Discontinuous Galerkin Method for Flow in Enlarged Fractured Carbonates

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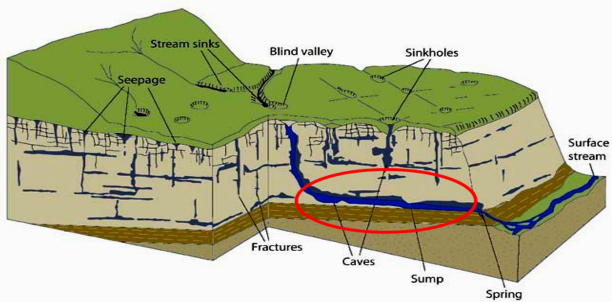
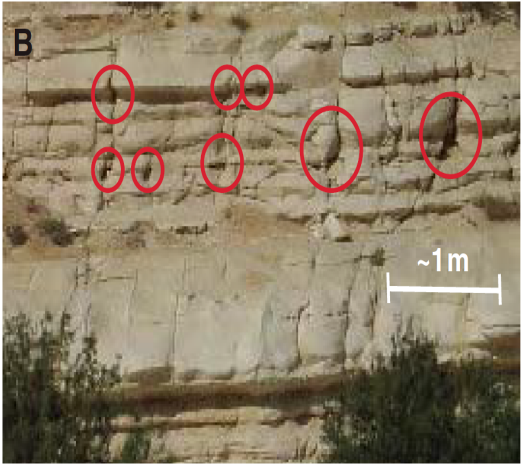
Different Levels of Karstification in Carbonate Rocks

- Vugs, large cavities, sinkholes, enlarged fractures, and conduit networks



Dissolution at Fracture Intersections



Localized aperture enlargement near fracture intersections

Mixed-Dimensional Flow Models 3D/2D/1D

Boon and Nordbotten. (2017), $\omega \in \Omega_n, \; n = d-1, \dots, 0$

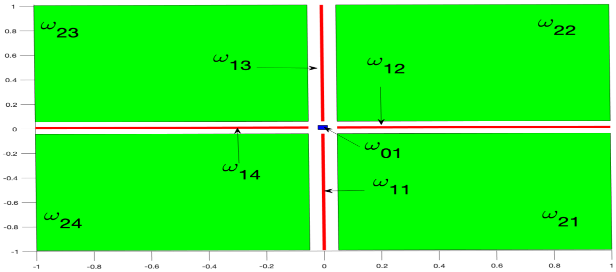
$$-\nabla_{\omega\tau} \cdot (K_{\omega\tau}(x)\nabla_{\omega\tau} p_\omega) = q_\omega - [\boldsymbol{\nu}_{\hat\omega} \cdot K_{\hat\omega\tau}(x)\nabla_{\hat\omega\tau} p_{\hat\omega}]_\omega \text{ in } \omega \in \Omega_n,$$

$$p_\omega = p_D \text{ at } \partial\omega_D,$$

$$-\boldsymbol{\nu}_\omega \cdot K_{\omega\tau}(x)\nabla_{\omega\tau} p_\omega = g_N \quad \text{at} \quad \partial\omega_N,$$

$$\boldsymbol{\nu}_\omega \cdot K_{\omega\tau}(x)\nabla_{\omega\tau} p_\omega = 0 \quad \text{at} \quad \partial\omega_I,$$

$$\boldsymbol{\nu}_{\hat\omega} \cdot K_{\hat\omega\tau}(x)\nabla_{\hat\omega\tau} p_{\hat\omega} = K_{\omega\nu}(x)(p_\omega - p_{\hat\omega}) \text{ on } \omega, \quad \hat\omega \in \hat\partial(\omega),$$

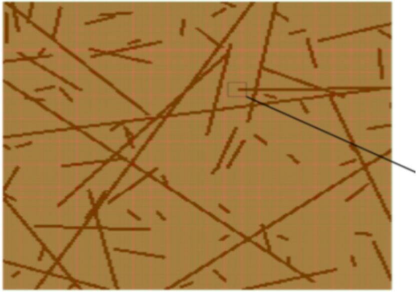


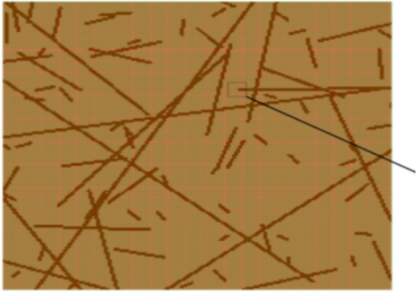
Particular Cases: Discrete Fracture Modeling: DFM

3D/2D Coupled Flow (Martin et al. (2005))

$$\left.\begin{aligned} \nabla\cdot \boldsymbol{u}_i &= 0 \\ \boldsymbol{u}_i &= -\mathbb{K}_i(S_i,\overline{p})(\nabla p_i \end{aligned}\right\} \text{ in } \Omega_i, i = 1,2$$

$$\left.\begin{aligned} \nabla_\tau \cdot \bar{\mathbf{U}}_j + \beta_j [\![p_j]\!] - \beta_{j+1} [\![p_{j+1}]\!] = 0 \\ \bar{\mathbf{U}}_j = -\mathbb{K}_{d,\tau,j} d_j \nabla_\tau \bar{p}_j \end{aligned}\right\} \text{ in } \gamma, j = 1,\cdots,n \qquad (1)$$





Particular Cases: 3D/1D Coupled Flow

Absence of Fractures: Ferraz et al (2021)) Landim et al (2023), Gjerde et al (2020)

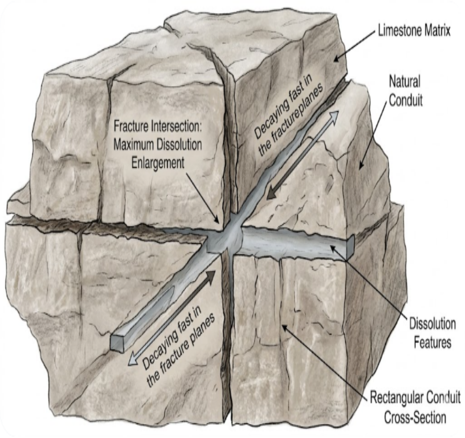
$$\begin{cases} \nabla \cdot \mathbf{v}_M = q_{cm}\delta_\Lambda & \text{in} \quad \Omega_M \\ \mathbf{v}_M = -\dfrac{K_M}{\mu}\nabla p_M & \\ \dfrac{\partial \tilde{v}_c}{\partial s} = -q_c & \text{in} \quad \gamma_c \\ \tilde{v}_c = -\dfrac{k_{cs}}{\mu}\nabla_s \widetilde{p}_c & , \end{cases}$$

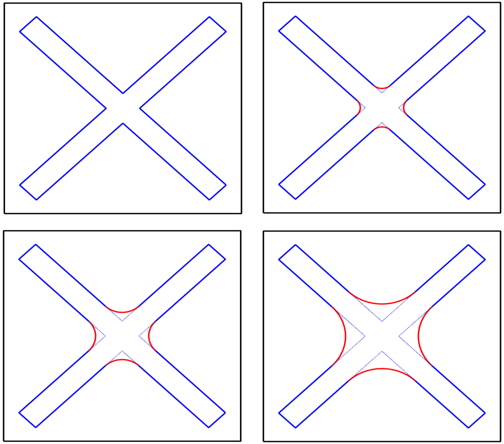
with

$$q_{cm} = \int_C \boldsymbol{v}_c \cdot \boldsymbol{n}_c \, dc, \qquad q_c(s) = \frac{1}{|A_c|}\int_C \boldsymbol{v}_c \cdot \boldsymbol{n}_c \, dc,$$

δ_Λ Dirac – Line source

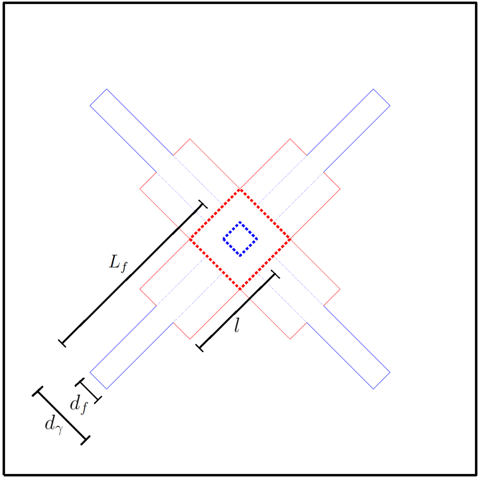
Study of the Influence of Aperture Enlargement at Fracture Intersections





Maximum enlargement at the intersection, rapidly decaying along the fracture planes.

Conduit Cross-Section Enlargement



Two parameter description: $\{d_f/d_{ref}, l/L_f\}$

Discretization 3D/2D/1D Model: SIPDG Method

Given $s_h^m \in V_h^k$, for $m = 0, 1, \dots, N-1$, find $p_h^{m+1} \in V_h^k$ such that, for all $v_h \in V_h^k$,

$$A_p^{DG}(p_h^{m+1}, v_h) = L_p^{DG}(v_h).$$

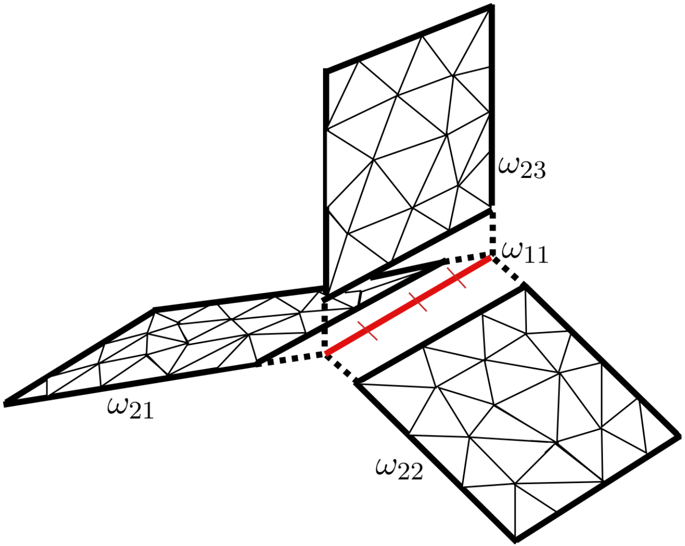
Bilinear form

$$\begin{aligned} A(\mathbf{u}, \mathbf{v}) = &\sum_{n=1}^{d} \sum_{\omega \in \mathbf{\Omega}_n} \int_{\mathcal{T}_h(\omega)} K_{\omega\tau} \nabla_{\omega\tau} u_\omega \cdot \nabla_{\omega\tau} v_\omega \\ &+ \sum_{n=1}^{d-1} \sum_{\omega \in \Omega_n} \Bigg[\int_\omega |\widehat{\partial}(\omega)| K_{\omega\nu} (u_\omega - \{u_{\widehat{\omega}}\}_\omega)(v_\omega - \{v_{\widehat{\omega}}\}_\omega) \\ &\qquad + \int_\omega \frac{K_{\omega\nu}}{|\widehat{\partial}(\omega)|} \sum_{i<j} (u_{\widehat{\omega}_i} - u_{\widehat{\omega}_j})(v_{\widehat{\omega}_i} - v_{\widehat{\omega}_j}) \Bigg]. \end{aligned}$$

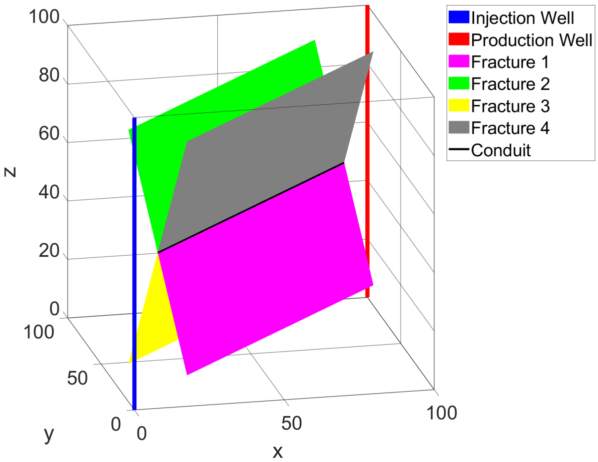
Linear form

$$L(\mathbf{v}) = \sum_{n=1}^{d} \sum_{\omega \in \mathbf{\Omega}_n} \left(\int_\omega q_\omega v_\omega + \int_{\partial\omega_N} g_N v_\omega \right).$$

Independent Computational Meshes



Numerical Example



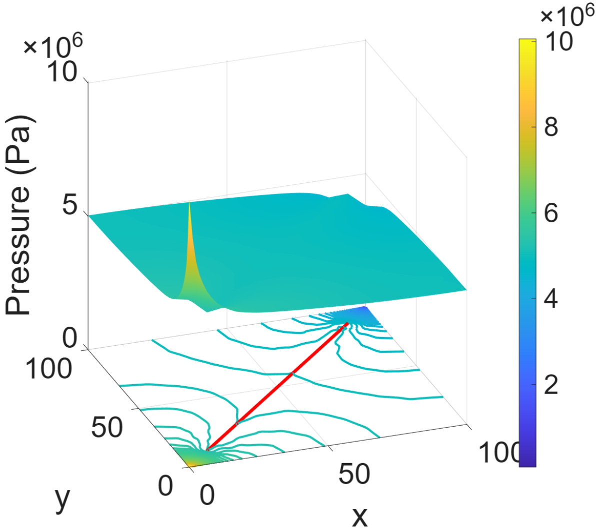
Configuration

- 3D matrix with four embedded 2D fractures
- 1D conduit along the shared fracture-intersection line
- Matrix permeability: 10^{-13} m^2
- Viscosity: 10^{-3} Pa · s
- Roughness parameter: $JRC = 16$

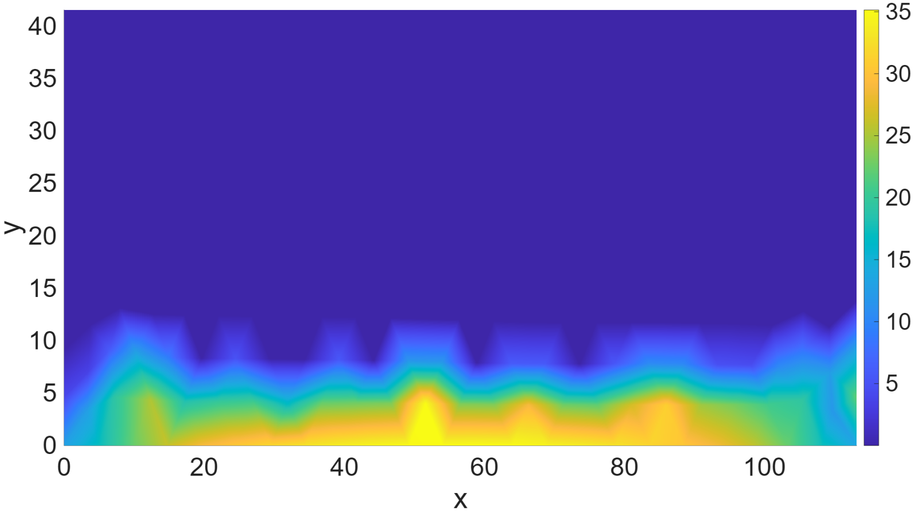
Quantity of interest

Productivity index and production gain versus localized aperture enlargement.

Pressure Field



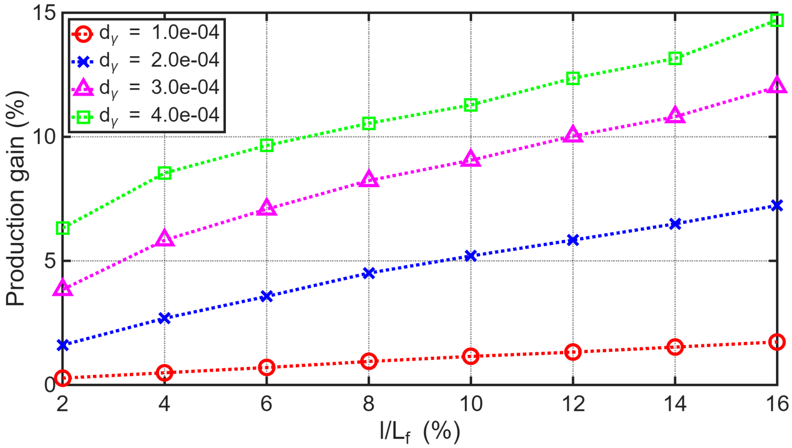
Velocity Norm in the Fracture Plane



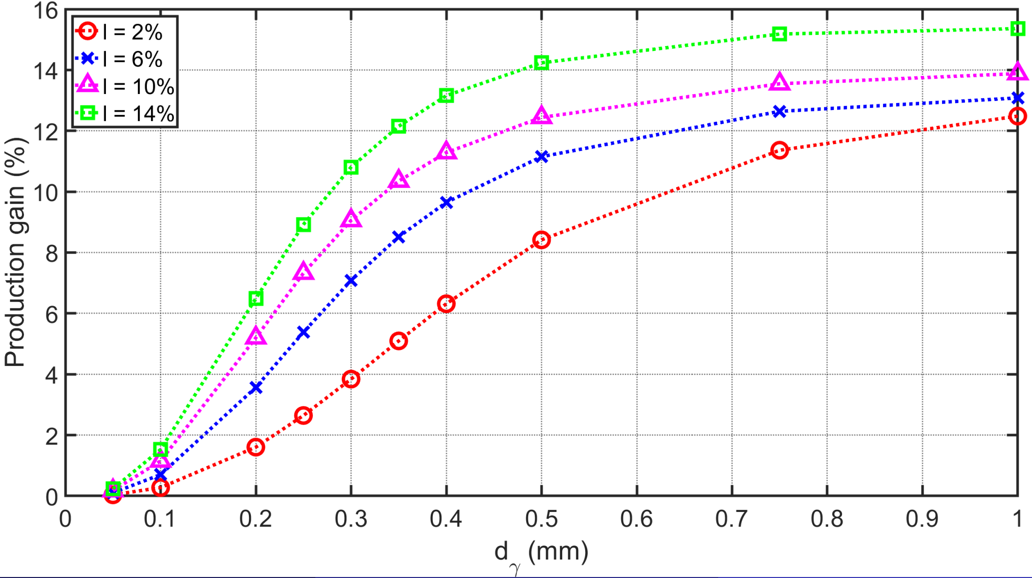
Producer **CONDUIT** Injector

Productivity Index: Effect of Karstified Length

$$PI := \frac{Q}{P_{inj} - P_{prod}}, \qquad d_f^{ref} = 5 \times 10^{-5}\,\text{m}.$$



Productivity Index: Effect of Aperture Enlargement



Takeaway Messages

Key findings

- DG is well suited for coupled 3D/2D/1D flow with independent meshes.
- Local dissolution at fracture intersections creates effective high-conductivity conduits.
- These conduits strongly alter pressure and velocity patterns.
- Productivity gains are mainly controlled by:
  - aperture enlargement ratio, d/d_{ref};
  - karstified fraction of the fracture, l/L_f.