Modeling the optimal foam injection slug in porous medium accounting adsorption effects

Grigori Chapiro

In collaboration with G. C. Fritis, P. S. Z. Paz, L. F. Lozano, and P. Bedrikovetski.

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Universidade Federal de Juiz de Fora

Federal University of Juiz de Fora, Brazil

Summary

Motivation

Applications of foam in porous media

Gas injection versatility

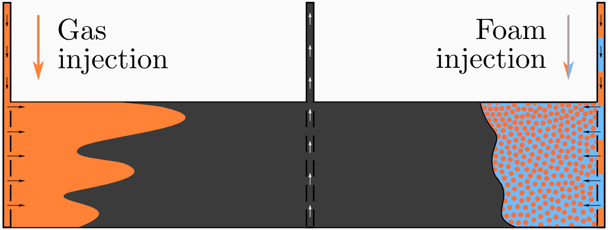
- Soil/aquifer remediation.
- EOR.
- CO_2 storage.
- Others.

Foam helps

- Increasing gas viscosity
- Reducing gravitational segregation
- Increase sweep efficiency

Gas injection challenges

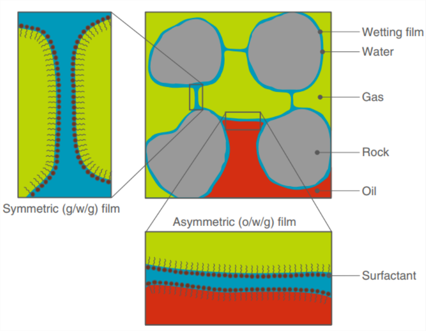
- Gravitational segregation
- Early gas breakthrough
- Low sweep efficiency



Cedro and Chapiro (2024), **Geoenergy Science and Engineering**

Foam in porous media and Surfactant adsorption

- Foam is a continuous aqueous phase and discontinuous gaseous phase
- Lamellae (liquid films) separate gas bubbles.
- Surfactant reduces interfacial tension.
- Adsorption can destabilize the foam.
- Loss of surfactant can make the injection economically infeasible.



Farajzadeh, et al. (2012), **Advances in Colloid and Interface Science**

Common strategies for foam injection

- Foam injection
- Gas/surfactant co-injection
- Surfactant Alternating Gas (SAG/FAWAG)

Advantages SAG over other injection procedures

- Decreasing facilities corrosion
- Improving injectivity
- In-situ foam formation

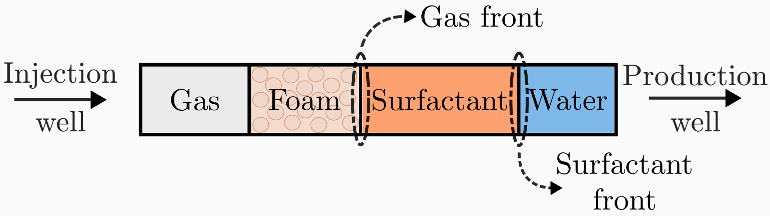


Figure 1: Schematic sequential injection.

Fritis et al., (2025)
International Journal of Non-Linear Mechanics

Mathematical Modeling

CMG/STARS model for foam flow with linear (Henry) adsorption

The surfactant linear adsorption can be described as

$$\begin{aligned} \partial_t S + \partial_x f(S, C) &= 0, \\ \partial_t \left[(S + \mathcal{A}) C \right] + \partial_x \left[f(S, C) C \right] &= 0, \end{aligned}$$

where the constant $\mathcal{A}$ represents the adsorption and is given by

$$\mathcal{A} = \frac{1}{1 - S_{wc} - S_{gr}} \left(S_{wc} + \frac{(1-\phi)\rho_s K_d^a}{\rho_w \phi} \right).$$

The gas's relative permeability in the presence of foam is given by

$$k_{rg}^f(S, C) = k_{rg}(S) / MRF(S, C)$$

MRF is the mobility reduction factor

$$MRF(S, C) = (1 + f_{mmob} F_1(C) \cdot F_2(S)).$$

$\mathbf{F_2}$ represents the water effects

$$F_2(S) = \frac{1}{2} + \frac{\arctan\left(epdry(SS_{ef} + S_{wc} - fmdry) \right)}{\pi},$$

where $S_{ef} = 1 - S_{wc} - S_{gr}$.

$\mathbf{F_1}$ describes the surfactant effects

$$F_1(C) = \begin{cases} \left(\dfrac{C_{max}\ C}{fmsurf} \right)^{epsurf}, & \text{if } C_{max}\ C < fmsurf, \\ 1, & \text{if } C_{max}\ C \geq fmsurf. \end{cases}$$

Computer Modeling Group (CMG) (2019) **CMG Ltd Calgary**

The gas-water flow in porous media with a chemical tracer and foam can be described as

$$\begin{aligned} \partial_t S + \partial_x f(S,C) &= 0, \\ \partial_t \left[(S+\mathcal{A})C\right] + \partial_x \left[f(S,C)C\right] &= 0. \end{aligned}$$

Assumptions:

- Fully saturated porous medium;
- Incompressible phases;
- Newtonian foam flow at local equilibrium.

We investigate the Riemann problem

$$(S(x,0),C(x,0)) = \begin{cases} (S_L, C_L), & \text{if } x < 0, \\ (S_R, C_R), & \text{if } x \geq 0, \end{cases}$$

Smoller (1994) **Springer-Verlag New York**

Isaacson (1989) **Rockefeller University**

Johansen and Winther (1988) **SIAM Journal on Mathematical Analysis**

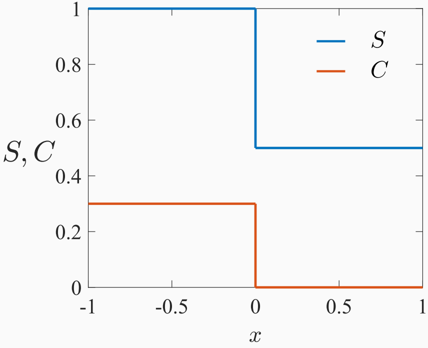


Figure 2: Schematic Riemann problem.

Analytical solution for linear adsorption using CMG-STARS model

The general Riemann problem solution is composed of constant states separated by waves

$$U_L \xrightarrow{W_1} U_1 \dots U_n \xrightarrow{W_n} U_R,$$

The waves can be

- Continuous - Rarefactions;
- Discontinuous - Shocks and contacts.

Our solution is composed of at most four states, connected by

- S-waves: keep C constant and follows Buckley-Leverett equation;
- C-waves: contacts that change C.

We classify the phase plane and describe every Riemann's problem solution according to left and right states (U_L and U_R).

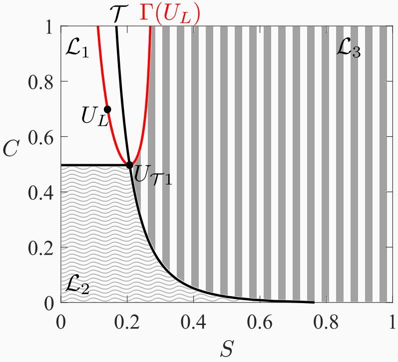


Figure 3: Phase plane classification for a fixed U_L.

Optimal surfactant slug

- Due to the high cost, we want to minimize the injected amount of surfactant.
- Chemical slug is usually defined as the injected amount of chemical.
- To create foam, we must inject gas after the surfactant slug.
- We consider the optimal slug as the smallest amount of surfactant that increases the time of breakthrough of gas, increasing water production.
- To obtain the optimal slug, we investigate
  - Slug injection time.
  - Surfactant concentration.

Ribeiro and Pires (2008) **SPE Annual Technical Conference and Exhibition**

Bakharev et al. (2020) **Journal of Computational and Applied Mathematics**

Hamid and Muggeridge (2018) **Computational Geosciences**

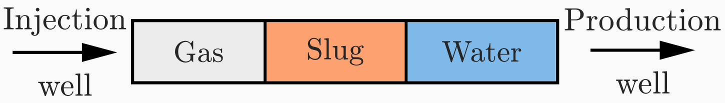


Figure 4: Schematic representation of a chemical slug followed by gas injection.

Investigating two sequences of Riemann problems

RP_2 is the gas injection, and the gas front velocity is given by

$$v_g(C) = \frac{f(U_R^2) - f(U_N^2)}{1 - S_N^2}.$$

RP_1 is the surfactant injection, and the constant surfactant front velocity is given by

$$v_C = \frac{f(U_R^1)}{S_R^l + \mathcal{A}} = \frac{1}{1 + \mathcal{A}}.$$

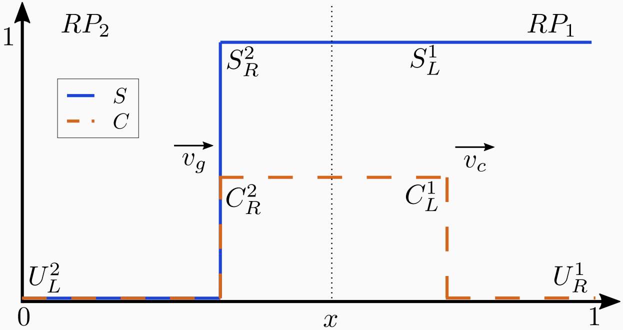


Figure 5: Schematic representation of a chemical slug.

Fritis et al., (2025) **International Journal of Non-Linear Mechanics**

Surfactant injection time

- The spatial variable x is normalized, so the producing well is in $x = 1$.
- We define the surfactant injection time as

$$T_C(C) = Tbt_C - Tbt_g(C) = \frac{1}{v_C} - \frac{1}{v_g(C)}.$$

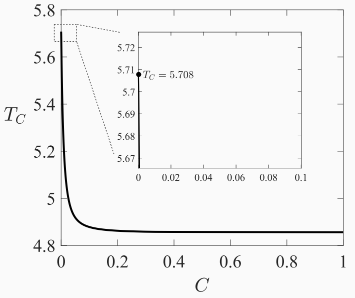


Figure 7: Surfactant injection time as a function of C.

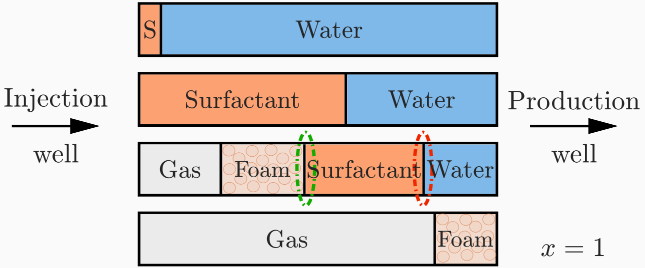


Figure 6: Schematic representation of the porous media after the chemical slug and gas injection.

- T_C represents the shortest time we need to inject the surfactant to guarantee that the gas does not break the surfactant front.

- The volume of injected surfactant is given by

$$Vol_C = T_C(S + \mathcal{A})C$$

- Our goal is to minimize the surfactant volume and maximize the gas breakthrough.
- We obtain a Pareto front.

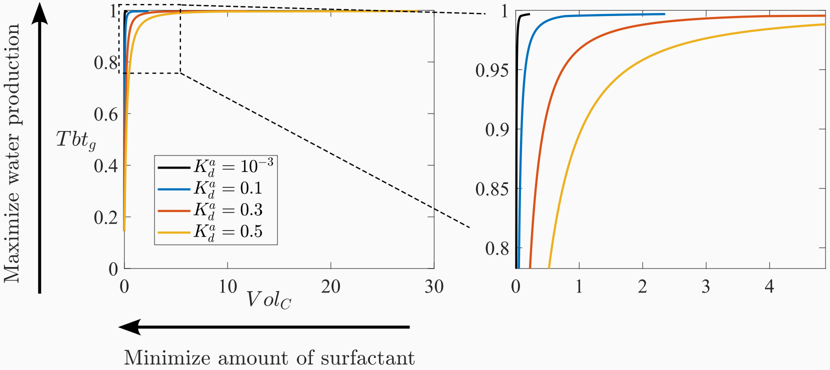


Figure 8: Time of breakthrough of gas as a function of the amount of surfactant. The left panel possesses arrows indicating the optimization objectives. The right panel is a zoom of the left panel in the dashed black square.

Extending the model

↬ Applications need an extension of the results.

Previous work (Fritis et al., 2025)

- Sequential injection with **non-interacting Riemann problems**.
- Assumed **Newtonian foam mobility**.
- Linear adsorption.
- Analytical solutions provided **optimal slug design** minimizing surfactant usage.
- Pareto analysis confirmed **economically feasible surfactant concentrations** below typical lab setups.

Generalization required for applications

- **Wave interaction** between gas and surfactant injections is now considered.
- Foam mobility now includes **non-Newtonian effects** for the CMG-STARS model.
- Analytical evolution of surfactant front derived, including **asymptotic position**.
- Applicable to both **linear and concave adsorption isotherms**.

Gas/water flow in porous media with surfactant adsorption

$$\begin{aligned} \partial_t S + \partial_x f &= 0, \\ \partial_t \left[S(C + \alpha) + A(C) \right] + \partial_x \left[f \ C \right] &= 0. \end{aligned}$$

$$S = \frac{S_w - S_{wc}}{1 - S_{wc} - S_{gr}}, \quad C = \frac{C_w}{C_{max}}$$

$$\begin{aligned} \alpha &= \frac{S_{wc}}{1 - S_{wc} - S_{gr}}, \\ A(C) &= \frac{1}{1 - S_{wc} - S_{gr}} \cdot \frac{\rho_s \cdot (1 - \phi)}{\phi} \cdot \frac{a(C)}{C_{max}}. \end{aligned}$$

We investigate the Riemann problem

$$(S(x,0), C(x,0)) = \begin{cases} (S_L, C_L), & \text{if } x < 0, \\ (S_R, C_R), & \text{if } x \geq 0, \end{cases}$$

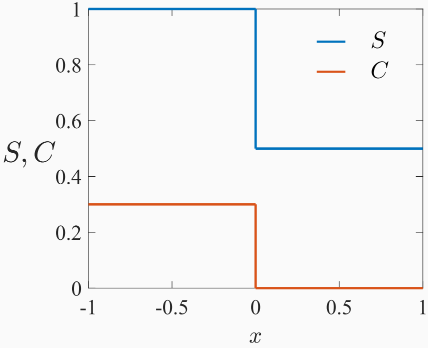


Figure 9: Initial condition Riemann problem.

CMG-STARS foam model with non-Newtonian effects

The mobility reduction factor is

$$FM(S_w, C_w, u) = \frac{1}{1 + fmmob\ F_{dry}(S_w)\ F_{surf}(C_w)\ F_{cap}(u)},$$

$F_{dry}(S_w)$ describes the water effects

$$F_{dry}(S_w) = \frac{1}{2} + \frac{\arctan\left(epdry(S_w - fmdry)\right)}{\pi},$$

$F_{surf}(C_w)$ describes the surfactant effects

$$F_{surf}(C_w) = \begin{cases} \left(\frac{C_w}{fmsurf}\right)^{epsurf}, & \text{if } C_w < fmsurf, \\ 1, & \text{if } C_w \geq fmsurf, \end{cases}$$

Zeng et al., (2016) **Industrial & Engineering Chemistry Research**

F_{cap} describes the non-Newtonian effects

$$F_{cap}(u) = \begin{cases} 1, & N_{ca}(u) < fmcap, \\ \left(\frac{fmcap}{N_{ca}(u)}\right)^{epcap}, & N_{ca}(u) > fmcap, \end{cases}$$

In this work, we adopt the simplified capillary number

$$N_{ca} = \frac{\mu_w\ u}{\sigma}. \qquad (1)$$

Ribeiro et al., (2025) **Water Resources Research**

We investigate the sequence of two Riemann problems

$$(S(x,T^J),C(x,T^J)) = \begin{cases} (0,0), & \text{if } x < 0, \\ (1,C^J), & \text{if } x \geq 0, \end{cases} \qquad (S(x,0),C(x,0)) = \begin{cases} (1,C^J), & \text{if } x < 0, \\ (1,0), & \text{if } x \geq 0, \end{cases}$$

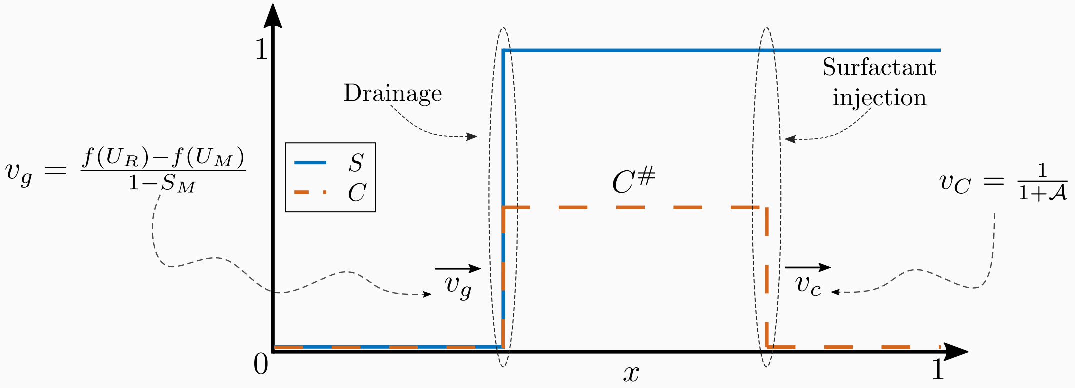


Figure 10: Schematic representation of the CO_2 and surfactant slug injection.

Solution for gas and surfactant injection

We adopt both Linear and Concave adsorption isotherms.

Gas injection

$U_L = (0,0)$ and $U_R = (1, C^J)$.

The solution is a C-wave with zero velocity, followed by a BL-type solution

$$v_g(C^J) = \frac{1 - f(U_M)}{1 - S_M}$$

Surfactant injection

$U_L = (1, C^J)$ and $U_R = (1,0)$.

The solution is a discontinuous C-wave with velocity

$$v_C^\delta(C^J) = \frac{1}{1 + \alpha + \delta(C^J)},$$

$$\delta(C^J) = \begin{cases} dA, & \text{linear adsorption model,} \\ h(C^J, 0), & \text{concave adsorption model.} \end{cases}$$

Riemann problems interaction

Characteristic intersection

- $X(t)$ denotes the surfactant front
- (x', t') is the intersection point
- Left and right state of surfactant jump:

$$U^-(t) = (S^-(t), C^J), \quad U^+(t) = (S^+(t), 0)$$

- Velocity of surfactant discontinuity:

$$\frac{dX(t)}{dt} = \frac{f(U^+(t))}{S^+(t) + \alpha + \delta(C^J)} = \frac{f(U^-(t))}{S^-(t) + \alpha + \delta(C^J)}$$

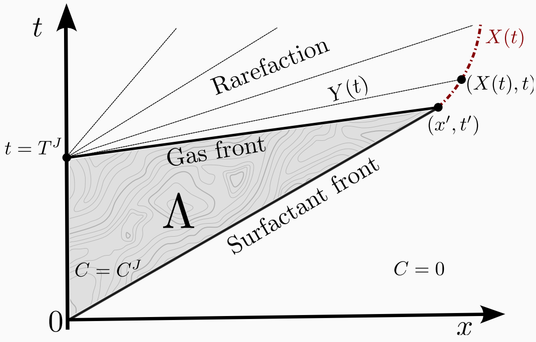


Figure 11: Schematic representation x-t plane with characteristic intersections

To calculate $U^-(t)$ and $U^+(t)$:

- We integrate the surfactant balance equation in Λ.
- Apply the Green theorem

$$\int_{\partial\Lambda} Cf \ dt - [C(S+\alpha) + A(C)]\, dx = 0. \tag{2}$$

- Let us define

$$\Gamma(U) = f(U) - \left[S + \alpha + \delta(C^J)\right] \cdot \partial_S f(U).$$

- $U^-(t)$ and $X(t)$ are calculated by

$$\frac{T^J}{t - T^J} = -\Gamma(U^-(t)), \qquad \frac{T^J}{X(t)} = -\frac{\Gamma(U^-(t))}{\partial_S f(U^-(t))}.$$

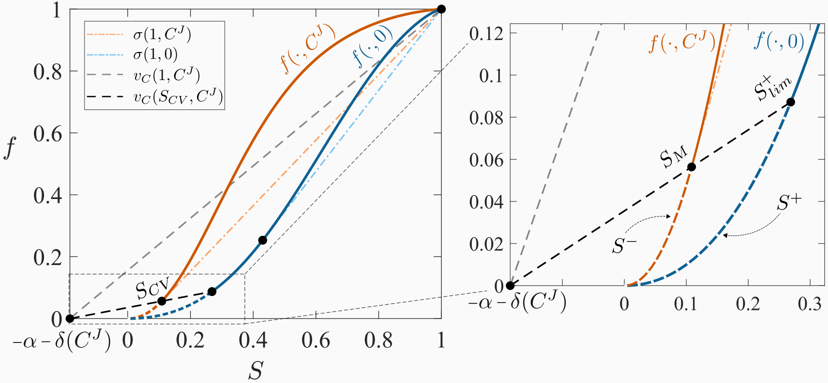


Figure 12: Geometrical representation of $S^-(t)$ and $S^+(t)$ in the S-f plane.

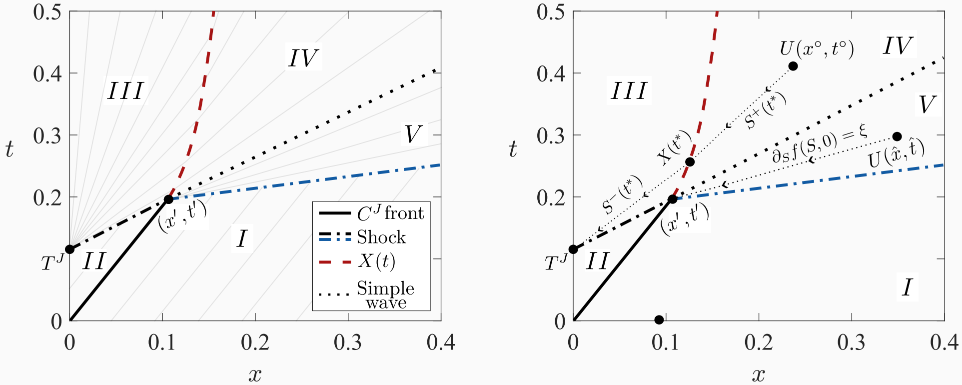


Figure 13: Schematic representation of the x–t plane, with characteristic curves shown as thin gray lines.

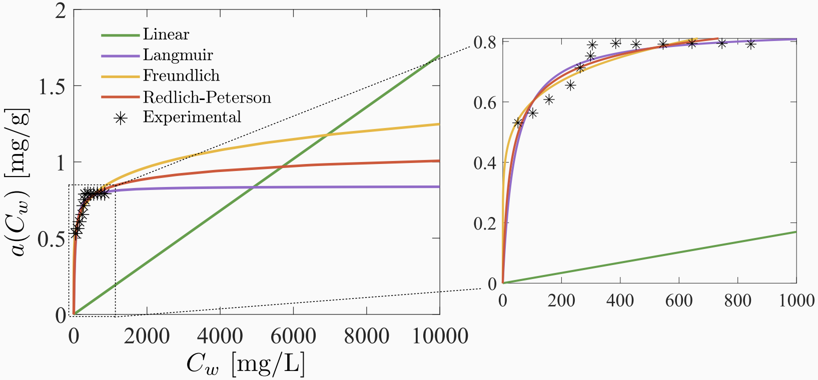
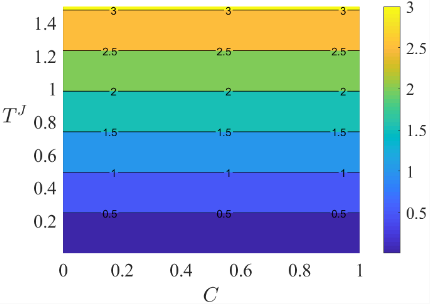


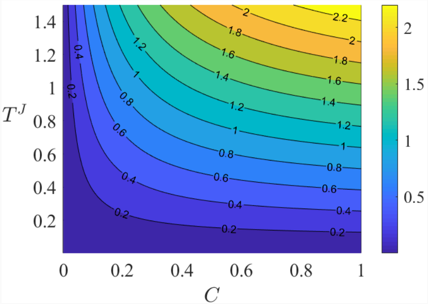
Figure 14: Adsorption isotherm profile for linear, Langmuir, Freundlich, and Redlich-Peterson models for CTAB surfactant.

The limiting position of the surfactant-front satisfies

$$\lim_{t\to\infty} X(t) = \lim_{t\to\infty} -T^J \cdot \frac{\partial_S f(U^-(t))}{\Gamma(U^-(t))} = \frac{T^J}{\alpha + \delta(C^J)}.$$



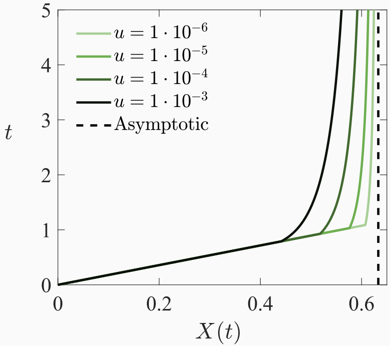
(a) Linear



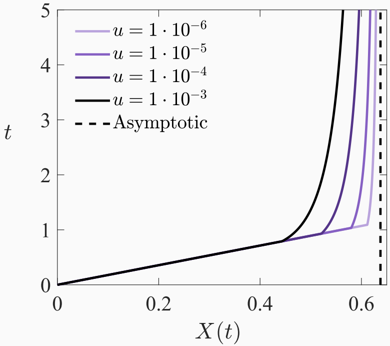
(b) Langmuir

Figure 15: Impact of the surfactant concentration and injected time on surfactant front position.

↬ Surfactant front position for distinct injection velocities



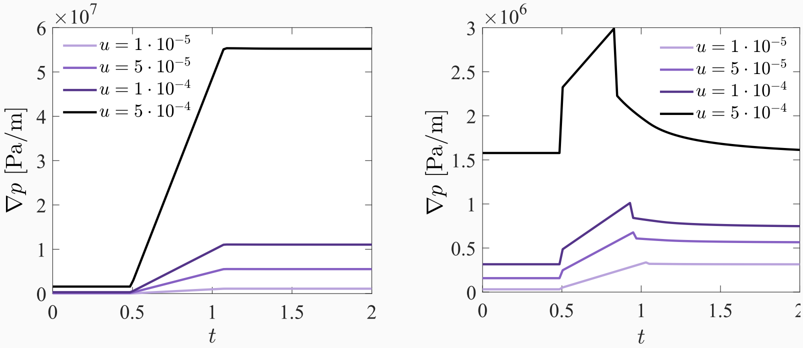
(a) Linear



(b) Langmuir

Figure 16: Impact of the injection velocity on the surfactant displacement

↬ Surfactant pressure gradients for distinct injection velocities



(a) Newtonian   **(b)** Non-Newtonian

Figure 17: Impact of the injection velocity on the pressure gradient for Langmuir adsorption.

Conclusions

Conclusions

- Analytical solution for foam displacement with surfactant adsorption formulated as **interacting injections**.
- Applicable to **linear and concave adsorption models** and compatible with **CMG-STARS** model.
- **Surfactant front reaches a finite asymptotic position**, controlled only by adsorption intensity and slug parameters (C^J, T^J).
- **Injection velocity and foam rheology affect only transient dynamics**, not the ultimate surfactant propagation.
- Linear and concave adsorption models show **similar qualitative behavior**, with quantitative differences mainly due to **slug concentration and duration**.
- **Slug design** (C^J, T^J) can be optimized to **delay or avoid surfactant breakthrough**, reducing chemical losses and operational issues.

Fritis et al. (2024) *SIAM Journal on Applied Math. 84 (2), 581-601*

Fritis et al. (2025) *International J. of Non-Linear Mech. 178, 105199*

Fritis et al. (2026) *Applied Math. Modelling, 116982*

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UNIVERSIDADE
FEDERAL DE JUIZ DE FORA

Laboratory of Applied Mathematics
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