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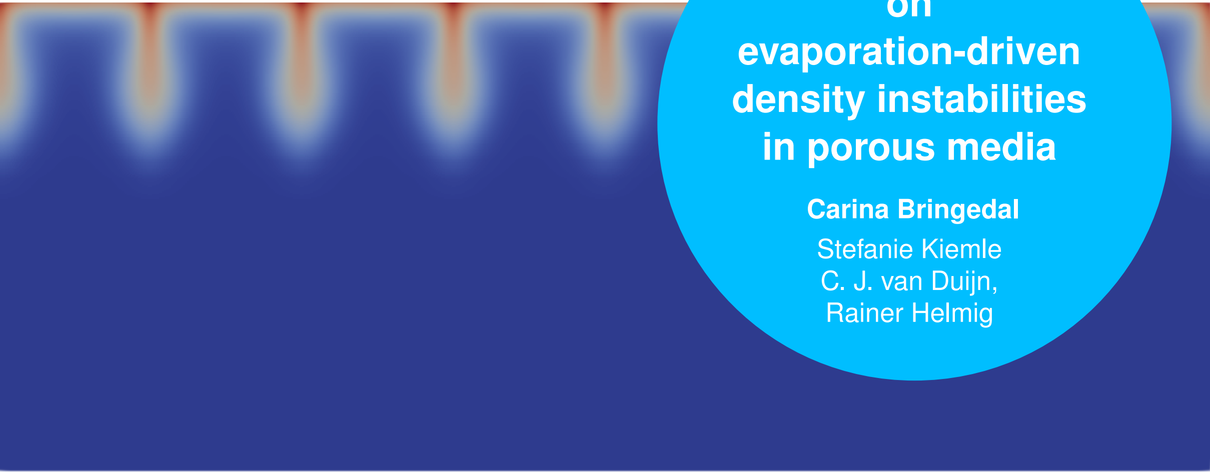

Impact of saturation on evaporation-driven density instabilities in porous media

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C. J. van Duijn,
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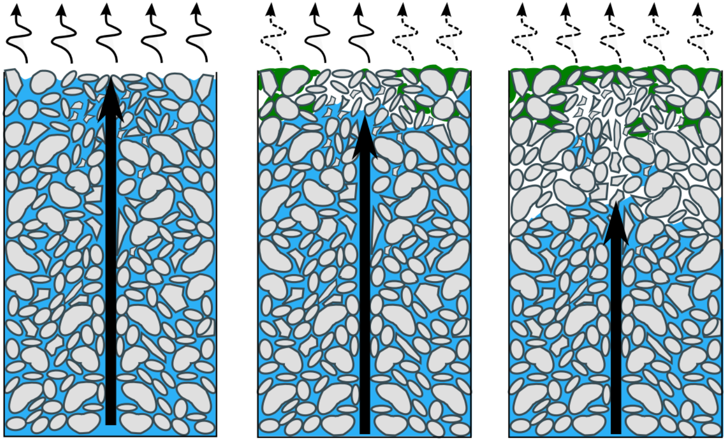
InterPore 19.05.2026

Motivation

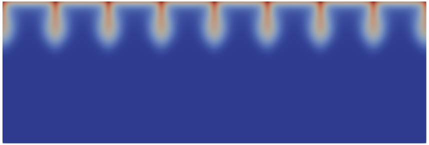
- Evaporation of saline water causes accumulation of salts near the upper boundary.
- Can eventually lead to salt precipitation and soil salinization.



V. A. Jambekhar et al., Transport in Porous Media, 2015, doi:10.1007/s11242-015-0516-7

Motivation

Setting also gravitationally unstable - saltier water also more dense. Can have density instabilities in the form of fingers, giving net downwards transport of salt.



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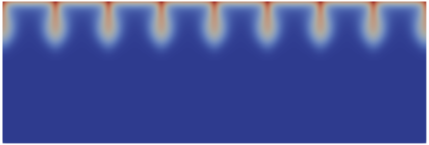


Question 1

When can these density instabilities occur?

Question 2

What is the influence of the water saturation on these instabilities?

C. Bringedal, S. Kiemle, C. J. van Duijn, R. Helmig, Impact of saturation on evaporation-driven density instabilities in porous media: mathematical and numerical analysis, Transport in Porous Media 152, 72 (2025), https://doi.org/10.1007/s11242-025-02207-y

Outline

Model for evaporation from porous media

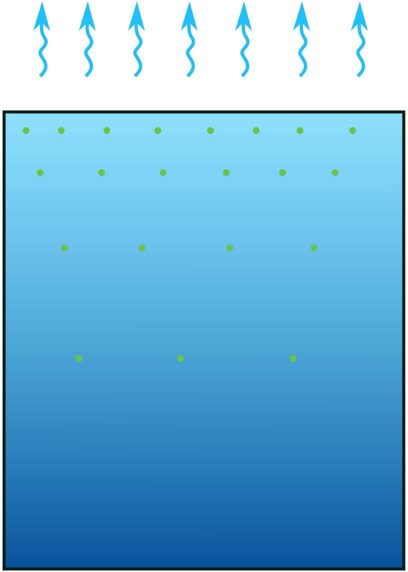
Linear stability analysis

Influence of saturation on density instabilities

Comparison to numerical simulations

Model for evaporation from porous media

- Darcy-scale approach.
- Evaporation from top of porous medium - water flux from top.
- Dissolved salts stay behind when water evaporates; hence, zero salt flux from top.
- Bottom connected to groundwater reservoir of constant (capillary) pressure and salt concentration.



Model for evaporation from porous media

Model equations for evaporation from partially saturated porous media:

$$\phi \partial_t S + \nabla \cdot \mathbf{Q} = 0,$$

$$\mathbf{Q} = \frac{k(S)}{\mu(X)} K(\nabla P_c - \rho(X) g \mathbf{e}_z),$$

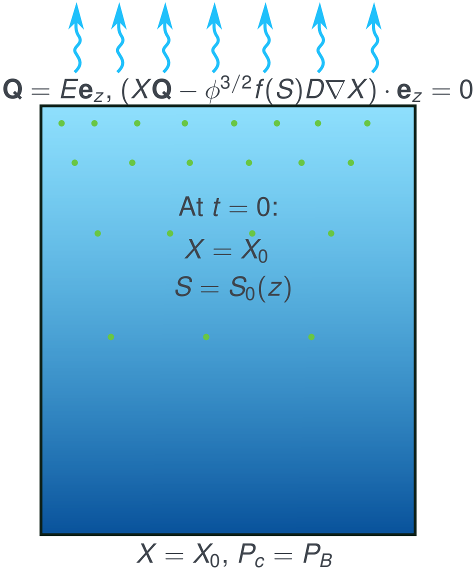
$$\phi \partial_t (SX) + \nabla \cdot (\mathbf{Q}X - \phi^{3/2} f(S) D \nabla X) = 0,$$

where

$$f(S) = S^{7/2}$$

$$k(S) = S^L(1 - (1 - S^{1/m})^m)^2,$$

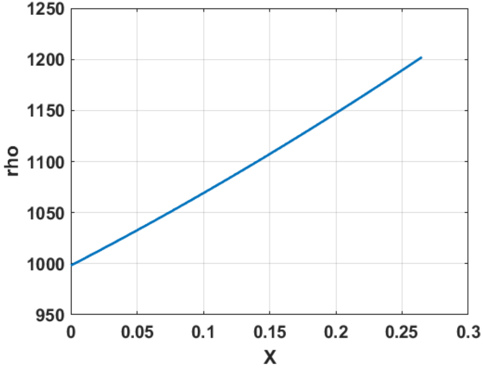
$$P_c(S) = \frac{1}{\alpha}(S^{-1/m} - 1)^{1/n}.$$

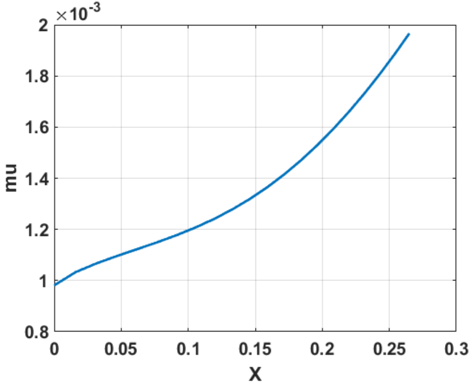


R. Helmig, *Springer*, 2011, doi:10.1007/978-3-642-60763-9
M. T. van Genuchten, *Soil Sci Soc Am J*, 1980, doi:10.2136/sssaj1980. 03615995004400050002x

Model for evaporation from porous media

Use that fluid density ρ and viscosity μ vary with salt mass fraction X:





Empirical relationships from Batzle and Wang (1992).

M. Batzle, Z. Wang, *Geophysics*, 1992, doi:10.1190/1.1443207

Model for evaporation from porous media

Use non-dimensional model equations and Kirchhoff potential as variable:

$$\Psi(\hat{P}_c) := \int_0^{\hat{P}_c} \tilde{k}(\xi)d\xi \text{ where } \tilde{k}(S(\hat{P}_c))$$

Then:

$$\partial_{\hat{t}} S(\Psi) + R_E \hat{\nabla} \cdot \hat{\mathbf{Q}} = 0,$$

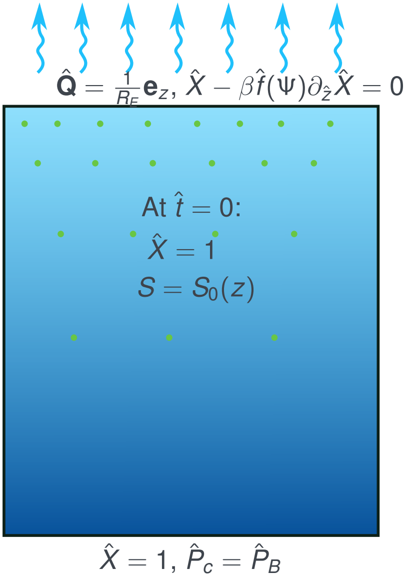
$$\hat{\mathbf{Q}} = \frac{\hat{k}(\Psi)}{\hat{\mu}(\hat{X})}(\hat{\nabla}\hat{P}_c(\Psi) - \hat{\rho}(\hat{X})\mathbf{e}_z),$$

$$\partial_{\hat{t}}(S(\Psi)\hat{X}) + \hat{\nabla} \cdot (R_E \hat{\mathbf{Q}}\hat{X} - \beta \hat{f}(\Psi)\hat{\nabla}\hat{X}) = 0,$$

where

$$R_E = \frac{Q_{\text{ref}}}{E}; \quad Q_{\text{ref}} = \rho_0 g K / \mu_0$$

$$\beta = \frac{\phi^{3/2} D}{EH}$$



Outline

Linear stability analysis

- To analytically find criteria for onset of instabilities, we perform a linear stability analysis:
  - Find a stable, time-dependent solution (‘the ground state’).
  - Perturb this solution and linearize the equations for perturbed quantities
  - Asking whether these perturbed quantities will grow or decay, results in a time-dependent eigenvalue problem, with R_E as eigenvalue.

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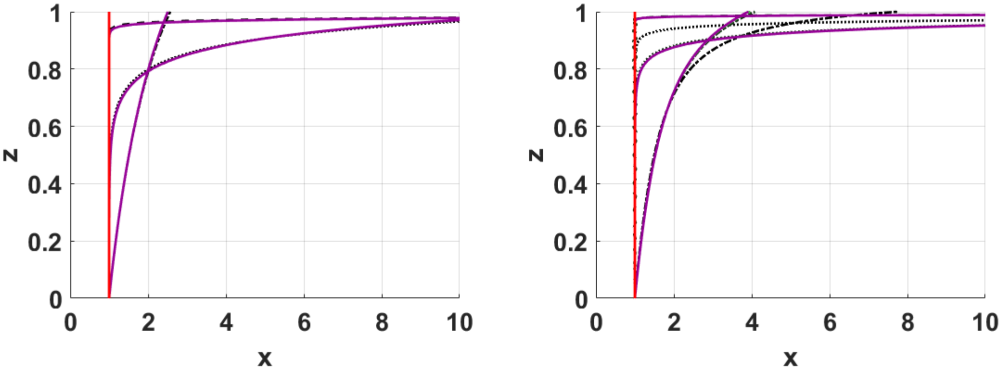
  - Perturb this solution and linearize the equations for perturbed quantities
  - Asking whether these perturbed quantities will grow or decay, results in a time-dependent eigenvalue problem, with R_E as eigenvalue.
- The linear stability analysis provides not only criteria for **whether** instabilities develop, but also **when**, since the eigenvalue problem is time-dependent.
- For given parameters, we can hence determine when instabilities develop.

Linear stability analysis: simplified ground state

Use simplified ground state:

- Fixed saturation (and Kirchhoff potential) corresponding to initial profile S_0.
- Fixed vertical velocity corresponding to evaporation $\hat{W}_\infty = \frac{1}{R_E}$.
- Only solve for salt concentration:

$$S(\Psi_0)\partial_{\hat{t}}\hat{X}_s + \partial_{\hat{z}}(\hat{X}_s - \beta\hat{f}(\Psi_0)\partial_{\hat{z}}\hat{X}_s) = 0, \quad 0 < \hat{z} < 1,$$



Linear stability analysis: eigenvalue problem

We then perturb and linearize the original ground state, and get an eigenvalue problem for perturbed velocity $\hat{w}$ and salt concentration $\hat{\chi}$:

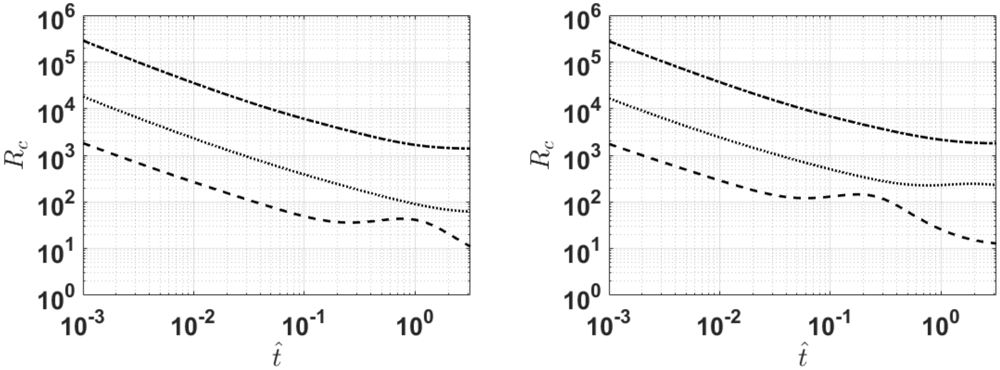
Linear stability analysis: eigenvalue problem

We then perturb and linearize the original ground state, and get an eigenvalue problem for perturbed velocity $\hat{w}$ and salt concentration $\hat{\chi}$:

$$
\widehat{\mathrm{EP}}
\begin{cases}
\text{Find the smallest } R_c(\hat{t}) = \min_{\hat{a}} R_E \text{ such that} & \\
\beta \check{f}(\Psi_0)\chi'' + (\beta \partial_{\hat{z}}(\check{f}(\Psi_0)) - 1)\chi' & \\
+\big(-\beta \hat{a}^2 \check{f}(\Psi_0) + \beta \check{f}'(\Psi_0)\partial_{\hat{z}}\hat{X}_s \big(\frac{\hat{\mu}'(\hat{X}_s)}{\hat{\mu}(\hat{X}_s)} \partial_{\hat{z}}\Psi_0 + \check{k}(\Psi_0)\hat{\rho}'(\hat{X}_s) - \check{k}(\Psi_0)\hat{\rho}(\hat{X}_s)\frac{\hat{\mu}'(\hat{X}_s)}{\hat{\mu}(\hat{X}_s)} \big)\big)\hat{\chi} & \\
= \frac{\hat{\mu}(\hat{X}_s)}{\hat{a}^2} \big(\check{S}'(\Psi_0)\partial_{\hat{t}}\hat{X}_s - \beta\partial_{\hat{z}}(\check{f}'(\Psi_0)\partial_{\hat{z}}\hat{X}_s) - \beta \check{f}'(\Psi_0)\partial_{\hat{z}}\hat{X}_s \check{k}'(\Psi_0)\hat{\rho}(\hat{X}_s)\big)\hat{w}' & \\
+(R_E \partial_{\hat{z}}\hat{X}_s - \beta \check{f}'(\Psi_0)\partial_{\hat{z}}\hat{X}_s \hat{\mu}(\hat{X}_s))\hat{w} & 0 < \hat{z} < 1 \\
\hat{w}'' + \big(\frac{\hat{\mu}'(\hat{X}_s)\partial_{\hat{z}}\hat{X}_s}{\hat{\mu}(\hat{X}_s)} - \check{k}'(\Psi_0)\hat{\rho}(\hat{X}_s)\big)\hat{w}' - \hat{a}^2\hat{w} & \\
= \frac{\hat{a}^2}{\hat{\mu}(\hat{X}_s)} \Big(\frac{\hat{\mu}'(\hat{X}_s)}{\hat{\mu}(\hat{X}_s)} \partial_{\hat{z}}\Psi^0 + \check{k}(\Psi_0)\hat{\rho}'(\hat{X}_s) - \check{k}(\Psi_0)\frac{\hat{\rho}(\hat{X}_s)\hat{\mu}'(\hat{X}_s)}{\hat{\mu}(\hat{X}_s)} \Big)\hat{\chi} & 0 < \hat{z} < 1 \\
\text{where } \hat{w} \text{ and } \hat{\chi} \text{ fulfill} & \\
\hat{w} = 0, \hat{\chi} - \beta \check{f}(\Psi_0)\hat{\chi}' - \beta \check{f}'(\Psi_0)\frac{\hat{\mu}(\hat{X}_s)}{\hat{a}^2}\partial_{\hat{z}}\hat{X}_s \hat{w}' = 0 & \hat{z} = 1 \\
\hat{w}' = 0, \hat{\chi} = 0 & \hat{z} = 0 \\
\text{has a non-trivial solution.} &
\end{cases}
$$

Linear stability analysis: critical Rayleigh number

Critical Rayleigh number as a function of non-dimensional time for various β:



Should only be trusted for large enough R_c (>100).
Also find corresponding critical wavelength $\Rightarrow$ preferred number of fingers at onset.

Outline

Influence of saturation on density instabilities

Onset of instabilities mainly determined by changes in salt profile, but still several competing effects.

$$\phi \partial_t (X S) + \nabla \cdot (\mathbf{Q} X - \phi^{3/2} f(S) D \nabla X) = 0,$$

Influence of saturation on density instabilities

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$$\phi \partial_t (XS) + \nabla \cdot (\mathbf{Q}X - \phi^{3/2} f(S) D \nabla X) = 0,$$

Lowering saturation leads to

- Lower storage term $\implies$ More unstable

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Lowering saturation leads to

- Lower storage term $\implies$ More unstable
- Less flow $\implies$ More stable

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- Less flow $\Longrightarrow$ More stable
- Less diffusion $\Longrightarrow$ More unstable

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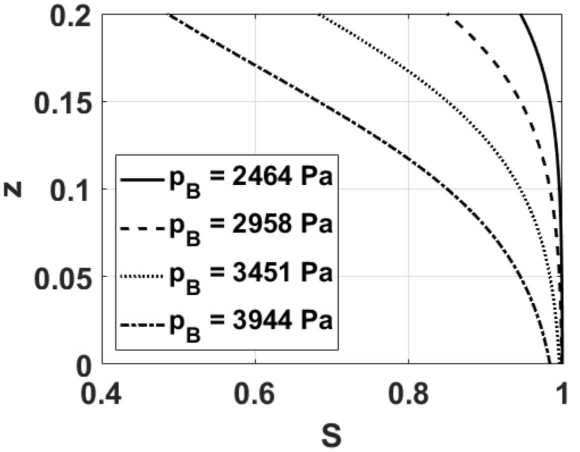
Lowering saturation leads to

- Lower storage term $\implies$ More unstable
- Less flow $\implies$ More stable
- Less diffusion $\implies$ More unstable

Everything at once $\implies$??

Influence of saturation on density instabilities

Vary bottom pressure to get different saturation profiles.



Influence of saturation on density instabilities

Bottom pressure	Constant viscosity		Varying viscosity	
	Onset time	Number of fingers	Onset time	Number of fingers
2464 Pa (base)	11 654 s	1	11 671 s	1
2958 Pa	12 415 s	1	12 452 s	1
3451 Pa	11 053 s	3	11 110 s	3
3944 Pa	7951 s	6	8042 s	6

- $R_E = 23\,984$, $\beta = 0.1823$.
- Very little influence of varying viscosity.
- No clear trend: slightly later onset times for slightly lower saturation, then earlier.
- More fingers (smaller wavelength) when lower saturation

Parameters from M. Bechtold et al, *Geophys Res Lett*, 2011, doi:10.1029/ 2011GL048147

Outline

Model for evaporation from porous media

Linear stability analysis

Influence of saturation on density instabilities

Comparison to numerical simulations

Comparison to numerical simulations

We solve the dimensional model equations:

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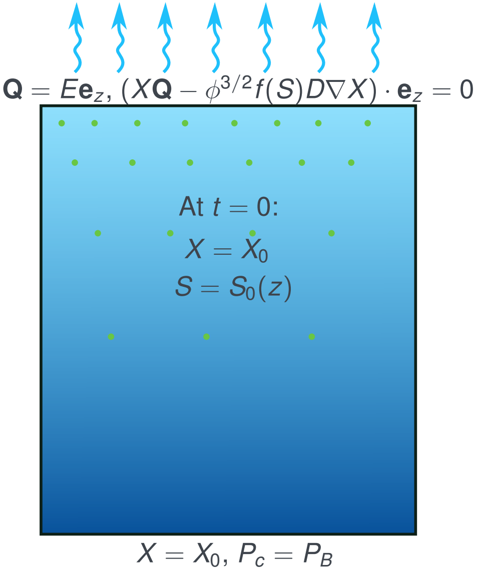
$$f(S) = S^{7/2}$$

$$k(S) = S^L (1 - (1 - S^{1/m})^m)^2,$$

$$P_c(S) = \frac{1}{\alpha} (S^{-1/m} - 1)^{1/n}.$$

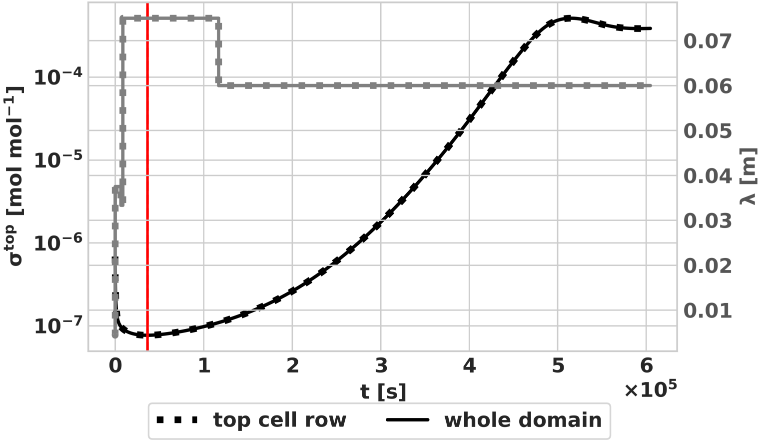
Solved using finite volumes and Euler backward in DuMu^X.



T. Koch et al., *Comput Math Appl*, 2021, doi:/10.1016/j.camwa.2020.02.012

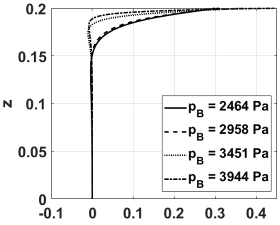
Comparison to numerical simulations

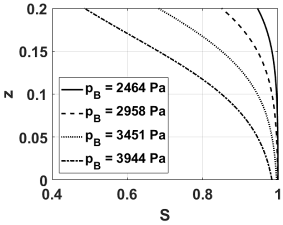
Numerical simulations ‘prefer’ shorter wavelengths (more fingers) than linear stability analysis.



Comparison to numerical simulations

For the four different bottom pressures and the four dominant wavelengths found by numerical simulations:

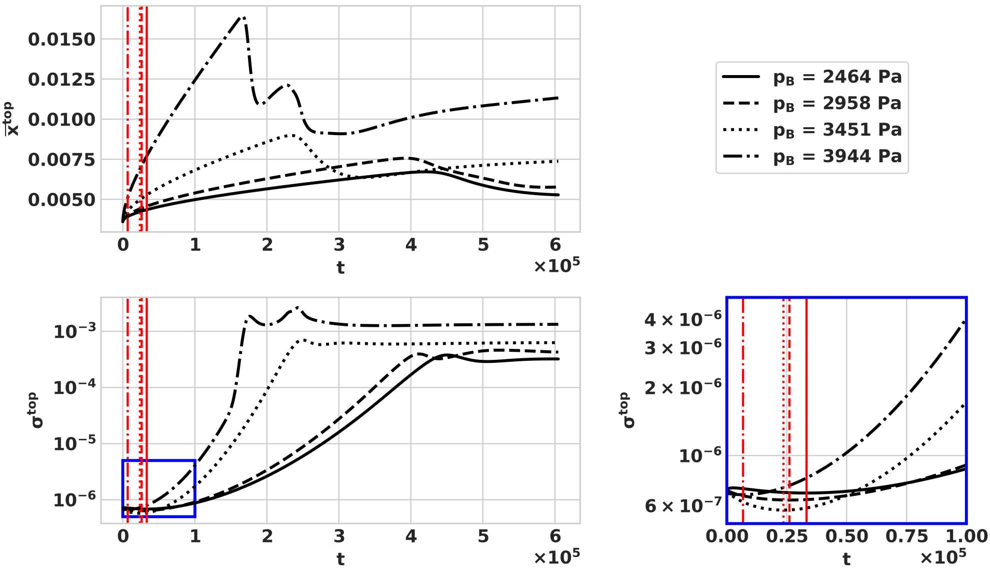




Bottom pressure	*N*	Numerical simulation		Linear stability analysis
		Top row	Eigenprofile	
2464 Pa (base)	8	73 050 s	33 250 s	22 693 s
2958 Pa	8	65 400 s	26 150 s	19 317 s
3451 Pa	18	38 500 s	23 650 s	20 899 s
3944 Pa	16	27 400 s	6 750 s	10 100 s

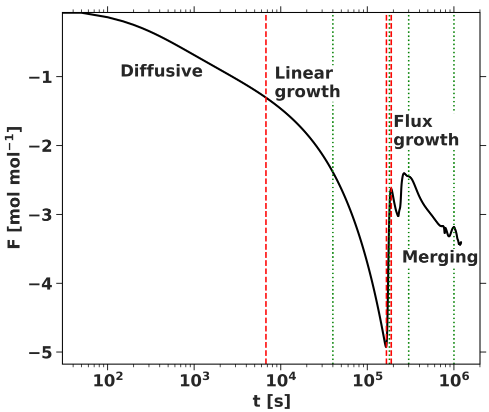
Overall - earlier onset times when saturation much reduced.

Further development of instabilities

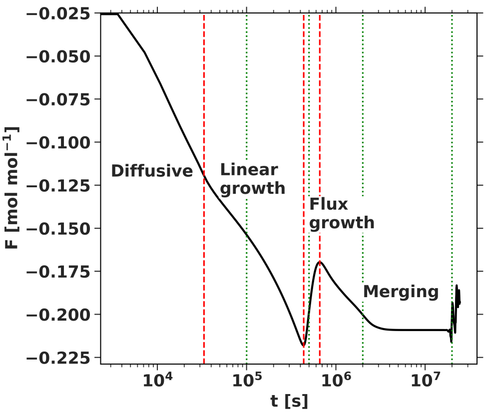


Further development of instabilities

Using dissolution flux at top to estimate strength of downwards salt transport and the different regimes as defined by Slim (2014):



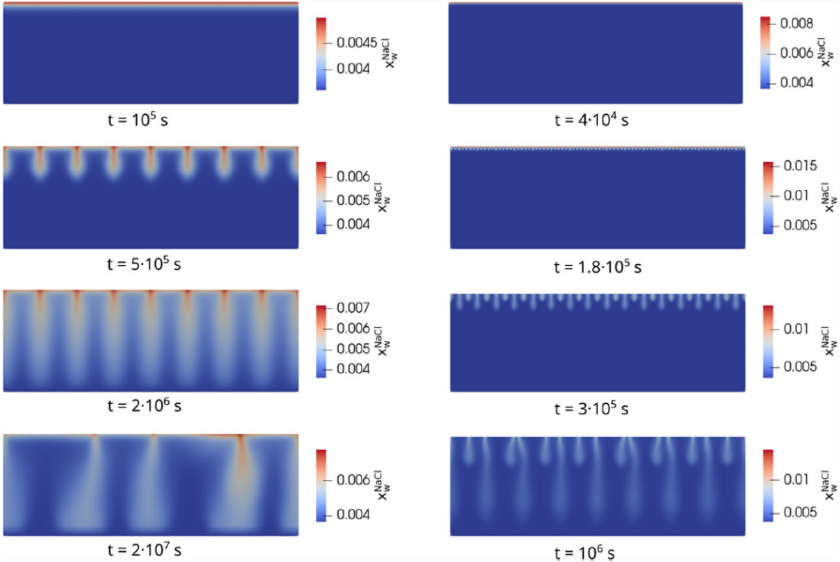
High saturation



Low saturation

A. C. Slim, *J Fluid Mech*, 2014, doi:10.1017/jfm.2013.673

Further development of instabilities



High saturation

Low saturation

Summary

Question 1: When can density instabilities occur?

Depends on size of evaporation rate, permeability, saturation profile etc., but can ‘easily’ be found by linear stability analysis (and further analyzed by numerical simulations).

Question 2: What is the influence of the saturation on these instabilities?

Several competing effects, but find overall that lower saturation gives earlier onset times.

- C. Bringedal, S. Kiemle, C. J. van Duijn, R. Helmig, Impact of saturation on evaporation-driven density instabilities in porous media: mathematical and numerical analysis, *Transport in Porous Media* 152, 72 (2025), doi:10.1007/s11242-025-02207-y
- Linear stability solver: doi:10.18419/darus-4711
- DuMu^x solver: doi:10.18419/darus-4610

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Thank you for your attention!

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Parameter choices for base case

Symbol	Value	Dimension
ϕ	0.41	-
K	$2.89 \cdot 10^{-11}$	m^2
D_w^{NaCl}	$1.5 \cdot 10^{-9}$	$m^2\ s^{-1}$
n	10.8	-
m	0.907	-
S_{wr}	0.122	-
L	0.73	-
p_B	2464	$kg\ m^{-1}\ s^{-2}$
α	$1.77 \cdot 10^{-4}$	$(kg\ m^{-1}\ s^{-2})^{-1}$
E	$1.08 \cdot 10^{-8}$	$m\ s^{-1}$
H	0.2	m
ℓ	0.6	m
g	9.8	$m\ s^{-2}$
$x_w^{NaCl}\vert_{t=0}$	0.0036	$mol\ mol^{-1}$

$$
\begin{aligned}
p_{\mathrm{ref}} &= H\rho_{\min} g, \\
Q_{\mathrm{ref}} &= K\rho_{\min} g/\mu_{\mathrm{ref}}, \\
t_{\mathrm{ref}} &= \phi H/E, \\
R_E &= \frac{Q_{\mathrm{ref}}}{E}, \\
\beta &= \frac{\phi^{3/2} D}{EH}, \\
\gamma &= \alpha p_{\mathrm{ref}}
\end{aligned}
$$

Symbol	Value	Dimension
p_{ref}	1972	Pa
Q_{ref}	$2.59 \cdot 10^{-4}$	$m\ s^{-1}$
t_{ref}	$7.59 \cdot 10^{6}$	s
R_E	23 984	-
β	0.1823	-
γ	0.349	-

Linear stability analysis: ground state

For partially saturated porous medium, problem highly coupled. Ground state only depends on $\hat{z}$ and $\hat{t}$:

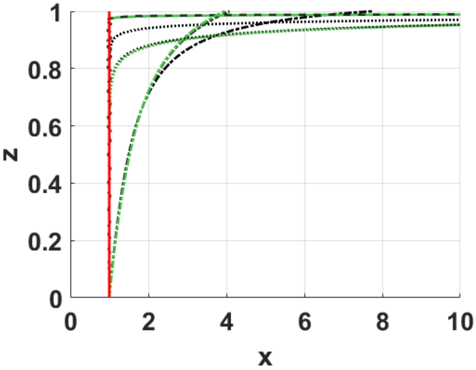
$$
\begin{aligned}
\partial_{\hat{t}}(S(\Psi_s)) + R_E \partial_{\hat{z}} \hat{W}_s &= 0, \quad 0 < \hat{z} < 1, \\
\partial_{\hat{t}}(S(\Psi_s)\hat{X}_s) + \partial_{\hat{z}}(R_E \hat{W}_s \hat{X}_s - \beta \hat{f}(\Psi_s)\partial_{\hat{z}} \hat{X}_s) &= 0, \quad 0 < \hat{z} < 1, \\
\hat{W}_s = \frac{\hat{k}(\Psi_s)}{\hat{\mu}(\hat{X}_s)}(\partial_{\hat{z}} \hat{P}_c(\Psi_s) - \hat{\rho}(\hat{X}_s)), & \quad 0 < \hat{z} < 1,
\end{aligned}
$$

& initial and boundary conditions.

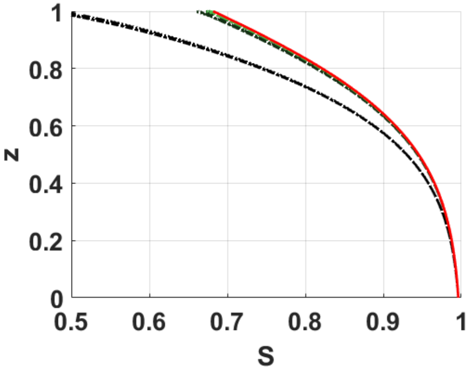
- Salt concentration, saturation and vertical velocity all develop with time.
- No explicit solution available, need to solve numerically.
- Problem: Ground state depends on R_E in not obvious way.

Linear stability analysis: ground state

- Very little influence of R_E in ground state behavior.
- Saturation and vertical velocity change marginally with time, except for low values of R_E.



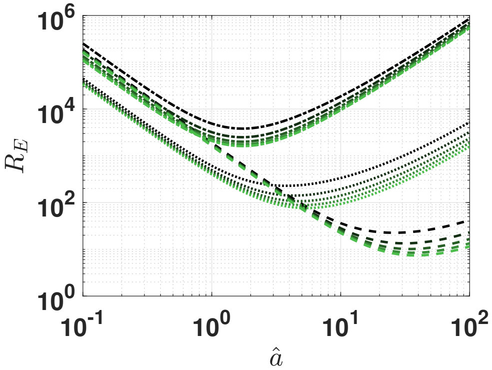
Salt concentration

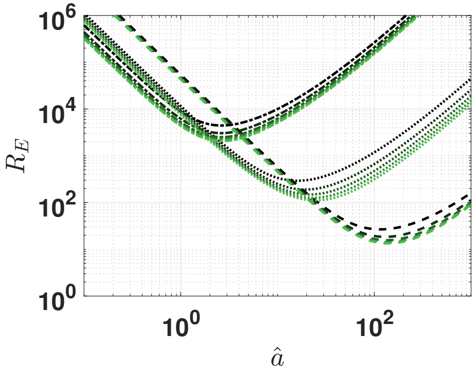


Saturation

Linear stability analysis: critical Rayleigh number

Rayleigh number as a function of wavenumber $\hat{a}$, for various time $\hat{t}$ and β:





Can find $R_c(\hat{t}) = \min_{\hat{a}} R_E$ and corresponding critical wavelength.

Influence of saturation on density instabilities

Change scaling of one term at the time:

$$\phi\partial_t(XS) + \nabla\cdot(\mathbf{Q}X - \phi^{3/2}f(S)D\nabla X) = 0,$$

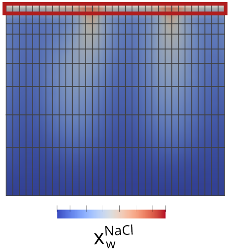
Case number	Constant viscosity		Varying viscosity	
	Onset time	Number of fingers	Onset time	Number of fingers
Base	11 654 s	1	11 671 s	1
Case 1	10 489 s	1	10 504 s	1
Case 2	8414 s	1	8426 s	1
Case 3	5998 s	1	6007 s	1
Case 4	17 804 s	1	17 850 s	1
Case 5	43 081 s	1	43 294 s	1
Case 6	193 211 s	1	195 669 s	1
Case 7	7317 s	1	7332 s	1
Case 8	3098 s	1	3105 s	1
Case 9	856 s	3	858 s	3

- R_E in range 1950 - 23 984.
- β in range 0.0177 - 0.1823.

Further development of instabilities

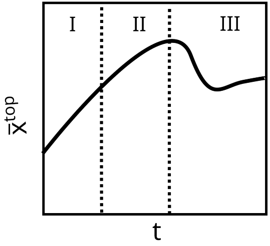
Numerical simulations can also address the development of salt concentration and instabilities after onset:

Numerical simulations



Post-Processing

Step 1: Mean salt concentration of top cell row



Step 2: Standard deviation of mean salt concentration