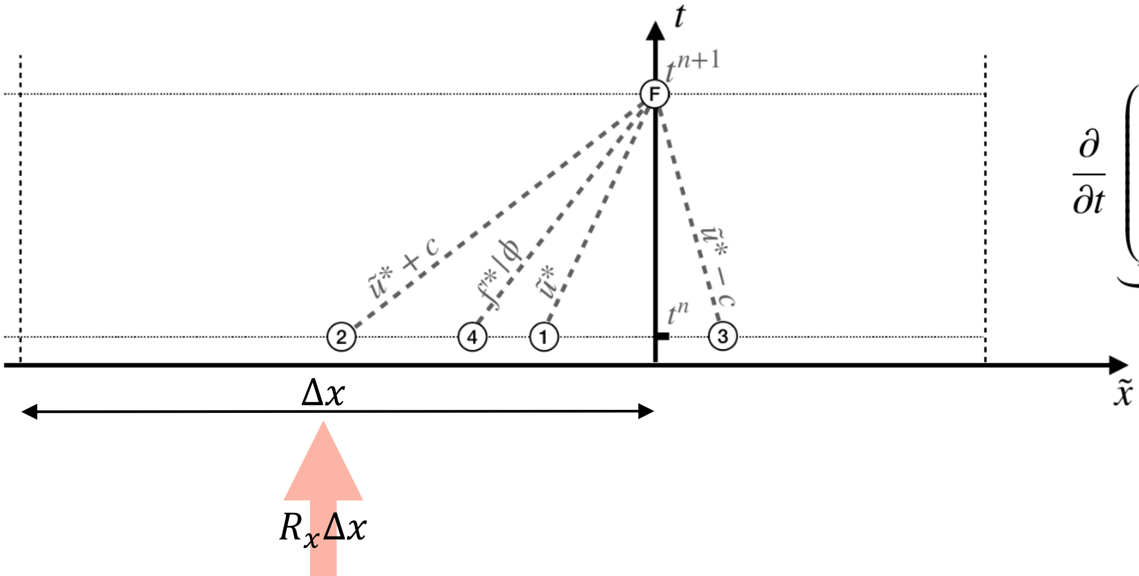


Artifacts at shale surface

- Hypothesis: Due to source terms
- Fix: Adapt Riemann solver

Rankine-Hugoniot-Riemann (RHR) solver



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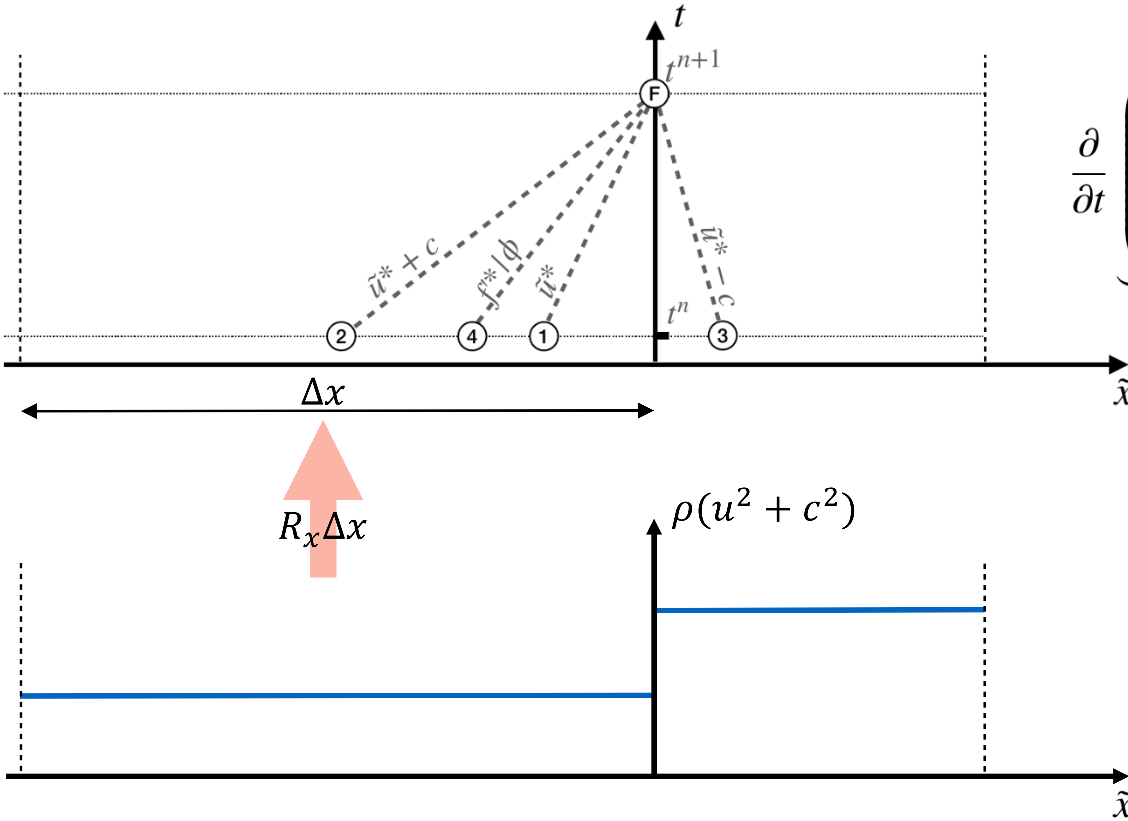

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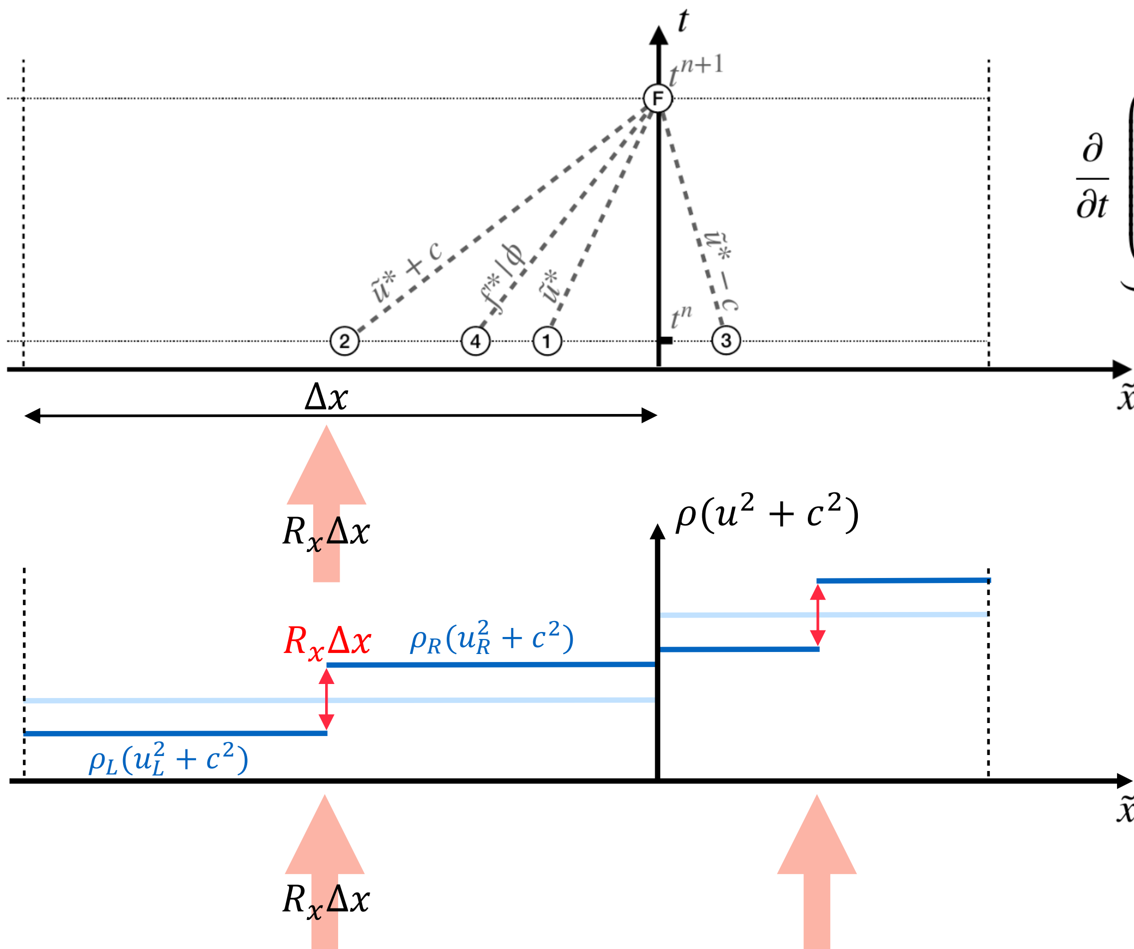

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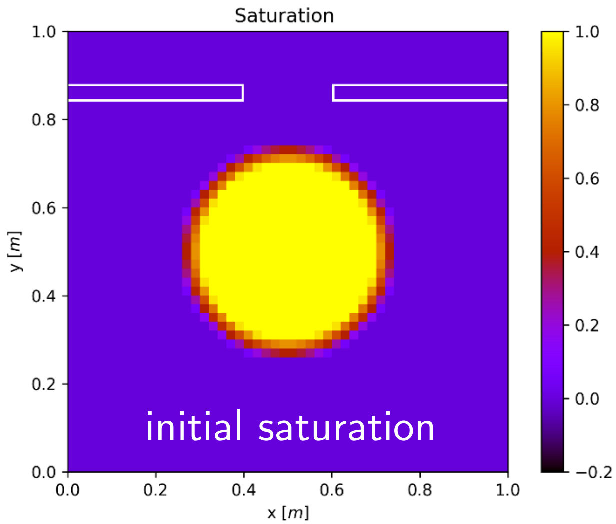
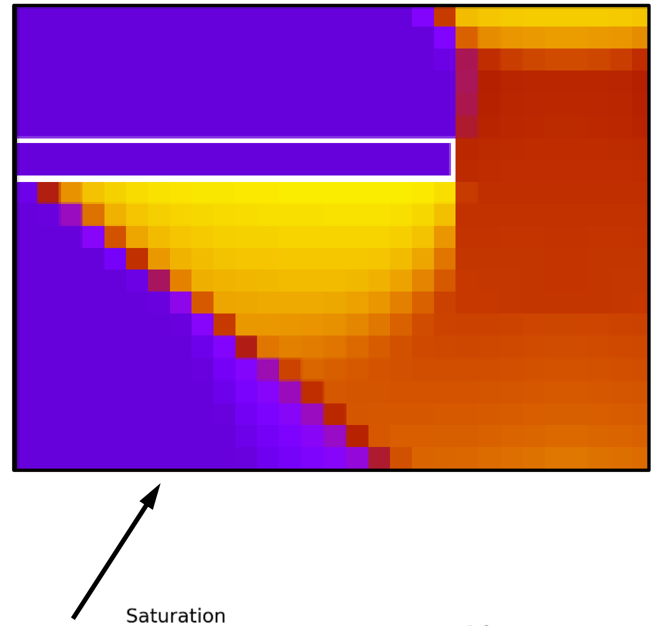
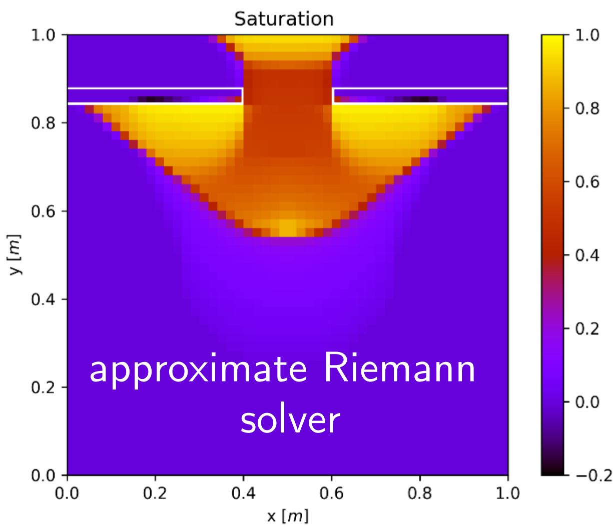
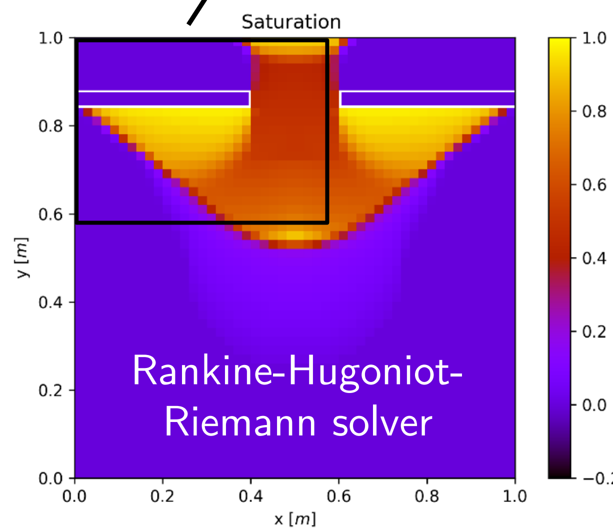
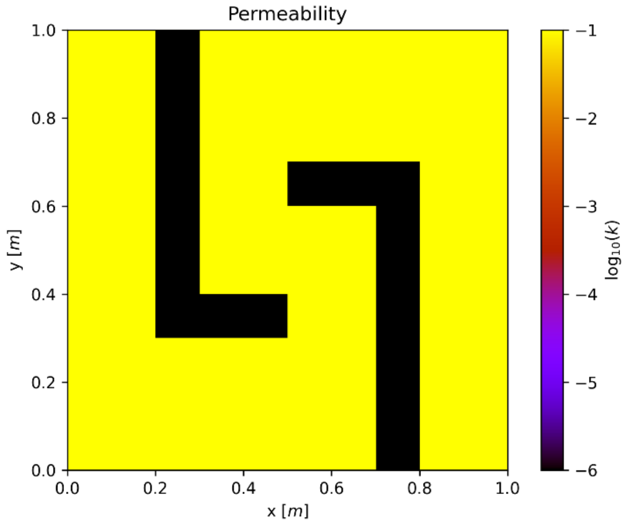






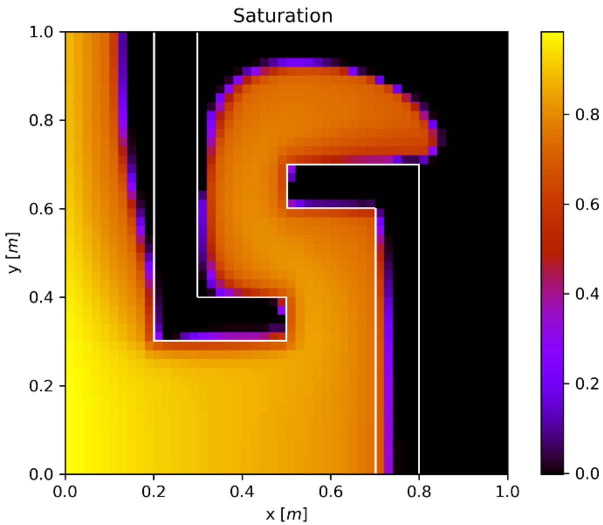


Case 3: Meander



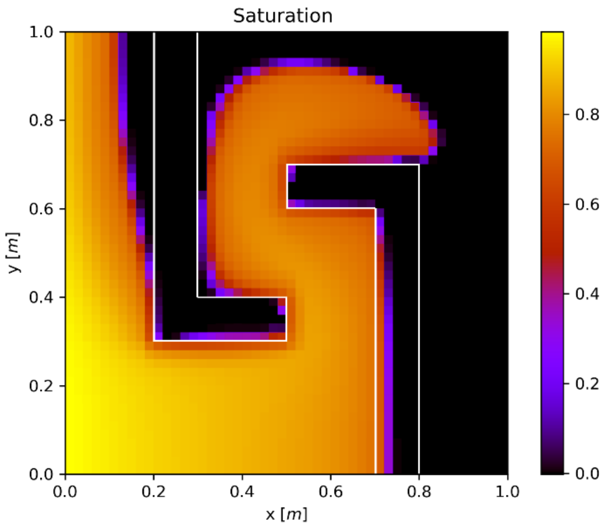
Case 3: Meander

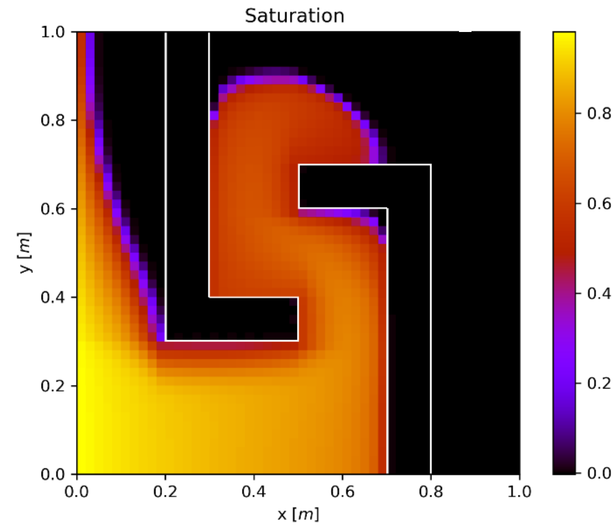
approximate
Riemann
solver



Case 3: Meander

approximate Riemann solver

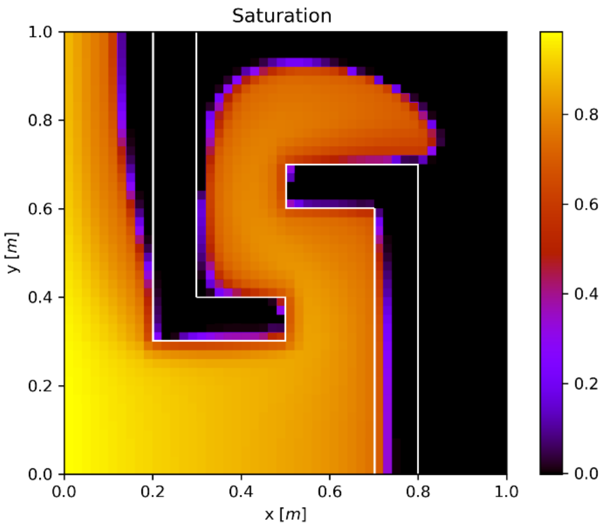


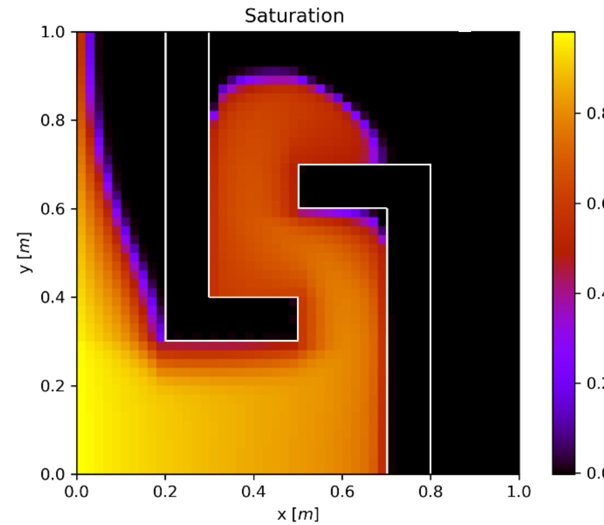


Rankine-Hugoniot-Riemann solver

Case 3: Meander

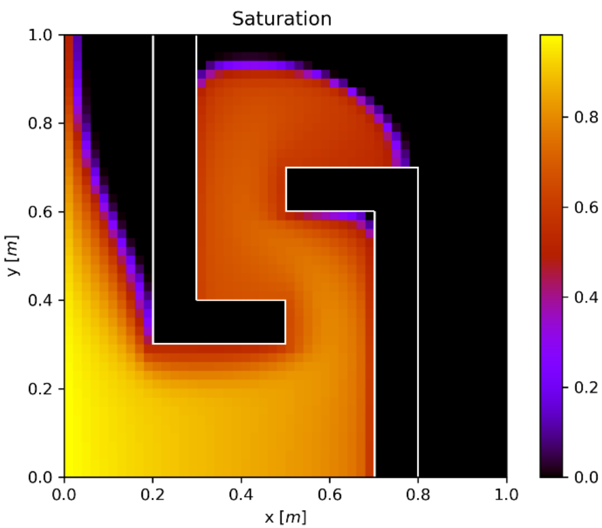
approximate Riemann solver



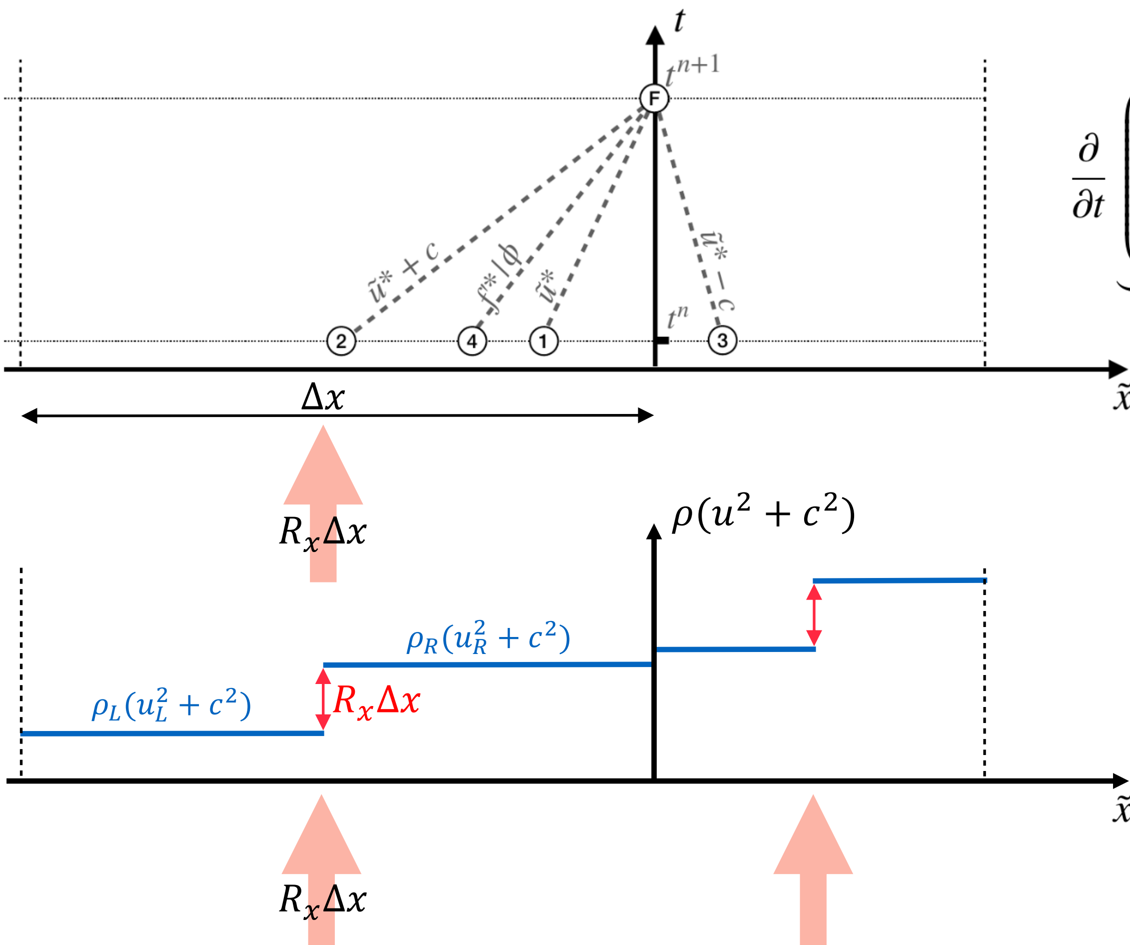


Rankine-Hugoniot-Riemann solver

Implicit Method



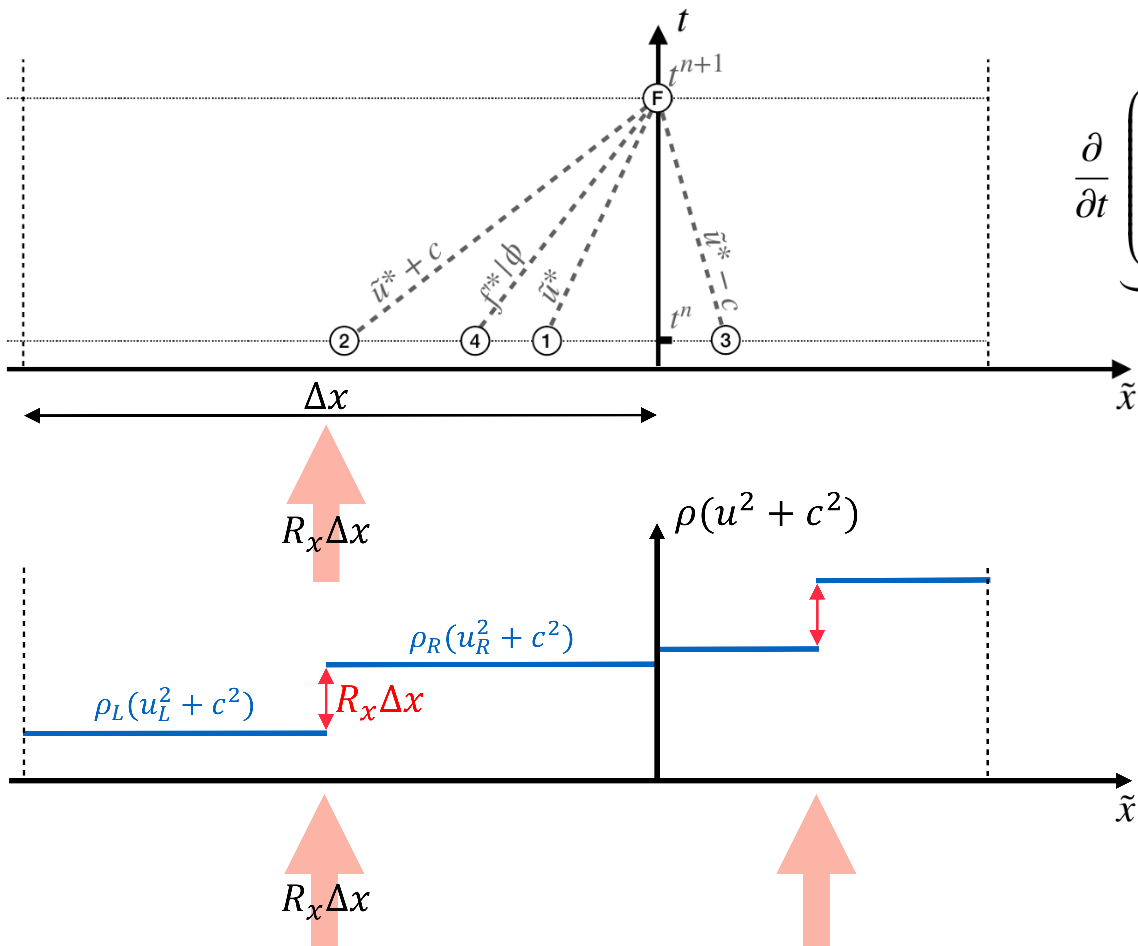
Rankine-Hugoniot-Riemann (RHR) solver



$p = c^2\rho$

$$\frac{\partial}{\partial t}\underbrace{\begin{pmatrix}\rho\\ \rho u\\ \rho v\\ \rho Y\end{pmatrix}}_{\boldsymbol{U}} + \frac{\partial}{\partial x}\underbrace{\begin{pmatrix}\rho u\\ \rho u^2 + p\\ \rho u v\\ F_x^Y\end{pmatrix}}_{\boldsymbol{F}_x} + \frac{\partial}{\partial y}\underbrace{\begin{pmatrix}\rho v\\ \rho v u\\ \rho v^2 + p\\ F_y^Y\end{pmatrix}}_{\boldsymbol{F}_y} = \underbrace{\begin{pmatrix}0\\ R_x\\ R_y\\ 0\end{pmatrix}}_{\boldsymbol{Q}}$$

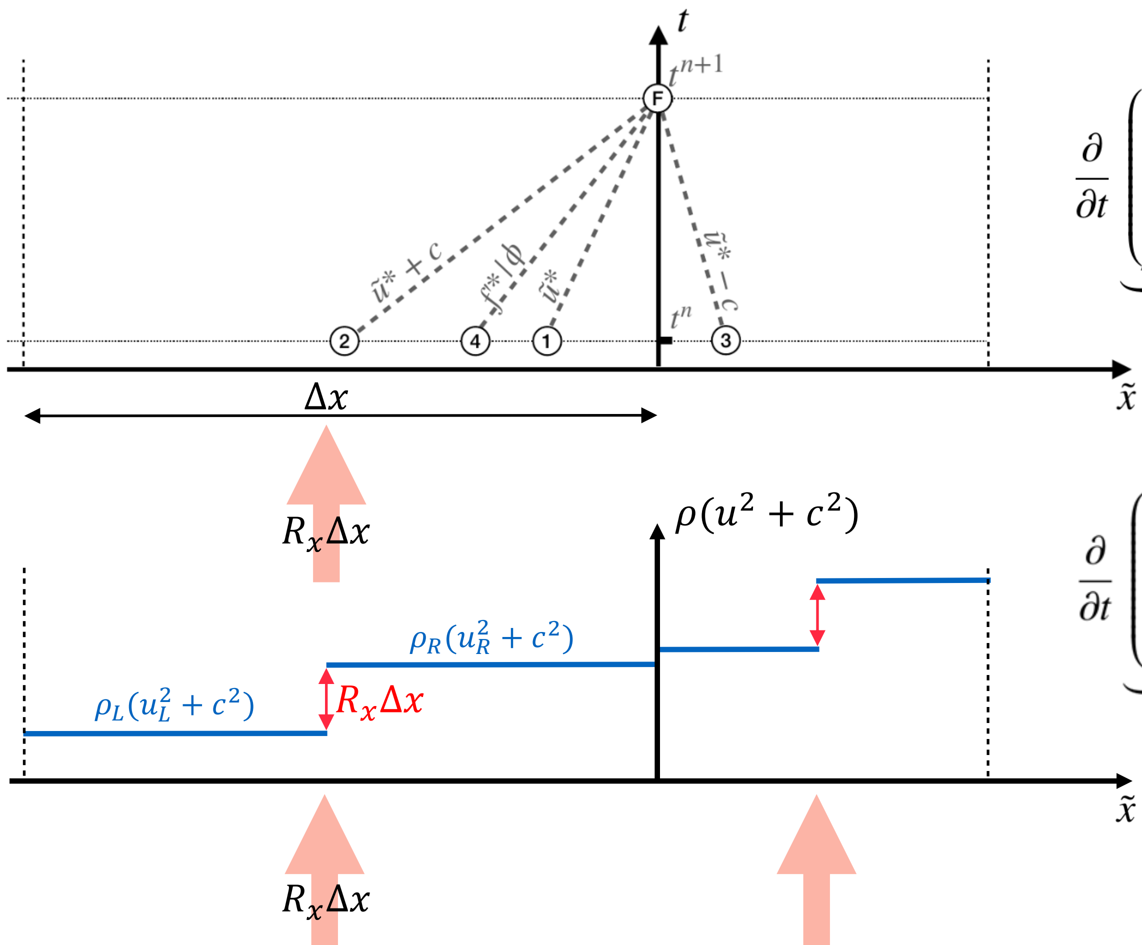
Rankine-Hugoniot-Riemann (RHR) solver



$p = c^2\rho$

$$\frac{\partial}{\partial t}\underbrace{\begin{pmatrix}\rho\\ \rho u\\ \rho v\\ \rho Y\end{pmatrix}}_{\boldsymbol{U}} + \frac{\partial}{\partial x}\underbrace{\begin{pmatrix}\rho u\\ \rho u^2 + p\\ \rho u v\\ F_x^Y\end{pmatrix}}_{\boldsymbol{F}_x} + \frac{\partial}{\partial y}\underbrace{\begin{pmatrix}\rho v\\ \rho v u\\ \rho v^2 + p\\ F_y^Y\end{pmatrix}}_{\boldsymbol{F}_y} = \underbrace{\begin{pmatrix}0\\ R_x\\ R_y\\ 0\end{pmatrix}}_{\boldsymbol{Q}}$$

Rankine-Hugoniot-Riemann (RHR) solver

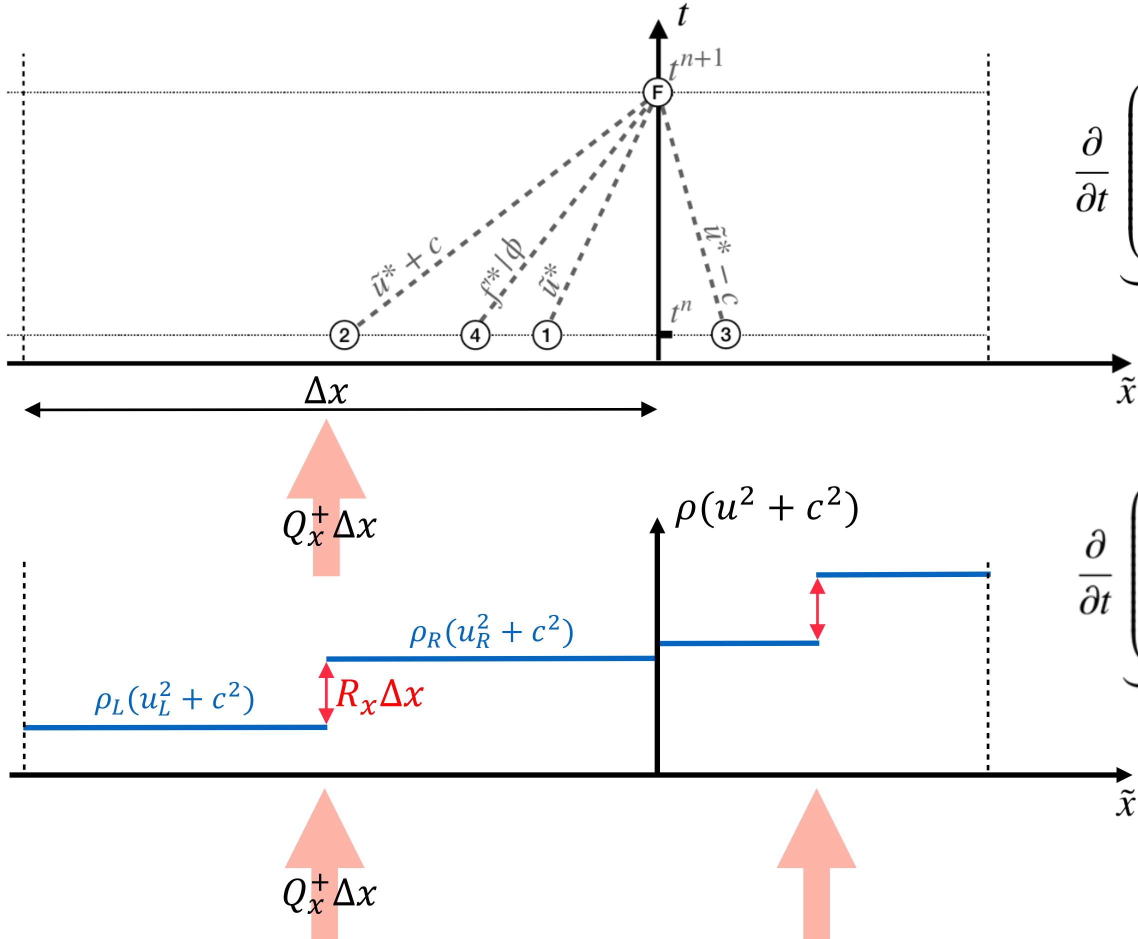


$p = c^2 \rho$

$$\frac{\partial}{\partial t}\underbrace{\begin{pmatrix}\rho\\ \rho u\\ \rho v\\ \rho Y\end{pmatrix}}_{\boldsymbol{U}} + \frac{\partial}{\partial x}\underbrace{\begin{pmatrix}\rho u\\ \rho u^2 + p\\ \rho u v\\ F_x^Y\end{pmatrix}}_{\boldsymbol{F}_x} + \frac{\partial}{\partial y}\underbrace{\begin{pmatrix}\rho v\\ \rho v u\\ \rho v^2 + p\\ F_y^Y\end{pmatrix}}_{\boldsymbol{F}_y} = \underbrace{\begin{pmatrix}0\\ R_x\\ R_y\\ 0\end{pmatrix}}_{\boldsymbol{Q}}$$

$$\frac{\partial}{\partial t}\underbrace{\begin{pmatrix}\rho\\ \rho u\\ \rho v\\ \rho Y\end{pmatrix}}_{\boldsymbol{U}} + \frac{\partial}{\partial x}\underbrace{\begin{pmatrix}\rho u\\ \rho u^2 + p\\ \rho u v\\ F_x^Y\end{pmatrix}}_{\boldsymbol{F}_x} = \underbrace{\begin{pmatrix}0\\ R_x\\ R_y\\ 0\end{pmatrix}}_{\boldsymbol{Q}} - \frac{\partial}{\partial y}\underbrace{\begin{pmatrix}\rho v\\ \rho v u\\ \rho v^2 + p\\ F_y^Y\end{pmatrix}}_{\boldsymbol{F}_y}$$

Rankine-Hugoniot-Riemann (RHR) solver



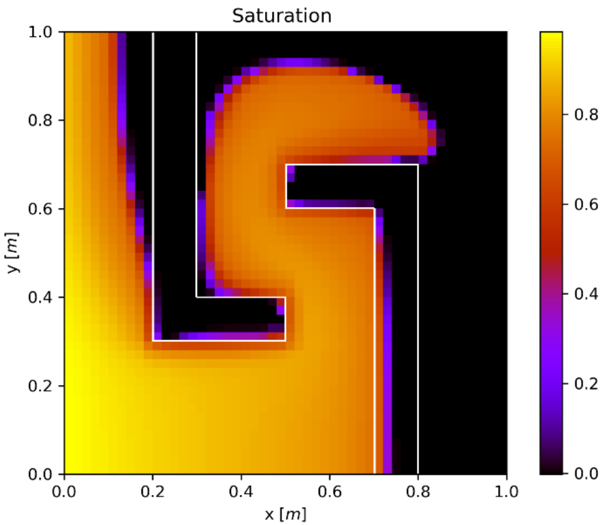
$p = c^2\rho$

$$\frac{\partial}{\partial t}\underbrace{\begin{pmatrix}\rho\\ \rho u\\ \rho v\\ \rho Y\end{pmatrix}}_{\boldsymbol{U}} + \frac{\partial}{\partial x}\underbrace{\begin{pmatrix}\rho u\\ \rho u^2 + p\\ \rho uv\\ F_x^Y\end{pmatrix}}_{\boldsymbol{F}_x} + \frac{\partial}{\partial y}\underbrace{\begin{pmatrix}\rho v\\ \rho vu\\ \rho v^2 + p\\ F_y^Y\end{pmatrix}}_{\boldsymbol{F}_y} = \underbrace{\begin{pmatrix}0\\ R_x\\ R_y\\ 0\end{pmatrix}}_{\boldsymbol{Q}}$$

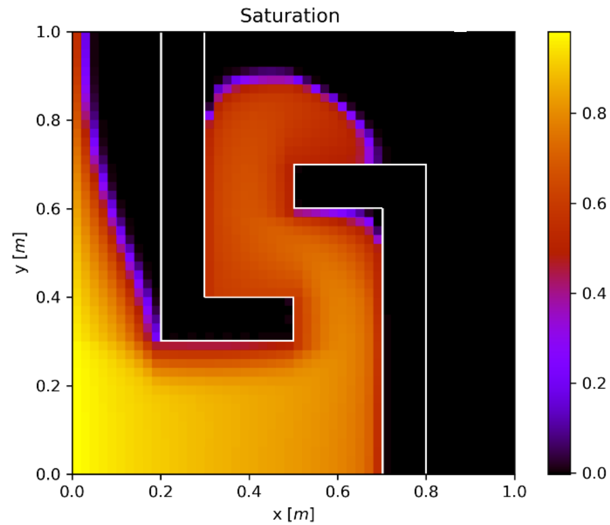
$$\frac{\partial}{\partial t}\underbrace{\begin{pmatrix}\rho\\ \rho u\\ \rho v\\ \rho Y\end{pmatrix}}_{\boldsymbol{U}} + \frac{\partial}{\partial x}\underbrace{\begin{pmatrix}\rho u\\ \rho u^2 + p\\ \rho uv\\ F_x^Y\end{pmatrix}}_{\boldsymbol{F}_x} = \underbrace{\underbrace{\begin{pmatrix}0\\ R_x\\ R_y\\ 0\end{pmatrix}}_{\boldsymbol{Q}} - \frac{\partial}{\partial y}\underbrace{\begin{pmatrix}\rho v\\ \rho vu\\ \rho v^2 + p\\ F_y^Y\end{pmatrix}}_{\boldsymbol{F}_y}}_{\boldsymbol{Q}^+}$$

Case 3: Meander

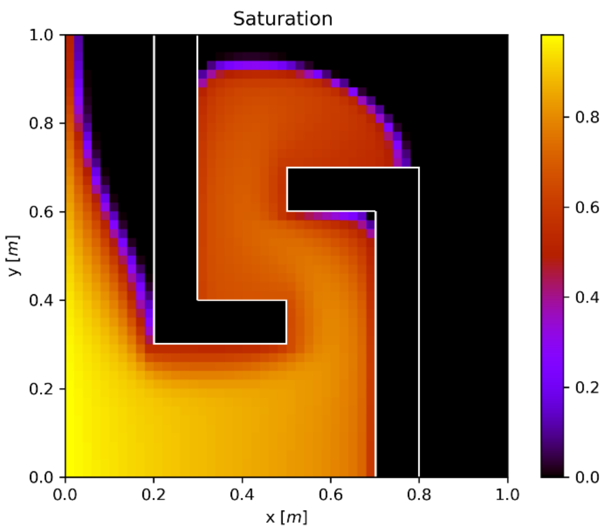
approximate Riemann Solver



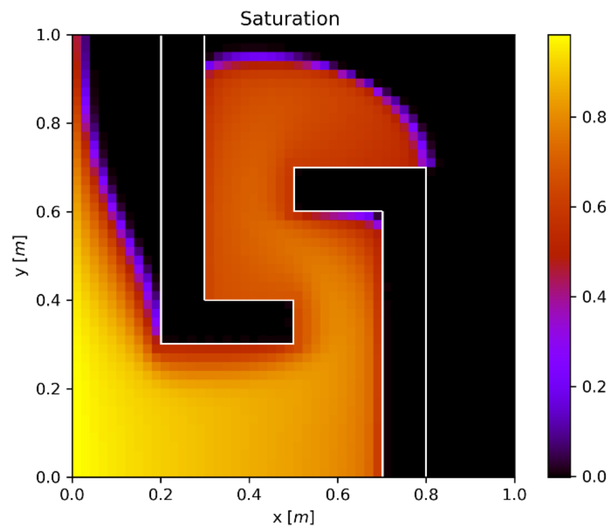
RHR Solver



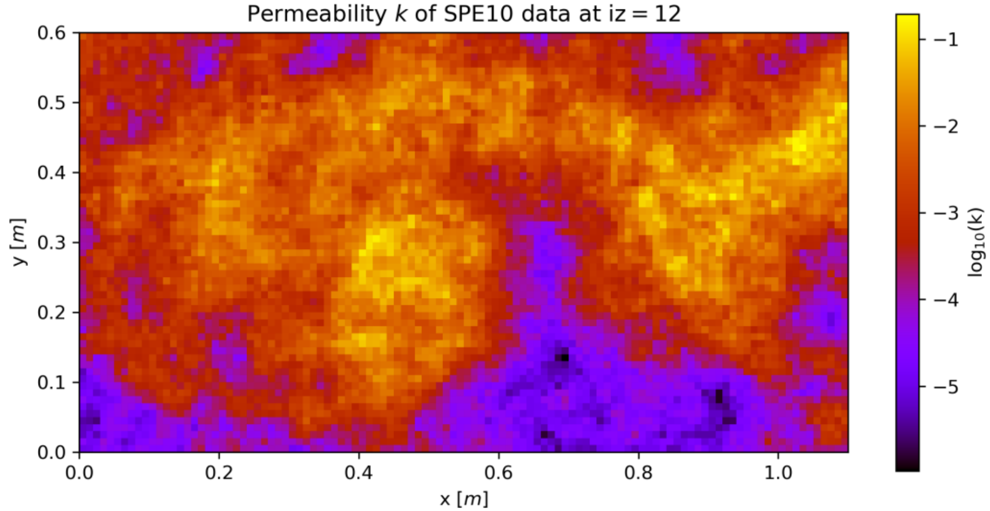
Implicit Method



RHR Solver with additional Source Term

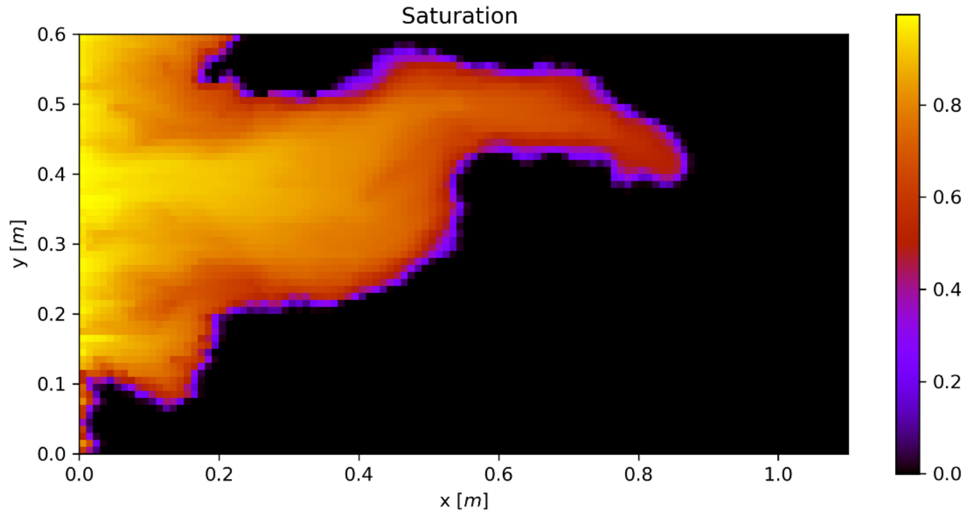


Case 4: SPE10 Top Layer

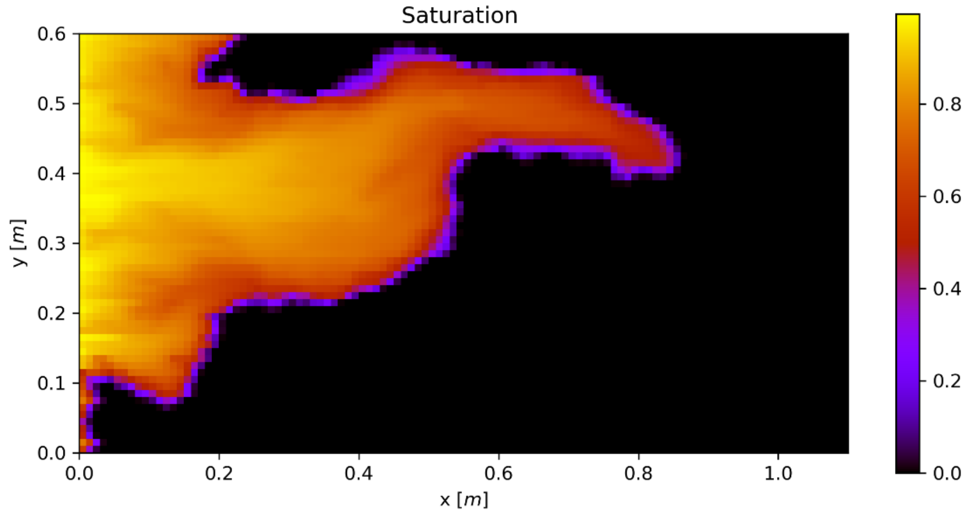


Case 4: SPE10 Top Layer

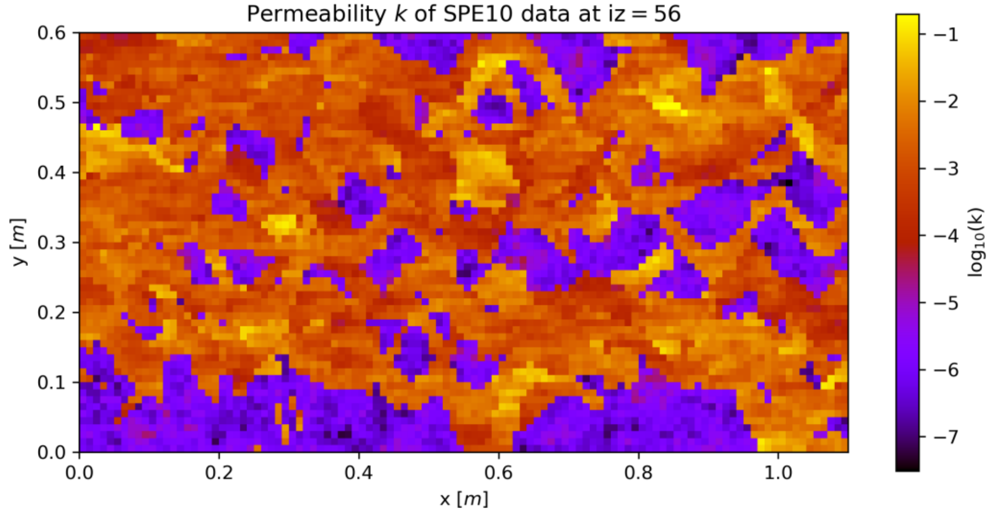
RHR Solver
with additional
Source Term



Implicit
Method

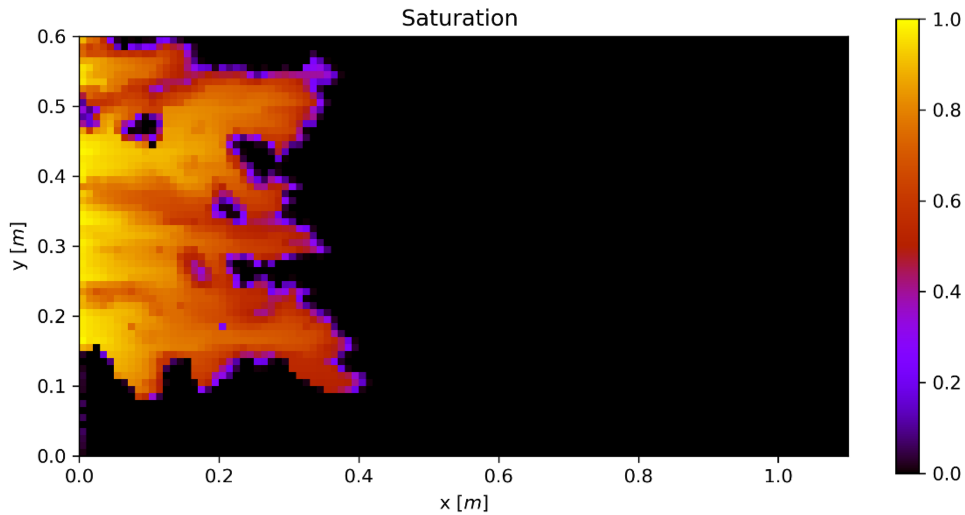


Case 5: SPE10 Bottom Layer

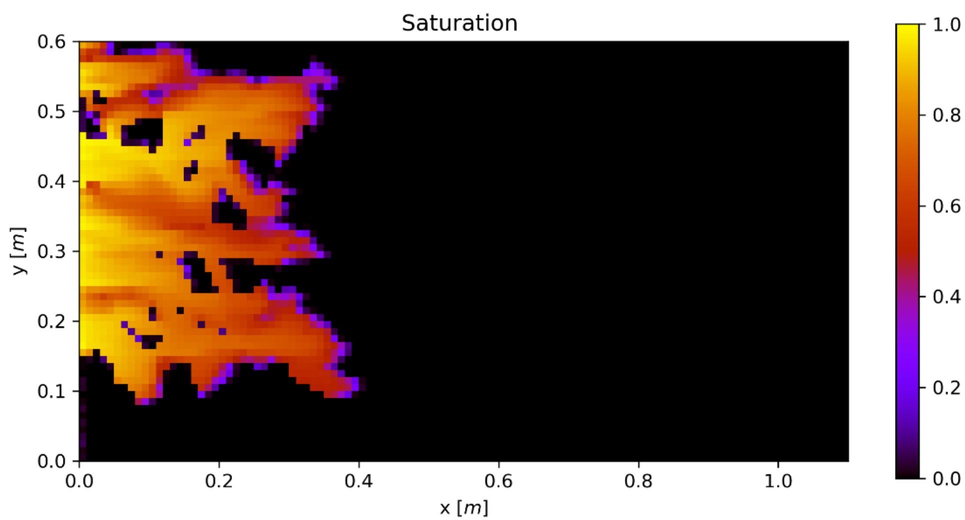


Case 5: SPE10 Bottom Layer

RHR Solver with additional Source Term



Implicit Method



Conclusions

- System of hyperbolic conservation laws describe coupled flow and transport
- Shale layers can be included thanks to RHR solver
- Two-dimensional effects with additional source term
- Proposed solution algorithm is well-suited for GPUs