ETH zürich

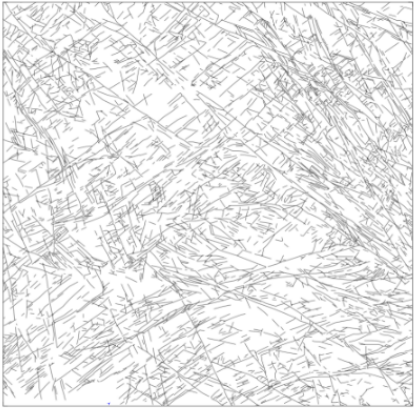
Institute of Fluid Dynamics

Efficient Flow and Transport in Fractured Porous Media using the Basis Function Method

Daniel Stalder, Shangyi Cao, Daniel W. Meyer, Patrick Jenny

22. May 2026, Interpore

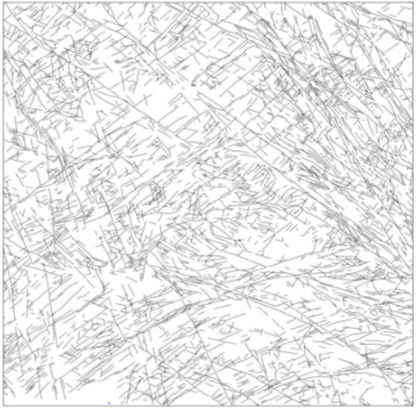
Motivation



Odling, N.E., 1997, Journal of Structural Geology

Motivation

- Meshless



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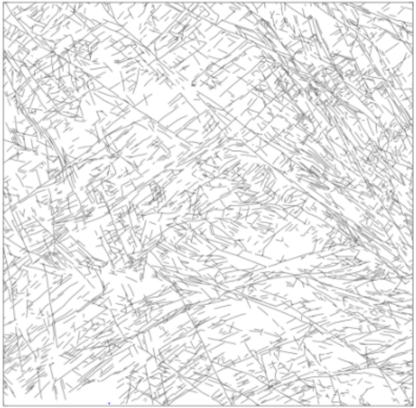
Motivation

- Meshless
- Efficient
- Few Degrees of Freedom (DoF)



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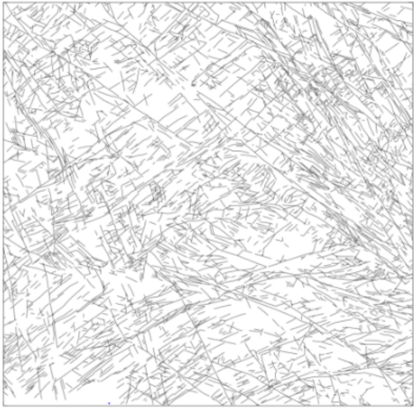
Motivation

- Meshless
- Efficient
- Few Degrees of Freedom (DoF)

Mechanics

Giulia Conti (3.2.MS07, Thursday 11:50 AM)



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Overview

Incompressible, single-phase, steady-state flow in porous media

$$\nabla \cdot \left(-\frac{k}{\mu} \nabla p \right) = 0$$

with permeability k, viscosity μ, and pressure p.

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Our Ansatz:

$$p(\mathbf{x}) = \sum_{i=1}^{N} w_i \hat{p}(\mathbf{T}_i^{G \to L} \mathbf{x}) + p_\infty(\mathbf{x})$$

Each fracture has the same basis function $\hat{p}$, but different weights $w_i \to$ 1 DoF per Fracture.

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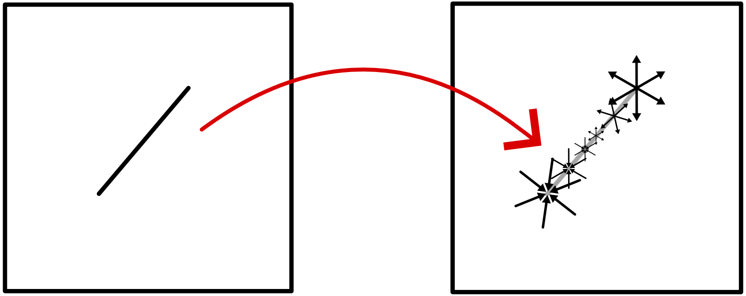
Solve a dense $N \times N$ system with N the number of fractures.

Theory



$$\nabla \cdot \left(-\frac{k}{\mu} \nabla p \right) = 0 \quad \Longrightarrow \quad -\frac{k_m}{\mu} \nabla^2 p = q$$

Theory

Green's Solution

$$\hat{p}(\mathbf{x}) = -\frac{\mu}{k_m} \int_{-1/2}^{1/2} \frac{\lambda_0(s)}{2\pi} \ln \|\mathbf{x} - s\hat{e}_x\| \, \mathrm{d}s$$

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Green's Solution

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Theory

Green's Solution

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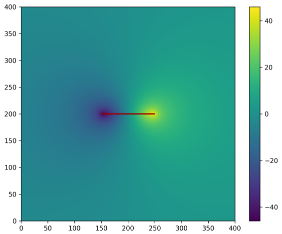
Collocation method to find the weights w_i

$$p(x_{\text{tip A}}) - p(x_{\text{tip B}}) = \Delta p$$

- Infinite conductivity: $\Delta p = 0$
- Finite conductivity: $\Delta p = F(a_f, k_f, l_f)$

Theory

$$p(\mathbf{x}) = \sum_{i=1}^{N} w_i \hat{p}(\mathbf{T}_i^{G \to L}\mathbf{x}) + p_\infty(\mathbf{x})$$

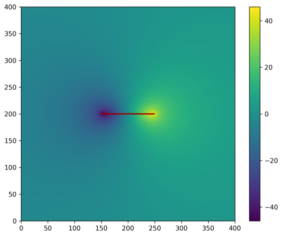


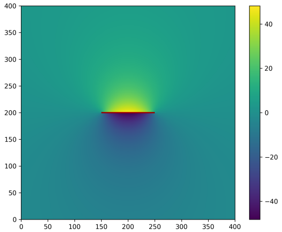
Theory

$$p(\mathbf{x}) = \sum_{i=1}^{N} w_i \hat{p}(\mathbf{T}_i^{G\to L}\mathbf{x}) + p_\infty(\mathbf{x})$$

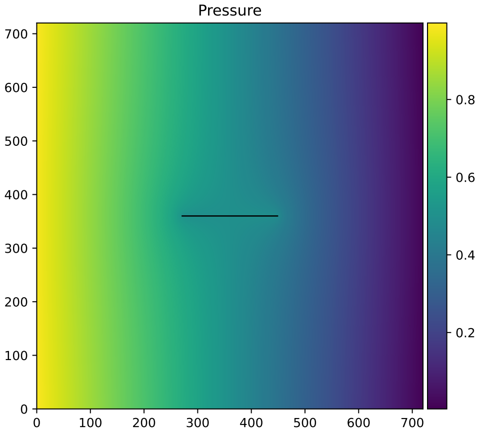
$$\Psi(\mathbf{x}) = \sum_{i=1}^{N} w_i \hat{\Psi}(\mathbf{T}_i^{G\to L}\mathbf{x}) + \Psi_\infty(\mathbf{x})$$

- Similar derivation for stream function Ψ
- Same weights w_i



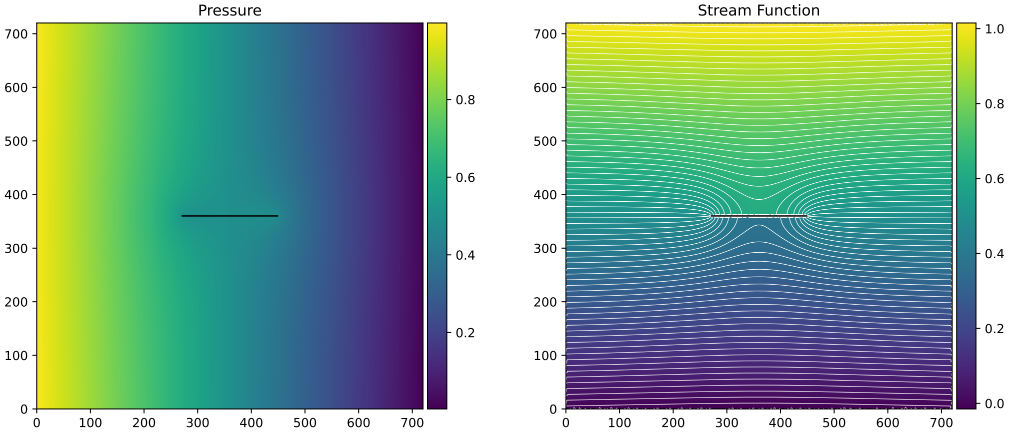


Results



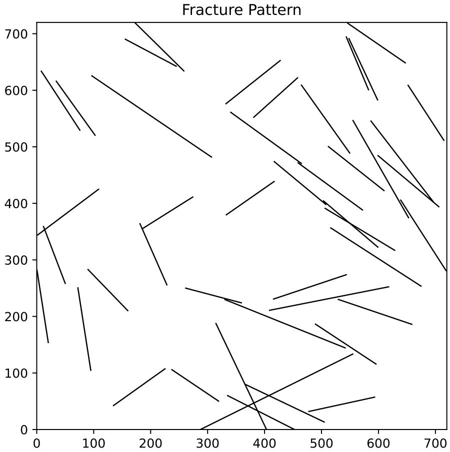
Pressure
700
600
500
400
300
200
100
0
0
100
200
300
400
500
600
700
0.8
0.6
0.4
0.2

Results



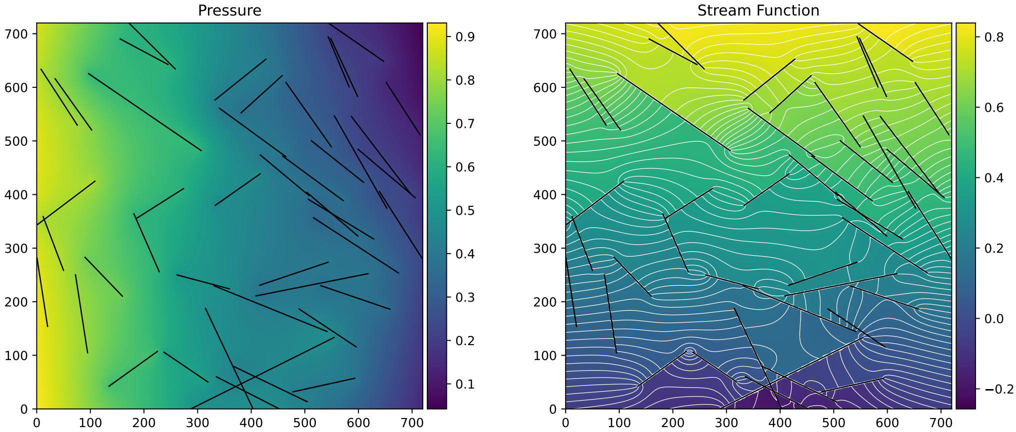
Pressure
700
600
500
400
300
200
100
0
0
100
200
300
400
500
600
700
0.8
0.6
0.4
0.2
Stream Function
700
600
500
400
300
200
100
0
0
100
200
300
400
500
600
700
1.0
0.8
0.6
0.4
0.2
0.0

Results



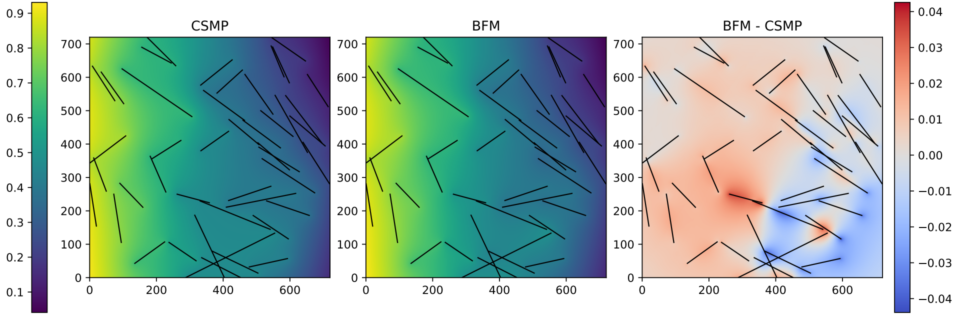
- Odling: 44 Fractures
- Meshing with GMSH
- Flow Simulation with CSMP++ (by Stephan Matthai and collegues)

Results

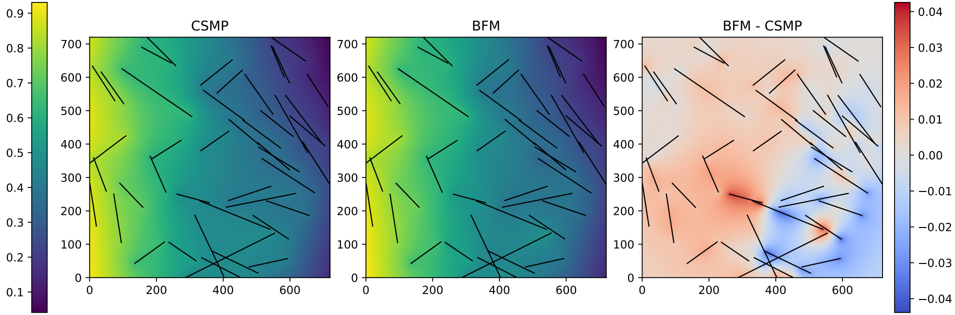


Pressure
Stream Function
0
100
200
300
400
500
600
700
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
−0.2
0.0
0.2
0.4
0.6
0.8

Results


CSMP
BFM
BFM - CSMP
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
100
200
300
400
500
600
700
0.04
0.03
0.02
0.01
0.00
−0.01
−0.02
−0.03
−0.04

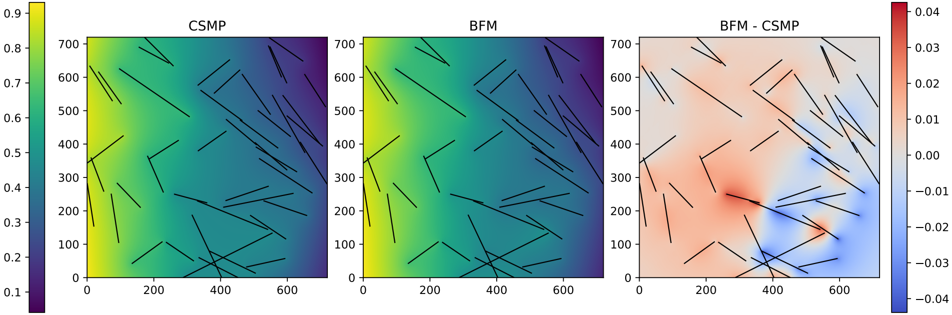
Results



CSMP

BFM

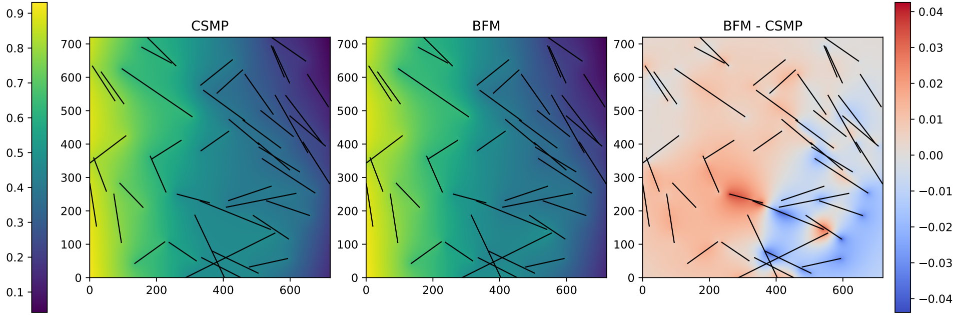
Results



CSMP Meshing (103s)

BFM

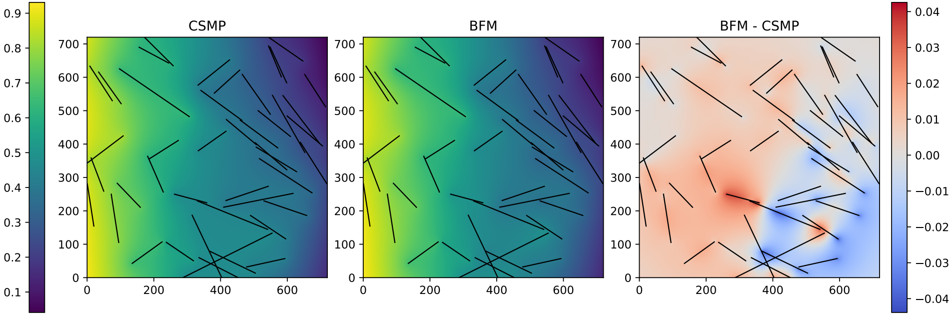
Results



CSMP Meshing (103s) Solver (133s) = 236s

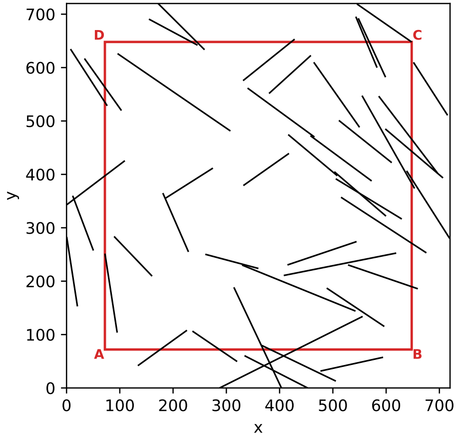
BFM

Results

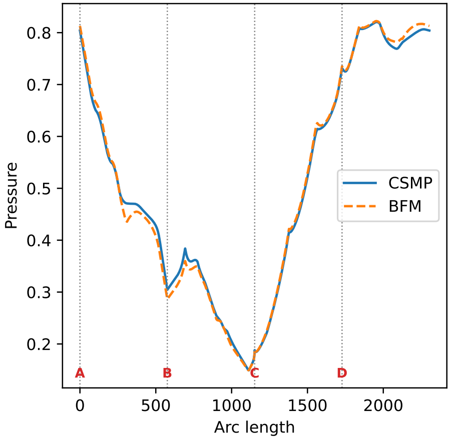

CSMP
BFM
BFM - CSMP
0.9 0.8 0.7 0.6 0.5 0.4 0.3 0.2 0.1
0 100 200 300 400 500 600 700
0 200 400 600
0.04 0.03 0.02 0.01 0.00 −0.01 −0.02 −0.03 −0.04


CSMP
Meshing (103s)
Solver (133s)
= 236s
BFM
1s

Results



700
600
500
400
300
200
100
0
y
0
100
200
300
400
500
600
700
x
A
B
C
D



0.8
0.7
0.6
0.5
0.4
0.3
0.2
Pressure
0
500
1000
1500
2000
Arc length
CSMP
BFM
A
B
C
D

Transport

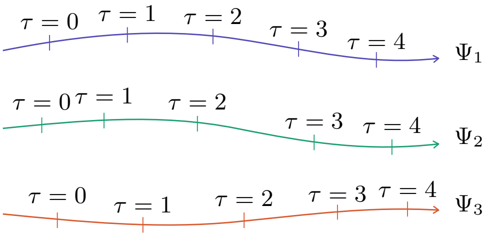
Time of Flight

$$\frac{\mathrm{D}\tau}{\mathrm{D}t} = 1 \qquad \Longleftrightarrow \qquad \cancel{\frac{\partial \tau}{\partial t}} + \frac{u_i}{\phi} \frac{\partial \tau}{\partial x_i} = 1$$

Transport

Time of Flight

$$\frac{D\tau}{Dt} = 1 \quad \Longleftrightarrow \quad \cancel{\frac{\partial \tau}{\partial t}} + \frac{u_i}{\phi}\frac{\partial \tau}{\partial x_i} = 1$$



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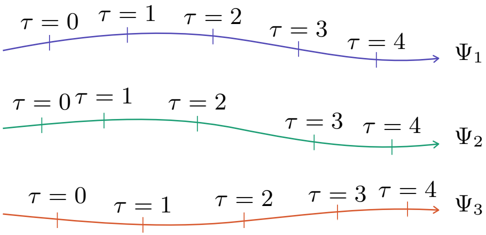


Passive Scalar Transport

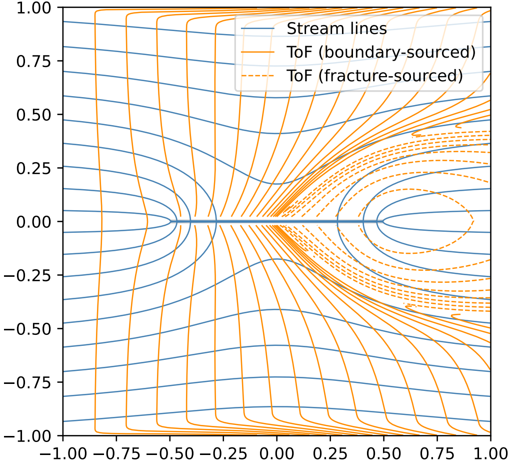
$$\frac{\partial c}{\partial t} + \frac{u_i}{\phi}\frac{\partial c}{\partial x_i} = 0 \quad \Longleftrightarrow \quad \frac{\partial c}{\partial t} + \frac{\partial c}{\partial \tau} = 0$$

Transport

1. Calculate τ for the domain

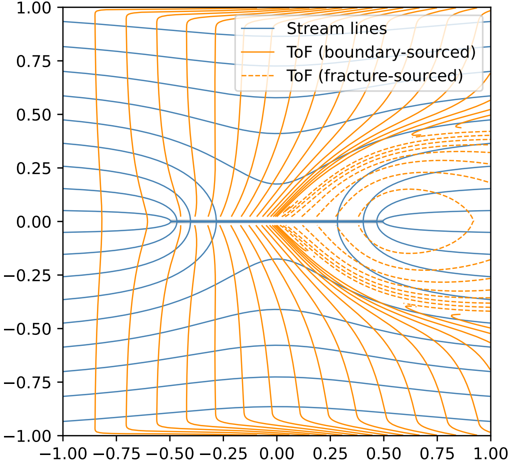
Transport

1. Calculate τ for the domain
2. Create a map $(x, y, t) \leftrightarrow (\Psi, \tau)$



Transport

1. Calculate τ for the domain
2. Create a map $(x, y, t) \leftrightarrow (\Psi, \tau)$
3. Transport is simply a shift in τ:
   $(\Psi, \tau) \rightarrow (\Psi, \tau + \Delta t)$



Transport

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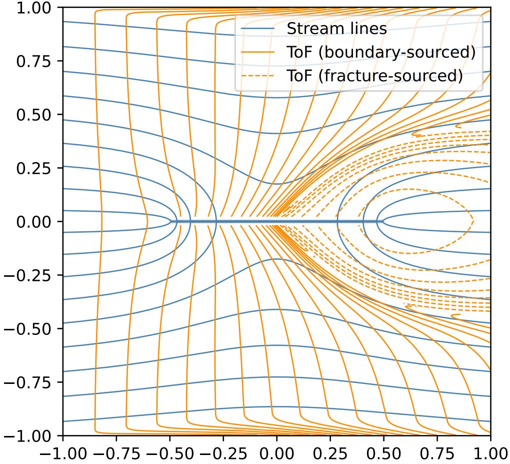
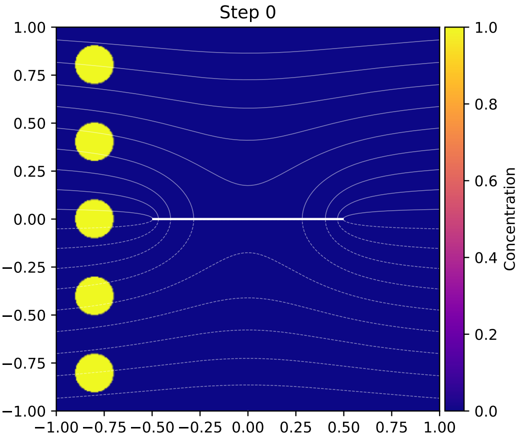
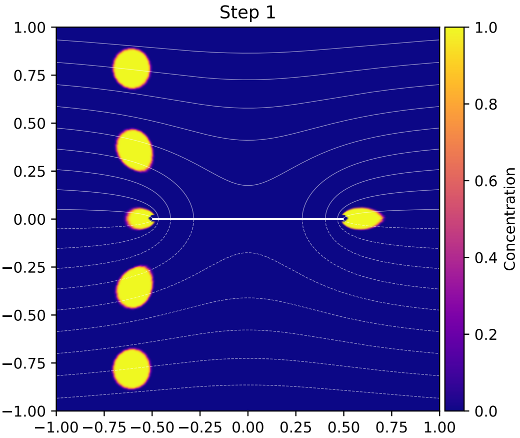
2. Create a map $(x, y, t) \leftrightarrow (\Psi, \tau)$
3. Transport is simply a shift in τ: $(\Psi, \tau) \rightarrow (\Psi, \tau + \Delta t)$
4. Map back to physical space: $(\Psi, \tau + \Delta t) \rightarrow (x', y', t + \Delta t)$



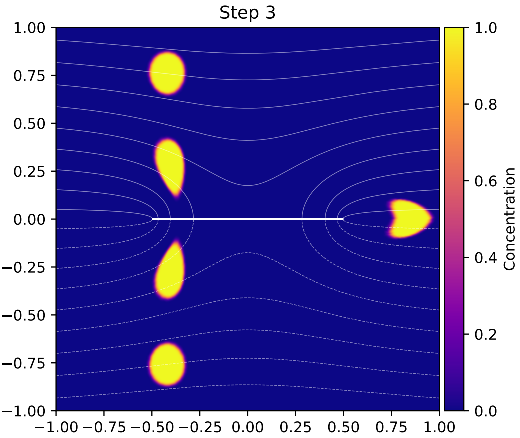
Result


Step 0
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
−1.00 −0.75 −0.50 −0.25 0.00 0.25 0.50 0.75 1.00
1.0
0.8
0.6
0.4
0.2
0.0
Concentration

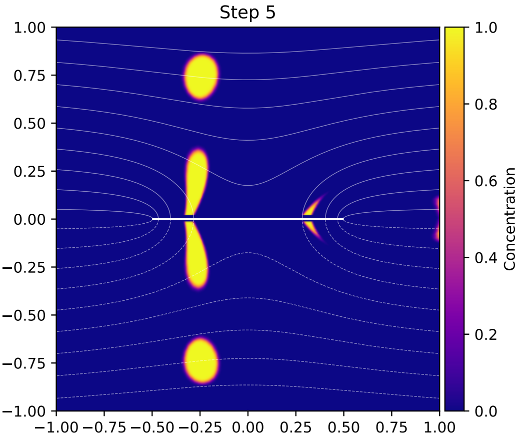
Result


Step 1
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
−1.00 −0.75 −0.50 −0.25 0.00 0.25 0.50 0.75 1.00
1.0
0.8
0.6
0.4
0.2
0.0
Concentration

Result

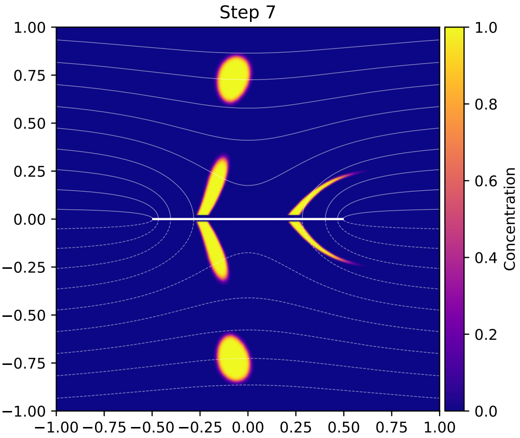

Step 3
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
−1.00 −0.75 −0.50 −0.25 0.00 0.25 0.50 0.75 1.00
1.0
0.8
0.6
0.4
0.2
0.0
Concentration

Result



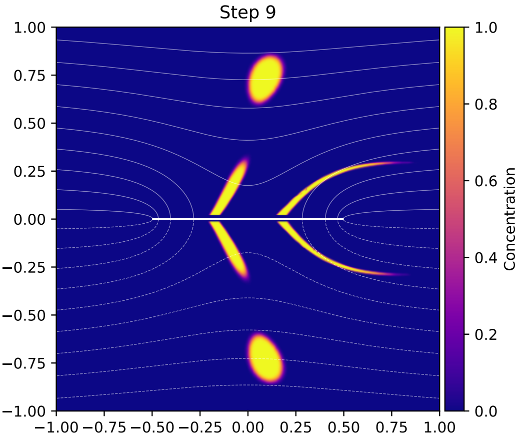
Step 5
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
−1.00 −0.75 −0.50 −0.25 0.00 0.25 0.50 0.75 1.00
1.0
0.8
0.6
0.4
0.2
0.0
Concentration

Result



Step 7
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
−1.00 −0.75 −0.50 −0.25 0.00 0.25 0.50 0.75 1.00
Concentration
1.0
0.8
0.6
0.4
0.2
0.0

Result


Step 9
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
−1.00 −0.75 −0.50 −0.25 0.00 0.25 0.50 0.75 1.00
1.0
0.8
0.6
0.4
0.2
0.0
Concentration

Conclusion and Outlook

Conclusion:

- Meshless
- Efficient
- Few DoF (1 per fracture)

Conclusion and Outlook

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- Few DoF (1 per fracture)

Work in Progress / Future Work:

- Passive Scalar Transport

Conclusion and Outlook

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- Coupling with Mechanics

Conclusion and Outlook

Conclusion:

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- Efficient
- Few DoF (1 per fracture)

Work in Progress / Future Work:

- Passive Scalar Transport
- Coupling with Mechanics
- 3D Extension

Conti G., Matthai S., Jenny P., 2025, Fracture Displacement Basis Function (FDBF) Method for Efficient Geomechanical Calculations of Fractured Rock, Int. J. Rock Mech. Min. Sci.