Building castles in Spain (and on any beach): from dream to mechanical considerations

How to take interfacial tension into account in an anisotropic extension of Terzaghi's stress for unsaturated soils

A sand castle at Cannon Beach, Oregon. https://commons.wikimedia.org/wiki/File:Sand_castle,_Cannon_Beach.jpg

Renaud Toussaint [1,2], Rashad Abbasov [1], Marwan Fahs [1], Eirik Grude Flekkøy[2], Knut Jørgen Måløy[2]

1: Institute Earth and Environment of Strasbourg, ITES, University of Strasbourg/CNRS
2: SFF PoreLab, The Njord Center, Dept of Physics, University of Oslo

Institut Terre & Environnement de Strasbourg | ITES | UMR 7063 de l'Université de Strasbourg & cnrs & ENGEES

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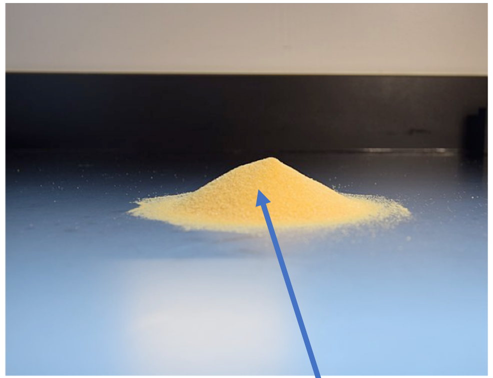
How to take interfacial tension into account in an anisotropic extension of Terzaghi's stress for unsaturated soils

UnSaturated sand

Saturated sand

A sand castle at Cannon Beach, Oregon.
https://commons.wikimedia.org/wiki/File:Sand_castle,_Cannon_Beach.jpg



Dry sandpile
https://commons.wikimedia.org/wiki/File:Sand_Pile.jpg

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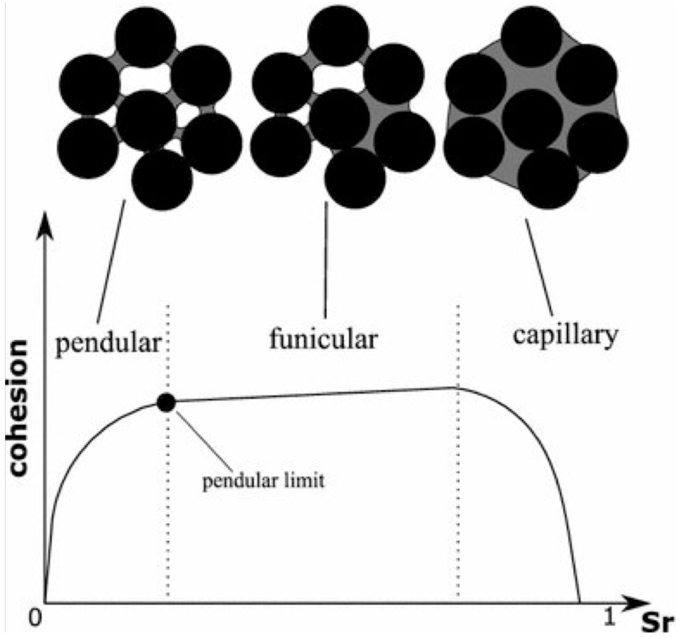
Unsaturated granular cohesion as a function of saturation



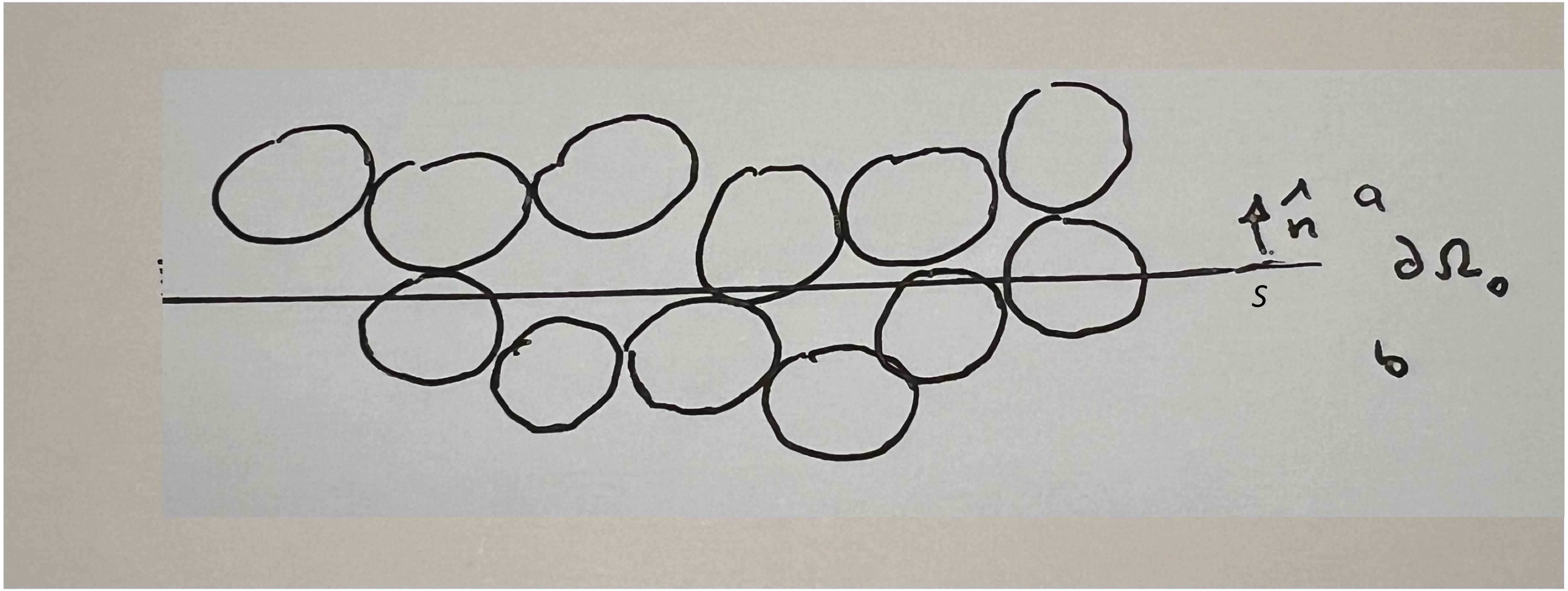
Example

Wang, JP., Li, X. & Yu, HS. A micro–macro investigation of the capillary strengthening effect in wet granular materials. *Acta Geotech.* **13**, 513–533 (2018). https://doi.org/10.1007/s11440-017-0619-0

Terzaghi's stress / effective stress concept

One Fluid

Contact Forces $F^c_{a \to b}$?

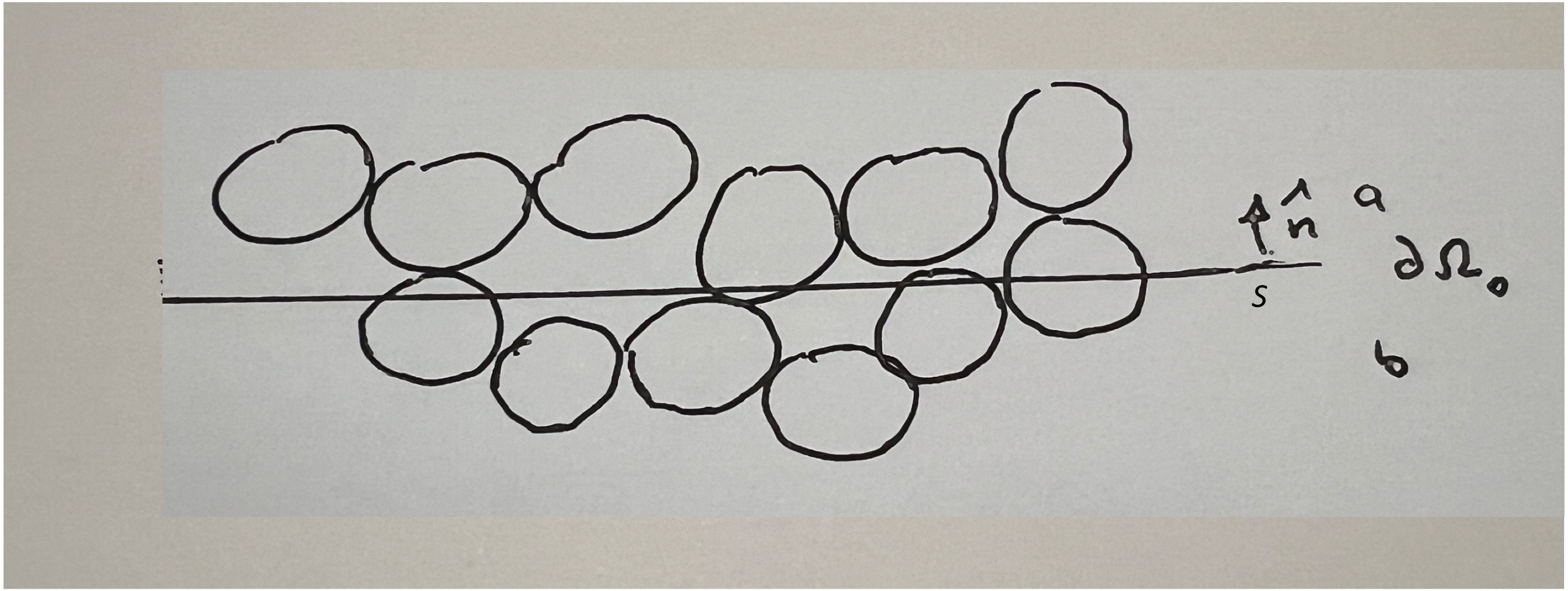


Control stability: sliding, rupture by exceeding cohesion.
Common description: Mohr-Coulomb criterion

Terzaghi’s stress / effective stress concept

One Fluid

Contact Forces $F^c_{a\to b}$?



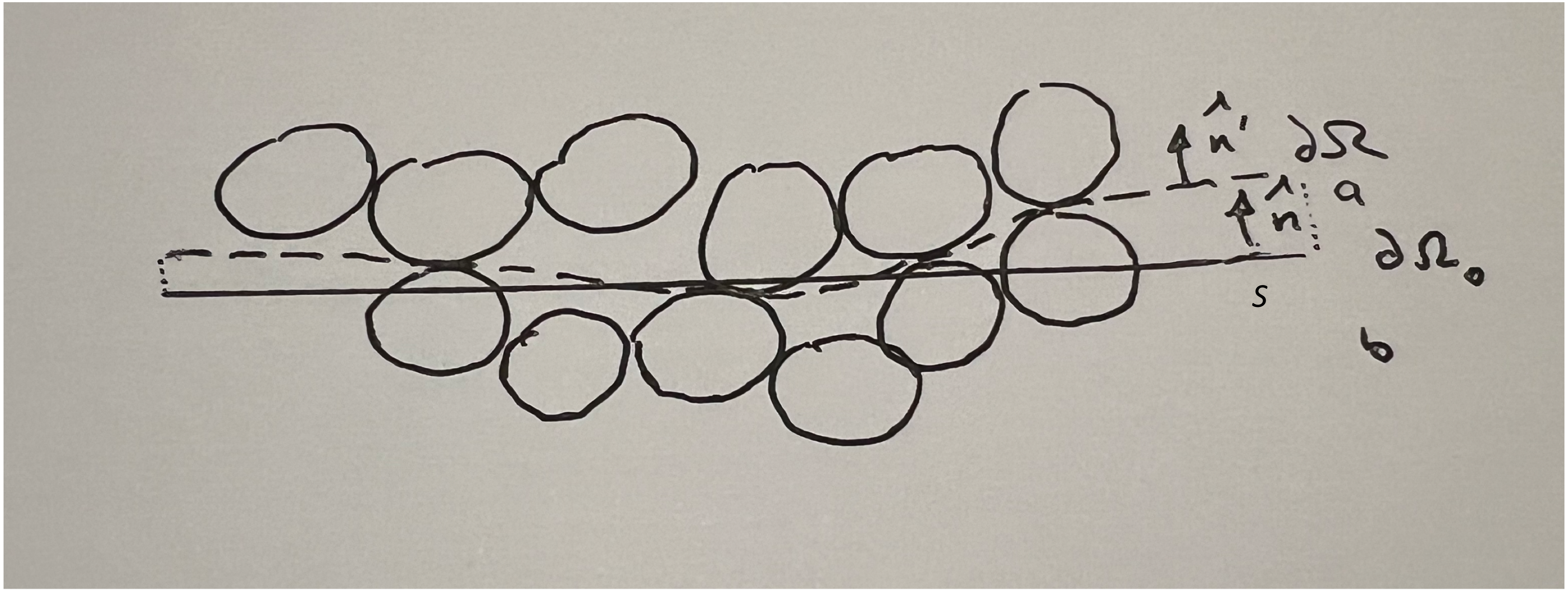
$$F^T_{a\to b} = \sigma^T \cdot \hat{n}S = -\sigma'^T \cdot \hat{n}S$$

Positive tension　　Positive compression

Terzaghi's stress / effective stress concept

One Fluid

Contact Forces $F^{c}_{a\rightarrow b}$?



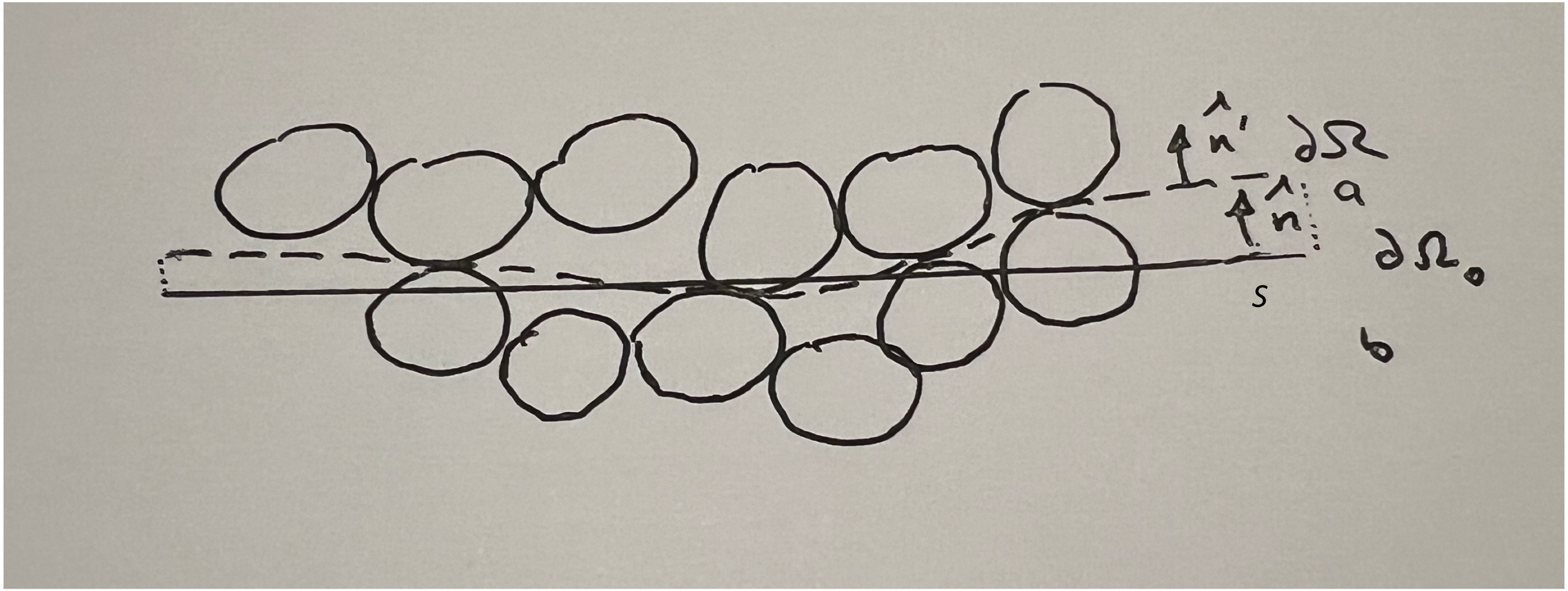
$$F^T_{a\rightarrow b} = -\sigma'^T \cdot \hat{n}S = -\int\int_{\partial\Omega} \sigma'^T \cdot \hat{n}' dS$$

(Mechanical equilibrium, $\partial_j \sigma^T_{ij} = 0$)

Terzaghi's stress / effective stress concept

One Fluid

Contact Forces $F^{c}_{a \to b}$?

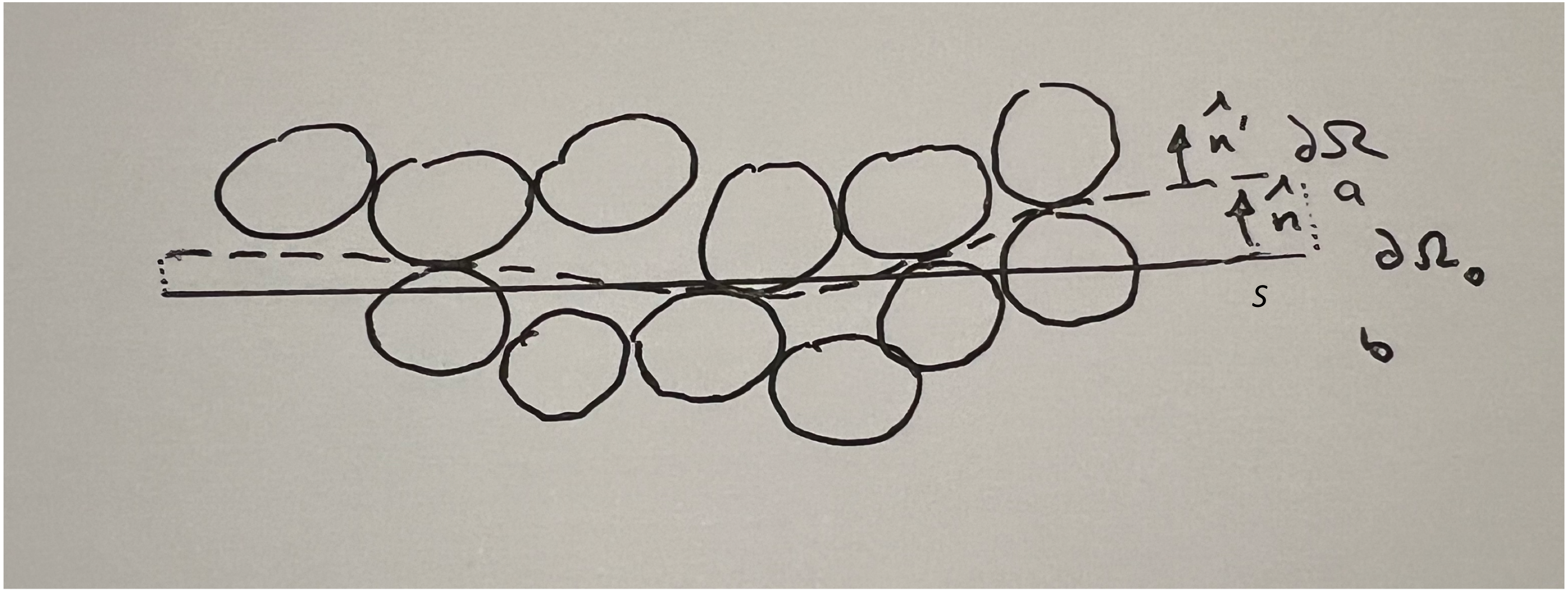


$$F^{T}_{a \to b} = -\sigma'^{T} \cdot \hat{n} S = -\int \int_{\partial\Omega} p \hat{n}' dS + F^{c}_{a \to b}$$

Terzaghi's stress / effective stress concept

One Fluid

Contact Forces $F^c_{a\to b}$?

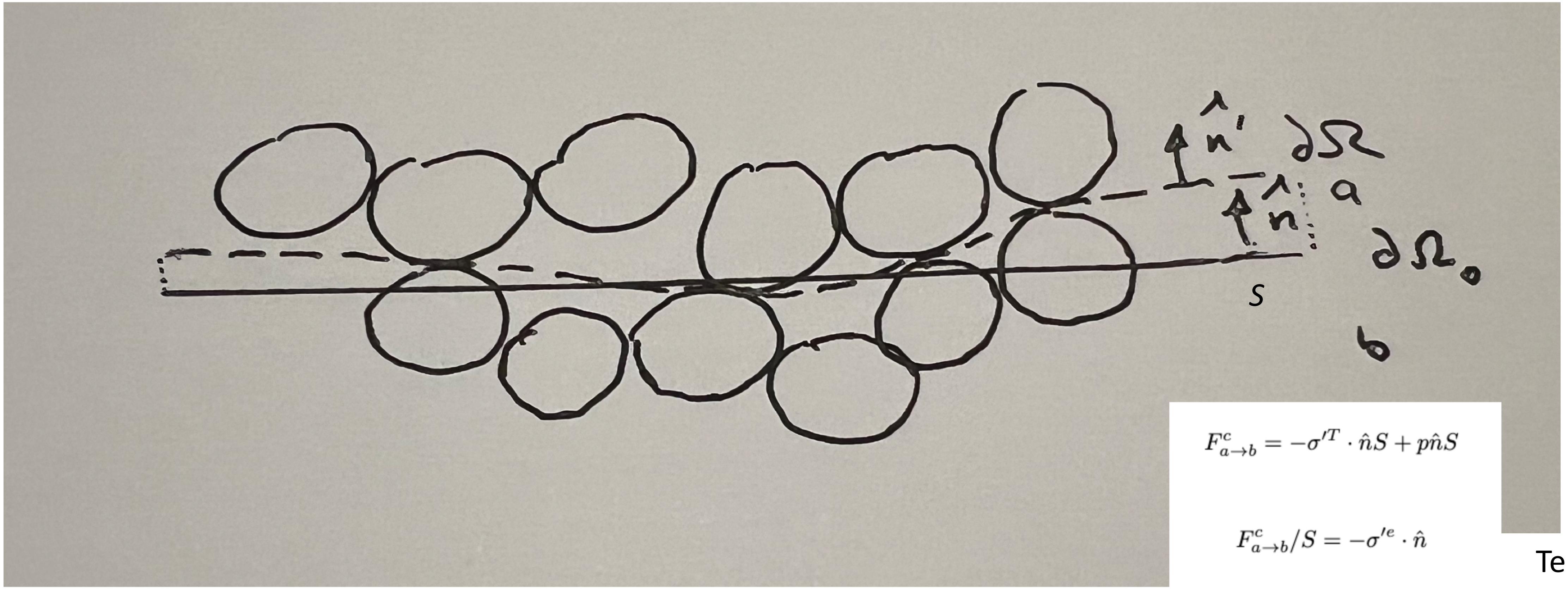


$$F^T_{a\to b} = -\sigma'^T \cdot \hat{n} S = -\int\int_{\partial\Omega} p\hat{n}' dS + F^c_{a\to b} = -p\hat{n}S + F^c_{a\to b}$$

Terzaghi's stress / effective stress concept

One Fluid

Contact Forces $F^c_{a\to b}$?



$$F^c_{a\to b} = -\sigma'^T \cdot \hat{n}S + p\hat{n}S$$

$$F^c_{a\to b}/S = -\sigma'^e \cdot \hat{n}$$

$$F^T_{a\to b} = -\sigma'^T \cdot \hat{n}S = -\int\int_{\partial\Omega} p\hat{n}'dS + F^c_{a\to b} = -p\hat{n}S + F^c_{a\to b}$$

Effective stress

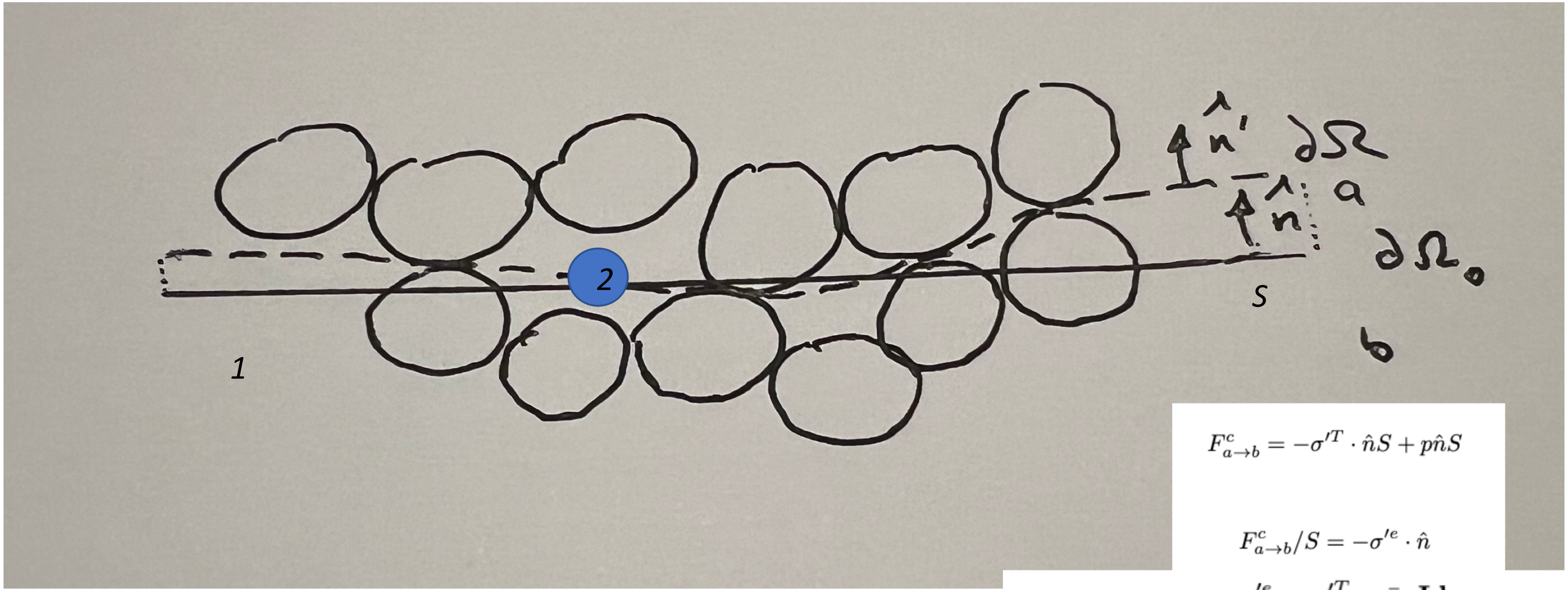
$$\sigma'^e = \sigma'^T - p\mathbf{Id}$$

Terzaghi:
Biot
coefficient
$\alpha=1$

Generalizing
Terzaghi's stress / effective stress concept

Two Immiscible Fluids, 1&2

Contact Forces $F^c_{a\to b}$?



$$F^c_{a\to b} = -\sigma'^T \cdot \hat{n}S + p\hat{n}S$$

$$F^c_{a\to b}/S = -\sigma'^e \cdot \hat{n}$$

$$\sigma'^e = \sigma'^T - \bar{p}_V \mathbf{Id}$$

The volume average of pore fluid pressure is

$$\bar{p}_V = (\bar{p}_1 S_1 + \bar{p}_2 S_2)$$

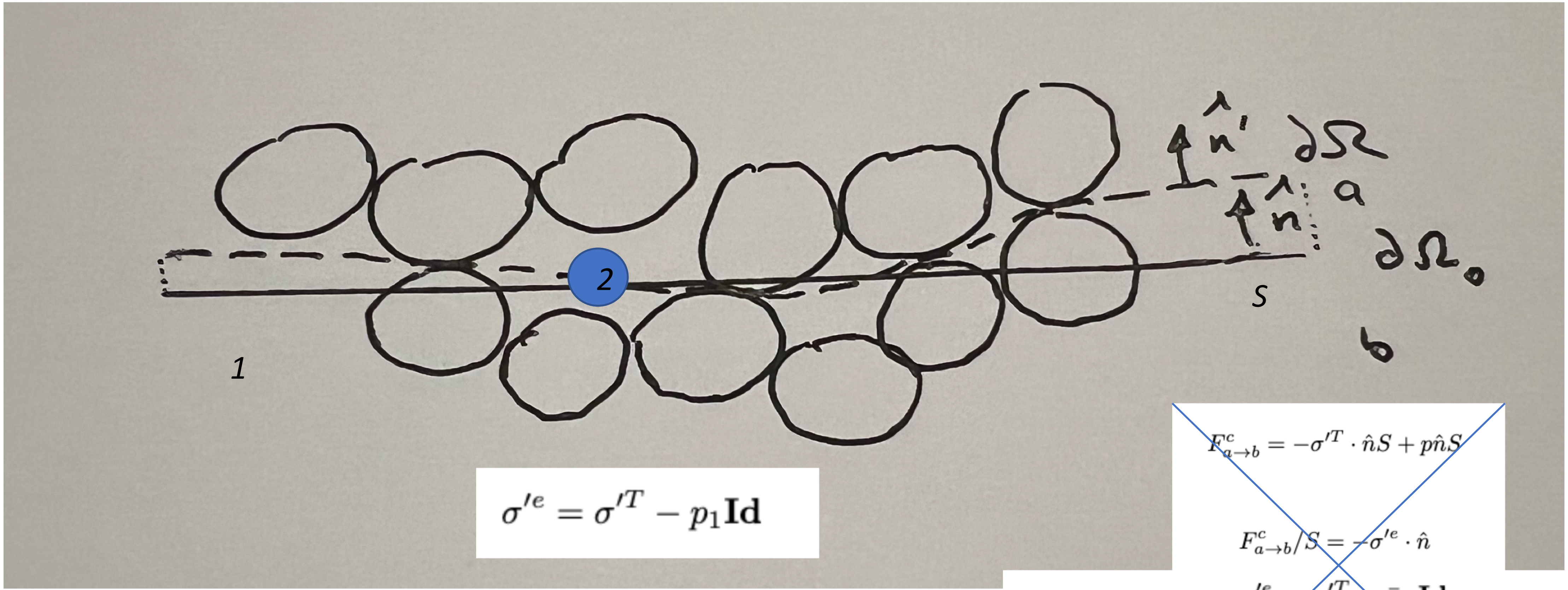
Effective stress ? (suction stress)

R. I. Borja and A. Koliji, On the effective stress in unsaturated porous continua with double porosity, Journal of the Mechanics and Physics of Solids 57, 1182 (2009).

Generalizing
Terzaghi's stress / effective stress concept

Two Immiscible Fluids, 1&2

Contact Forces $F^c_{a\to b}$?



$$\sigma'^e = \sigma'^T - p_1 \mathbf{Id}$$

$$F^c_{a\to b} = -\sigma'^T \cdot \hat{n} S + p\hat{n}S$$

$$F^c_{a\to b}/S = -\sigma'^e \cdot \hat{n}$$

$$\sigma'^e = \sigma'^T - \bar{p}_V \mathbf{Id}$$

The volume average of pore fluid pressure is

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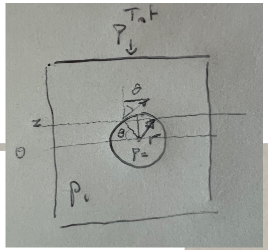
Effective stress ? Missing term: surface tension

R. I. Borja and A. Koliji, On the effective stress in unsaturated porous continua with double porosity, Journal of the Mechanics and Physics of Solids 57, 1182 (2009).

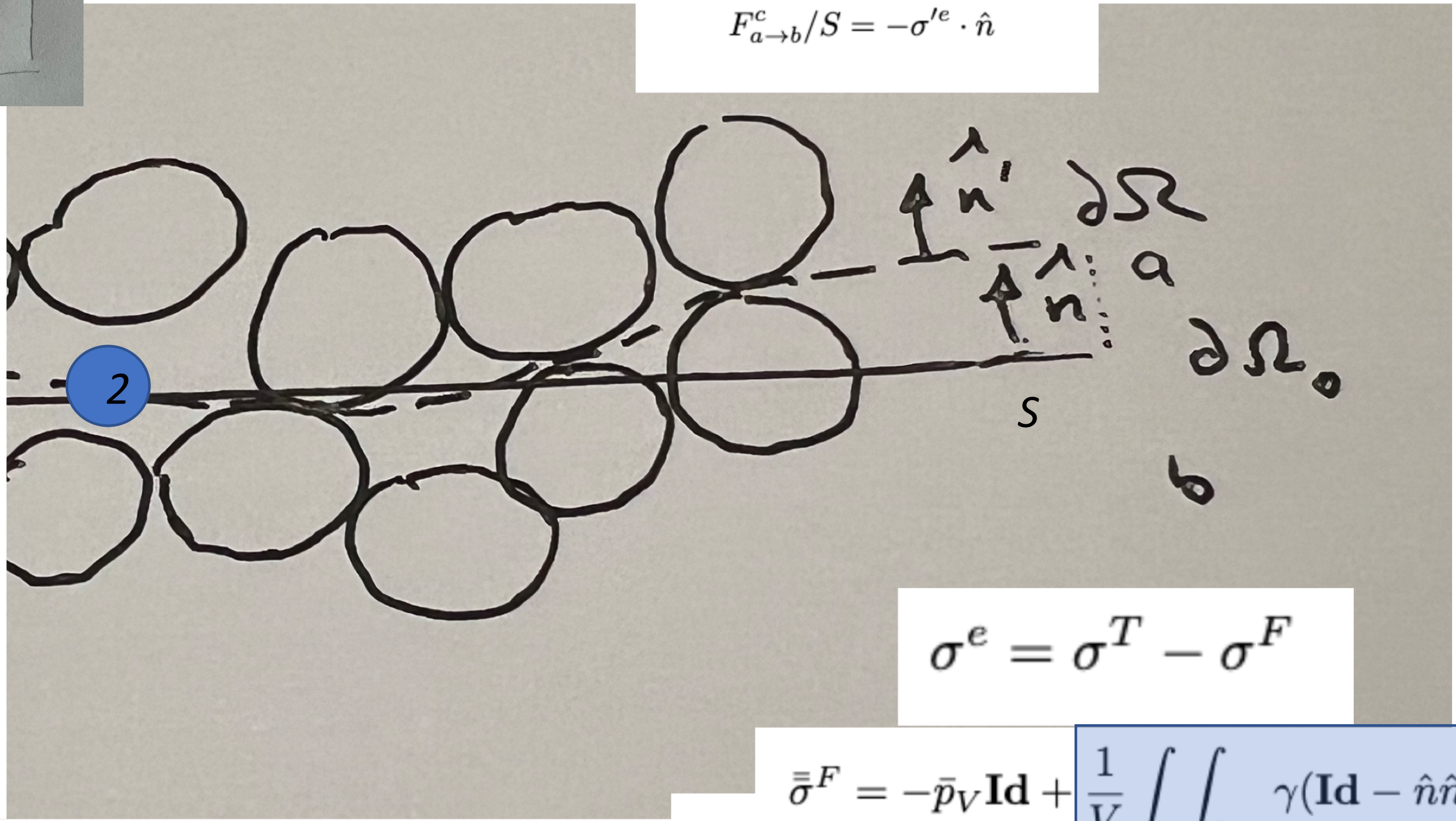
Generalizing
Terzaghi's stress / effective stress concept

Two Immiscible Fluids, 1&2



Contact Forces $F^c_{a\to b}$?

$$F^c_{a\to b}/S = -\sigma'^e \cdot \hat{n}$$



$$F_z = \bar{\sigma}_{zz}(z_c)S\hat{e}_z =$$
$$-\int\int_{z=z_c} p(x,y,z)dxdy\hat{e}_z + \int_{z=z_c,r=R} \gamma dl\cos(\theta)\hat{e}_z$$
$$= -\int\int_{z=z_c,r_c>R\cos(\theta)} p_1 dxdy\hat{e}_z$$
$$-\int\int_{z=z_c,r<R\cos(\theta)} p_2 dxdy\hat{e}_z$$
$$+\int_{z=z_c,r=R\cos(\theta)} \gamma dl\cos(\theta)\hat{e}_z$$
$$= (-p_1(S-\pi R^2\cos^2(\theta)) - p_2\pi R^2\cos^2(\theta)$$
$$+\gamma 2\pi R\cos(\theta))\cos(\theta)\hat{e}_z$$
$$= -p^T S\hat{e}_z$$

so that

$$\bar{\sigma}_{zz}(z_c) = -p_1 = -p^T \qquad (25)$$

$$\sigma^e = \sigma^T - \sigma^F$$

$$\bar{\bar{\sigma}}^F = -\bar{p}_V \mathbf{Id} + \frac{1}{V}\int\int_{\Omega_{12}} \gamma(\mathbf{Id} - \hat{n}\hat{n})dS$$

The volume average of pore fluid pressure is

$$\bar{p}_V = (\bar{p}_1 S_1 + \bar{p}_2 S_2)$$

Effective stress ? Missing term: surface tension

Potentially anisotropic fluid stress contribution, fabric tensor of interfaces...

In 1943, Karl Terzaghi defined effective stress as stress representing "that part of the total stress which produces measurable effects such as compaction or an increase of the shearing resistance" (Terzaghi 1943). Bishop's (1959) often cited effective stress approach involves a modified form of Terzaghi's effective stress written as follows:

Saturated Condition

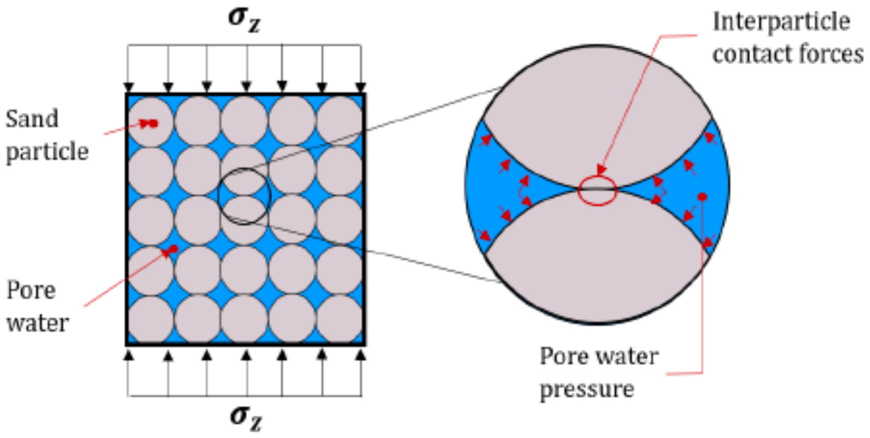
$$\boldsymbol{\sigma}_{eff} = \boldsymbol{\sigma} - p_w.\boldsymbol{I}$$

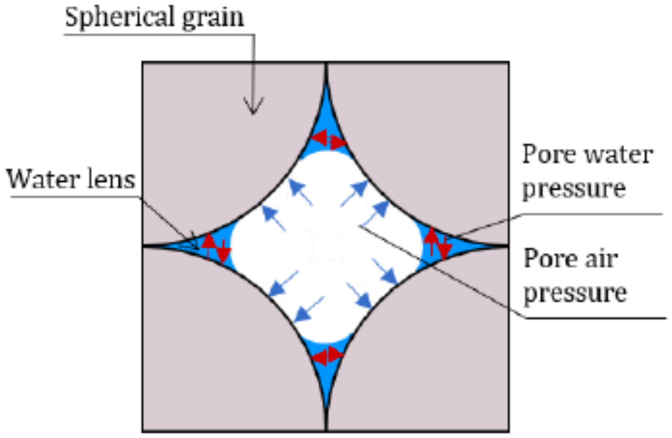
Unsaturated Condition

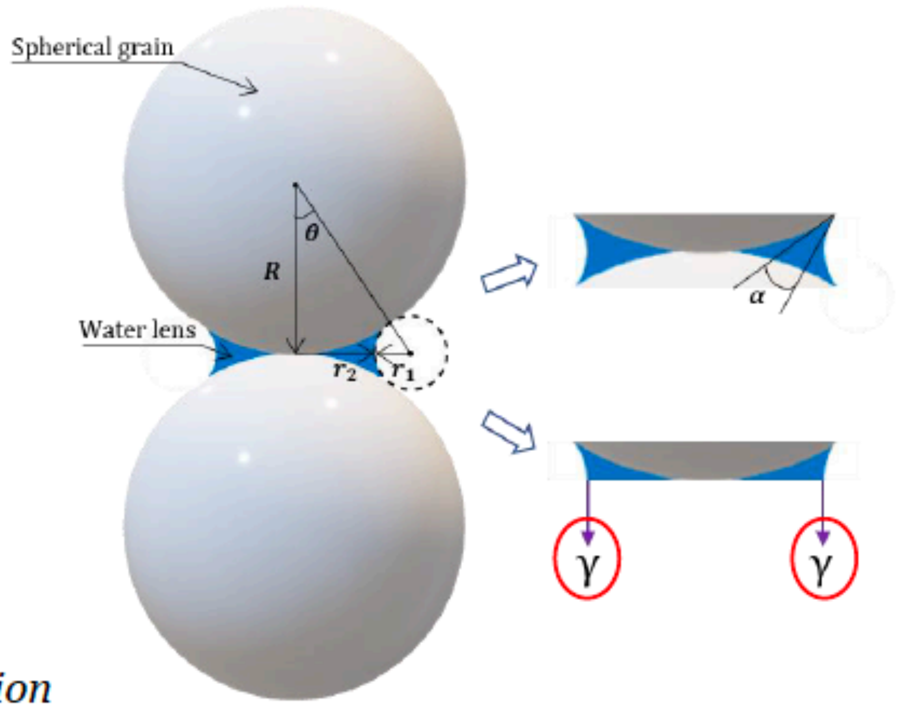
$$\boldsymbol{\sigma}_{eff} = (\boldsymbol{\sigma} - p_a.\boldsymbol{I}) + \chi(S_w)(p_a - p_w).\boldsymbol{I}$$

Conventions

+ *Compressive forces* − *Tensile forces*







Unsaturated Condition

$$\boldsymbol{\sigma_{eff}} = (\boldsymbol{\sigma} - p_a.\boldsymbol{I}) + \chi(S_w)(p_a - p_w).\boldsymbol{I} + \;?$$

Net normal stress

Suction stress

Direct Effect of Surface Tension

Research Gaps:

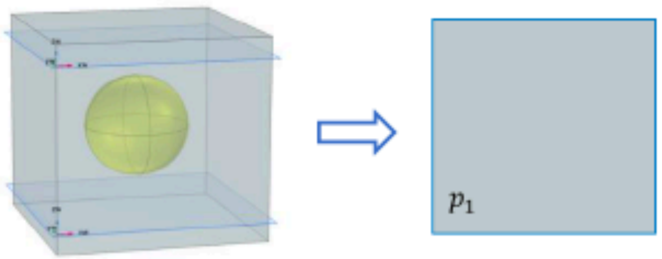
Houlsby, 1997	Hutter et al., 1999
Borja, 2006	Zhao et al., 2010
Lu et al. (2010)	Nikooee et al. (2013)

Despite the precedence for ignoring or simplifying interfacial forces' contribution to effective stress, no convincing physical justification has been reported for doing so.

Objectives:

- **Asses the impact of the "direct effect of surface tension" term on RILS HM models**

$z_c > R$ or $z_c < -R$



$$\bar{F}_z = -\iint_{z=z_c} -p(x,y,z)dxdy\hat{e}_z$$

$$\bar{F}_z = -p_1 S\hat{e}_z = \bar{\sigma}_{zz}(z_c)S\hat{e}_z$$

$$\bar{\sigma}_{zz}(z_c) = -p_1 = -p^T$$

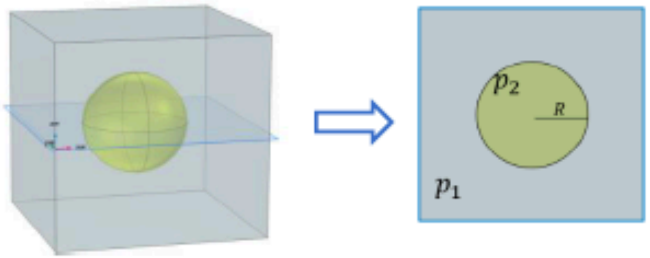
1

Young–Laplace equation

$$p_2 - p_1 = \frac{2\gamma}{R}$$

2

$z_c = 0$



$$\bar{F}_z = -\iint_{z=0,r>R} p_1 dxdy\hat{e}_z - \iint_{z=0,r<R} p_2 dxdy\hat{e}_z + \int_{z=0,r=R} \gamma dl\hat{e}_z$$

$$\bar{F}_z = [-p_1(S-\pi R^2) - p_2\pi R^2 + \gamma 2\pi R]\hat{e}_z$$

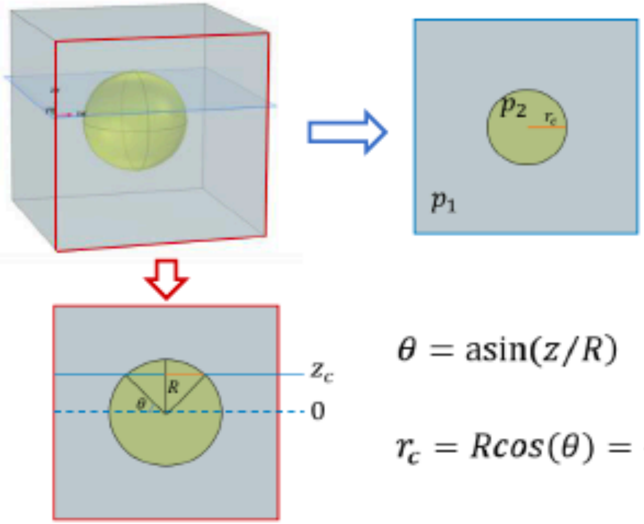
$$\bar{F}_z = -p^T S\hat{e}_z$$

$$\bar{\sigma}_{zz}(z=0) = -p^T$$

$$\bar{\sigma}_{zz}(z=0) = -p_T - \frac{2\gamma}{R}\frac{\pi R^2}{S}$$

3

$-R < z_c < R$



$$\bar{F}_z = -p^T S\hat{e}_z$$

$$\bar{\sigma}_{zz}(z=z_c) = -p^T$$

$$\bar{\sigma}_{zz}(z=z_c) = -p_T - \frac{2\gamma}{R}\frac{\pi(R^2 - z_c^2)}{S}$$

$$\theta = \text{asin}(z/R)$$

$$r_c = R\cos(\theta) = \sqrt{R^2 - z^2}$$

4

$$\boldsymbol{\sigma_{eff}} = \boldsymbol{\sigma} - p_{av}.\boldsymbol{I} + \frac{1}{V}\iint_{\Omega_{12}} \gamma(.\boldsymbol{I} - \hat{n}\hat{n})dS$$

↓

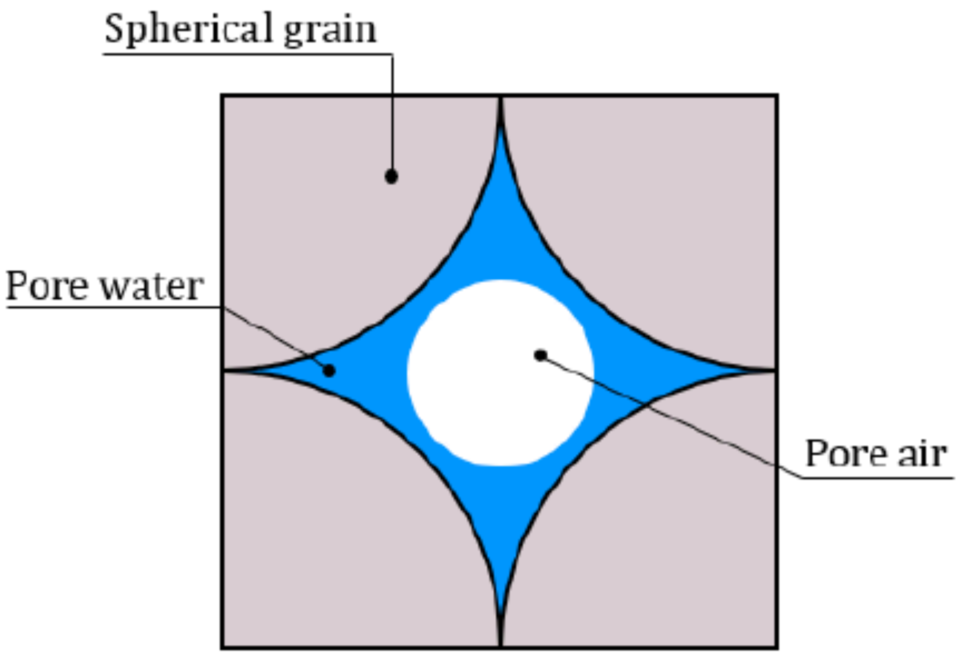
$$\boldsymbol{\sigma_{eff}} = \boldsymbol{\sigma} - p_{av}.\boldsymbol{I} + \frac{2\gamma}{R}\left(\frac{4\pi R^3}{3V}\right).\boldsymbol{I}$$

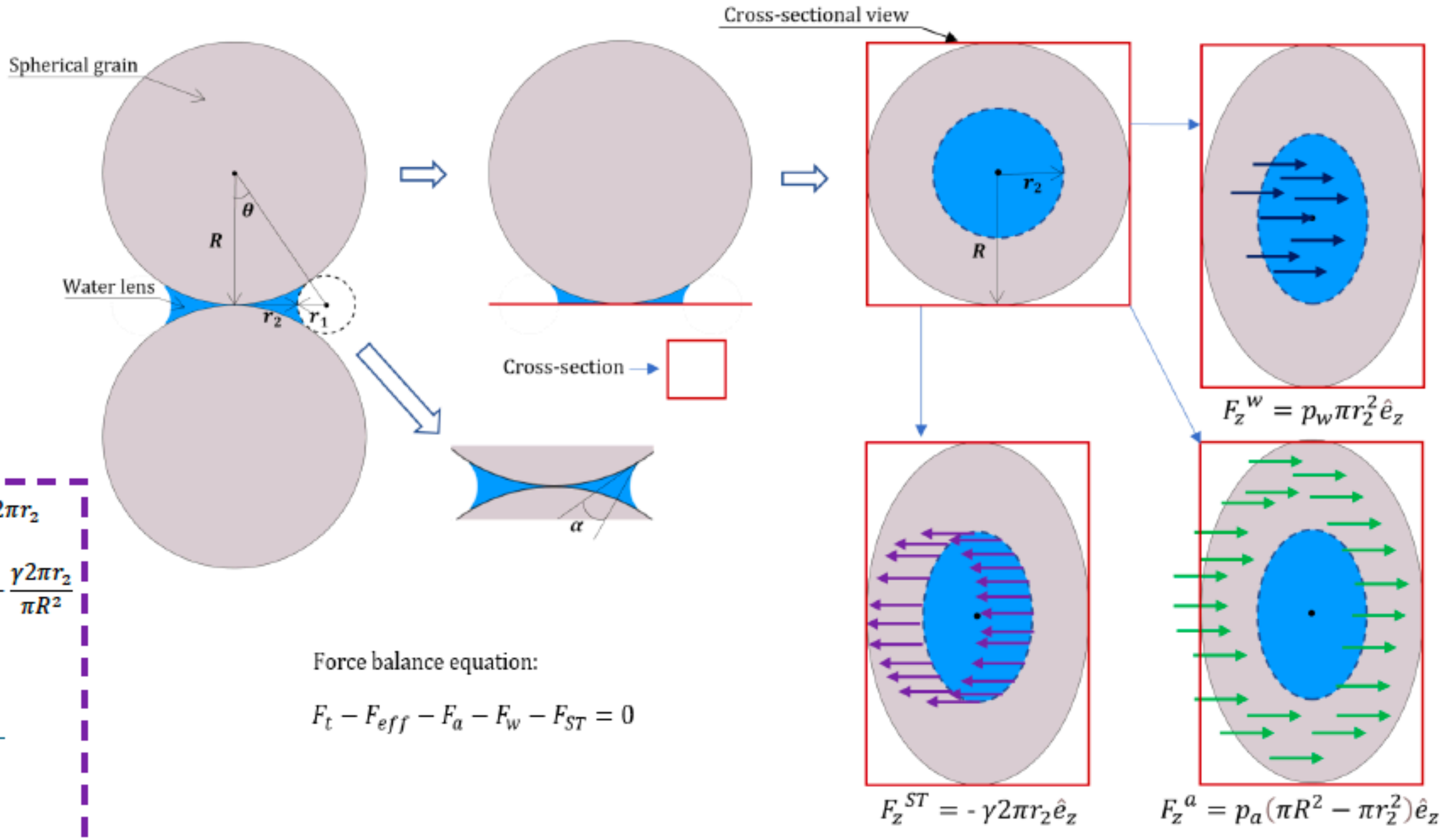
↓

$$\boldsymbol{\sigma_{eff}} = (\boldsymbol{\sigma} - p_a.\boldsymbol{I}) + \chi(S_w)(p_a - p_w).\boldsymbol{I} + \frac{2\gamma}{R}\left(\frac{4\pi R^3}{3V}\right).\boldsymbol{I}$$

↓

$$\boldsymbol{\sigma_{eff}} = \boldsymbol{\sigma} + \boldsymbol{\sigma^F} = \boldsymbol{\sigma} - p^T.\boldsymbol{I}$$





Force balance equation:

$$F_t - F_{eff} - F_a - F_w - F_{ST} = 0$$

$$F_{eff} = F_t - p_a(\pi R^2 - \pi {r_2}^2) - p_w \pi {r_2}^2 + \gamma 2 \pi r_2$$

$$\frac{F_{eff}}{\pi R^2} = \frac{F_t}{\pi R^2} - \frac{p_a \pi R^2}{\pi R^2} + \frac{p_a \pi {r_2}^2}{\pi R^2} - \frac{p_w \pi {r_2}^2}{\pi R^2} + \frac{\gamma 2 \pi r_2}{\pi R^2}$$

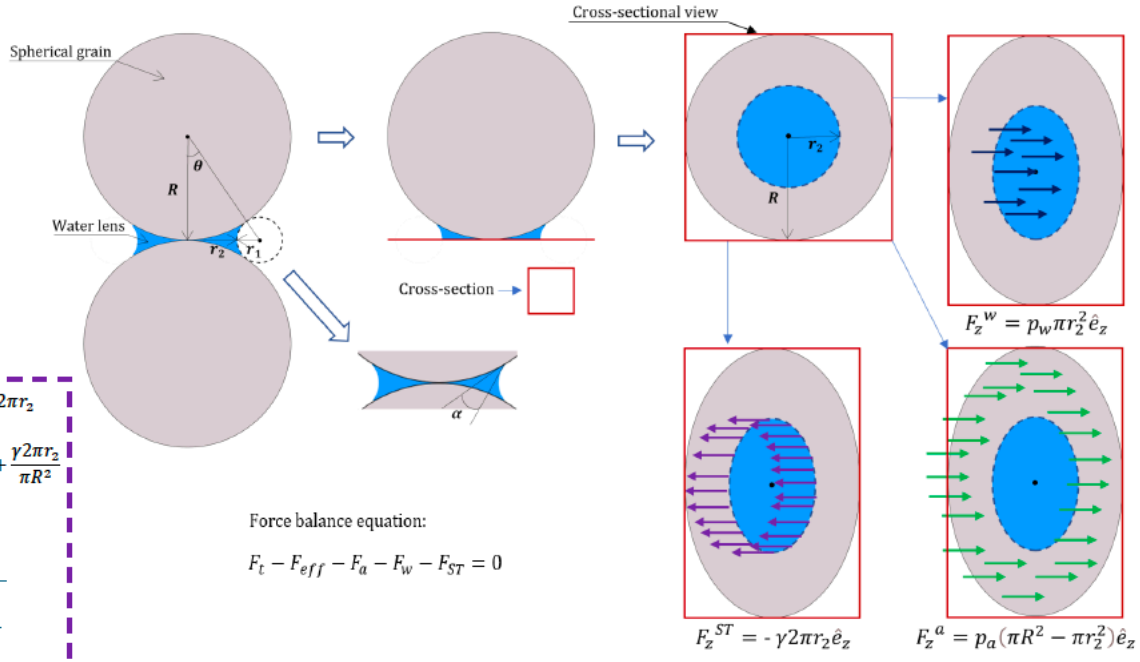
$$\sigma_{eff} = \sigma_t - p_a + \frac{\pi {r_2}^2}{\pi R^2}(p_a - p_w) + \frac{\gamma 2 r_2}{R^2}$$

Capillary stress

$$\sigma_{eff} = \sigma_t - p_a + \chi(S_w)(p_a - p_w) + \frac{\gamma 2 r_2}{R^2}$$

- $\sigma_t - p_a$: Net normal stress
- $\chi(S_w)(p_a - p_w)$: Suction stress
- $\frac{\gamma 2 r_2}{R^2}$: Direct effect of Surface tension

This direct effect term depends on the location of the capillary bridges. The volume average of this term defining the effective stress is anisotropic:
More cohesion along directions through more capillary bridges



$$F_{eff} = F_t - p_a(\pi R^2 - \pi {r_2}^2) - p_w \pi {r_2}^2 + \gamma 2\pi r_2$$

$$\frac{F_{eff}}{\pi R^2} = \frac{F_t}{\pi R^2} - \frac{p_a \pi R^2}{\pi R^2} + \frac{p_a \pi {r_2}^2}{\pi R^2} - \frac{p_w \pi {r_2}^2}{\pi R^2} + \frac{\gamma 2\pi r_2}{\pi R^2}$$

$$\sigma_{eff} = \sigma_t - p_a + \underbrace{\frac{\pi {r_2}^2}{\pi R^2}(p_a - p_w) + \frac{\gamma 2 r_2}{R^2}}_{\text{Capillary stress}}$$

$$\sigma_{eff} = \underbrace{\sigma_t - p_a}_{\text{Net normal stress}} + \underbrace{\chi(S_w)(p_a - p_w)}_{\text{Suction stress}} + \underbrace{\frac{\gamma 2 r_2}{R^2}}_{\text{Direct effect of Surface tension}}$$

In the absence of more detailed description, possible approximation:

. No cohesion (interfaces) at full or zero saturation, presence of interface proportional to saturation product

. Characteristic length given by permeability

$$\sigma'^e = \sigma'^T - \bar{p}_V \mathbf{Id} + (\gamma/\sqrt{\kappa}) S_1 S_2$$

The volume average of pore fluid pressure is

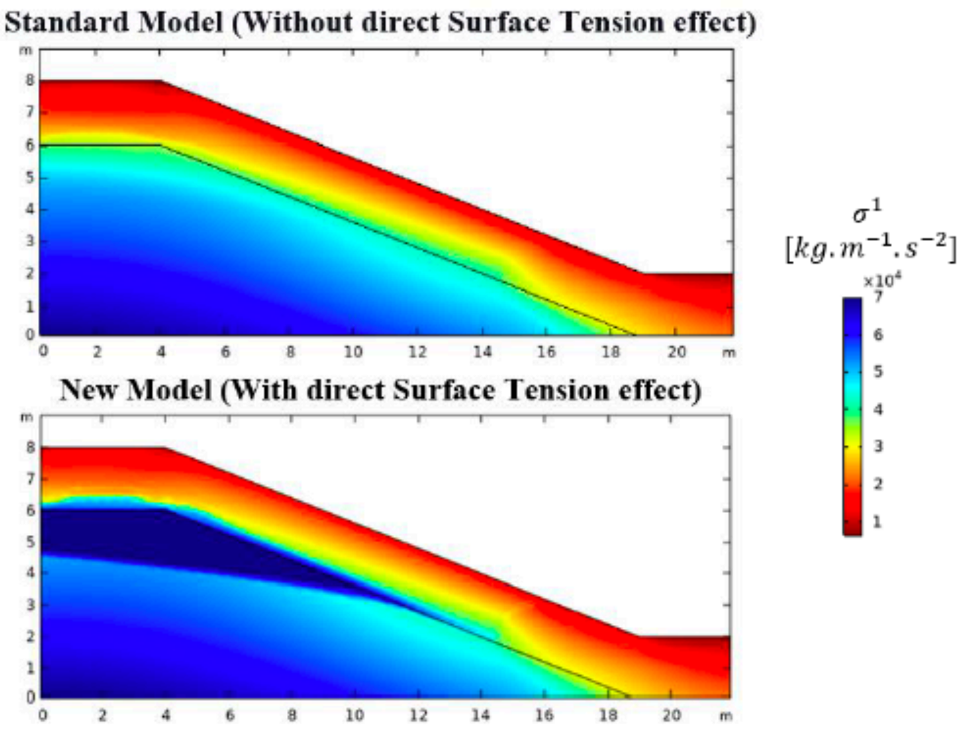
$$\bar{p}_V = (\bar{p}_1 S_1 + \bar{p}_2 S_2)$$

$$\sigma^e = \sigma^T + \bar{p}_V \mathbf{Id} - (\gamma/\sqrt{\kappa}) S_1 S_2$$

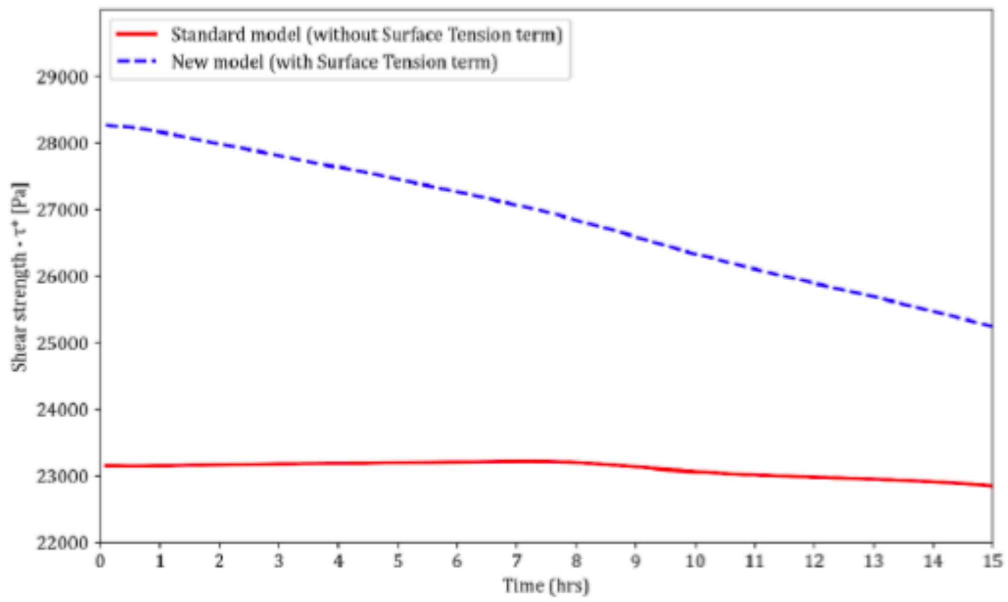
. With better description, anisotropic tensor, function of interface density

$$\sigma'^e = \sigma'^T - \bar{p}_V \mathbf{Id} + (\gamma/\sqrt{\kappa}) S_1 S_2 \mathbf{g}_{shape}$$

Numerical study:
impact on the stability of slope during rainfall



Maps of the effective stress distribution for the last time (i.e. 15th hour) of the intensive rainfall event with the standard model and the new model

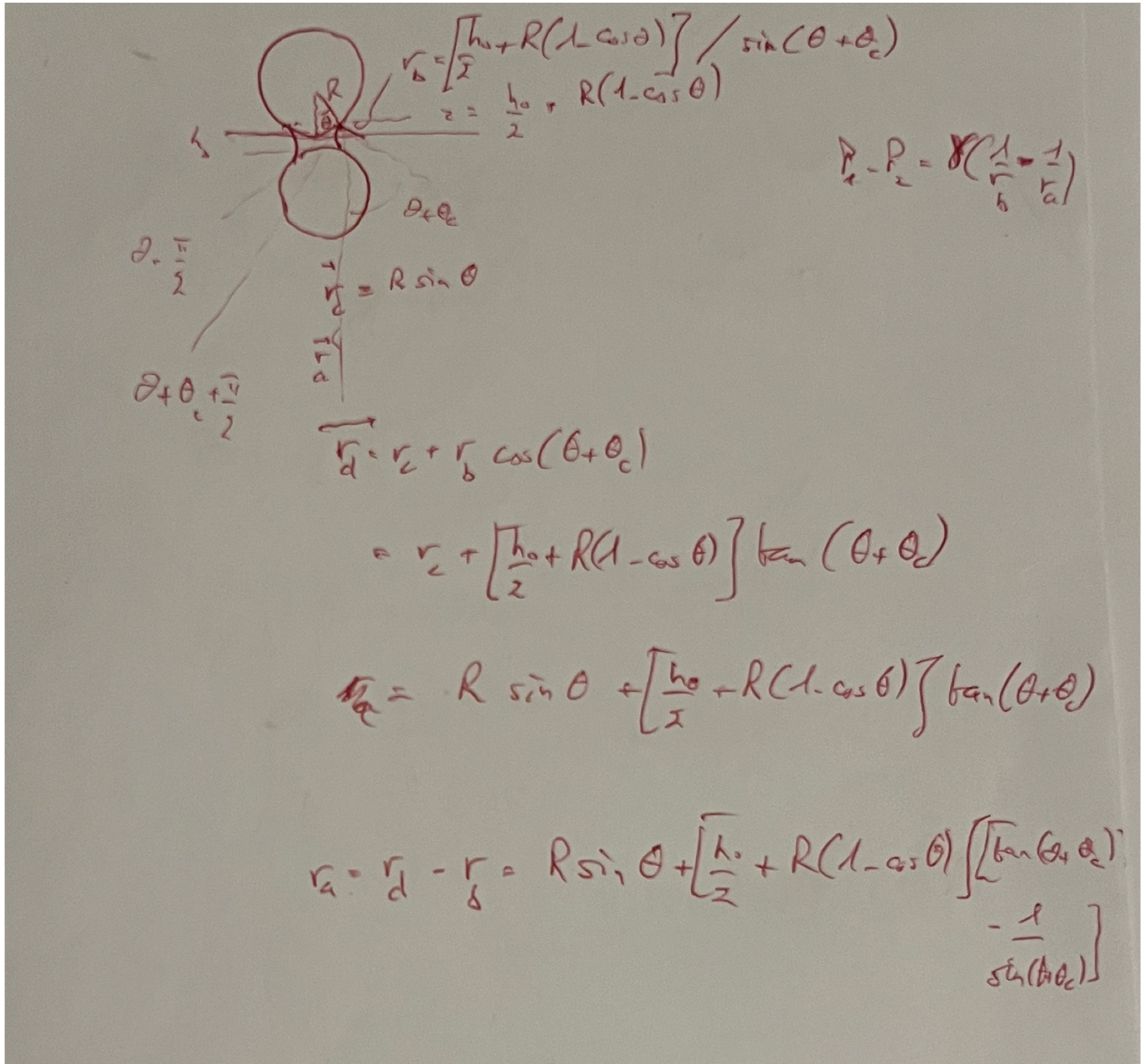


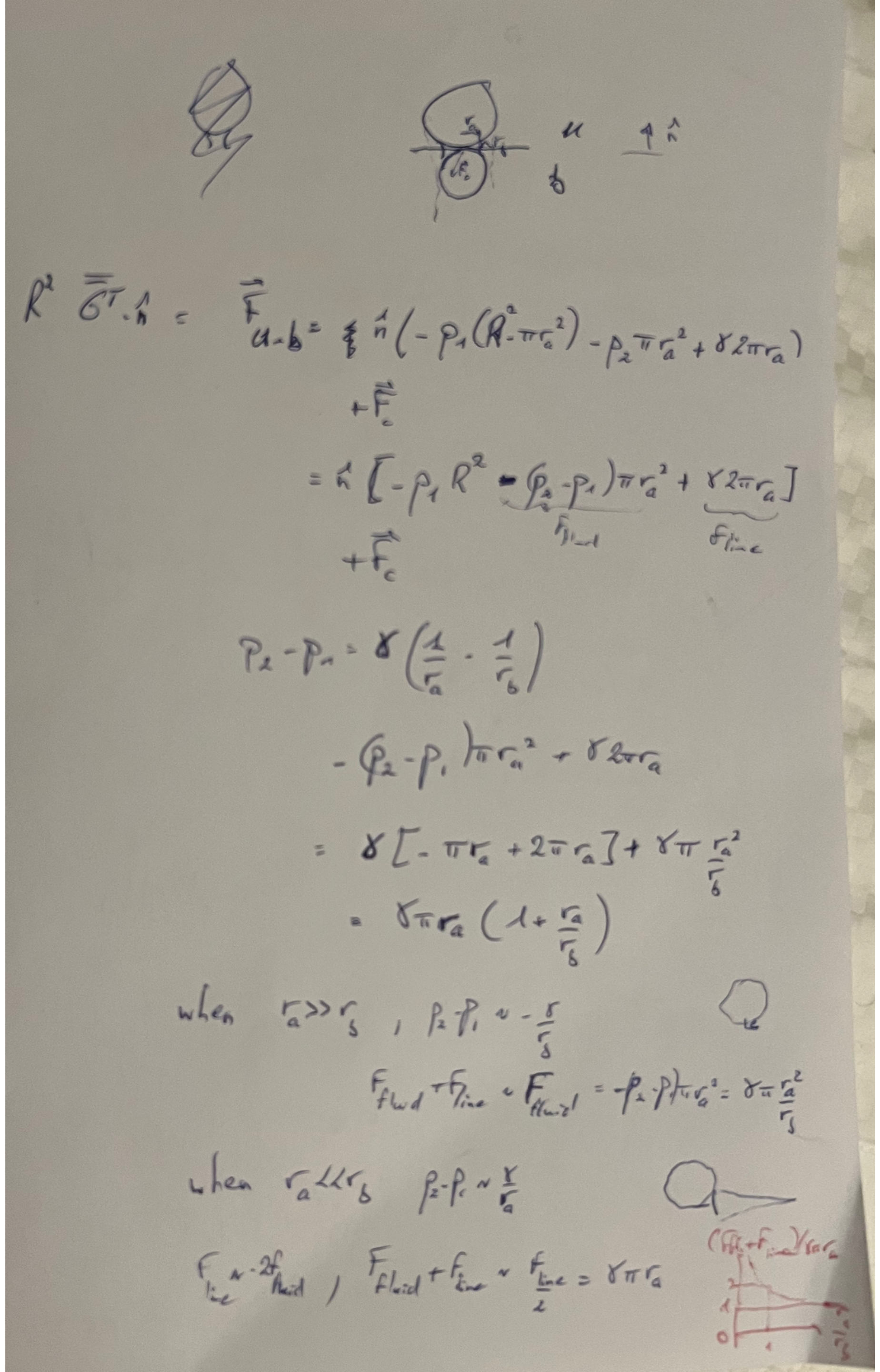
Plots of the shear strength predicted by the standard and new hydro-mechanical models over 15 hours of an intensive rainfall event

Conclusions

- Taking into account surface tension in effective stress amounts to add a term representing the stress supported by the interfaces
- This contribution is not negligible
- It is potentially anisotropic. It depends on the geometry of the interfaces
- These interfaces result from the history of the flow
- Some crude approximations can be used to estimate this term, or deeper models producing the micromechanics are needed

- Particular expression, capillary bridges between spherical grains





- Particular expression, capillary bridges between spherical grains