The traveling wavefront for foam flow in multi-layer porous media

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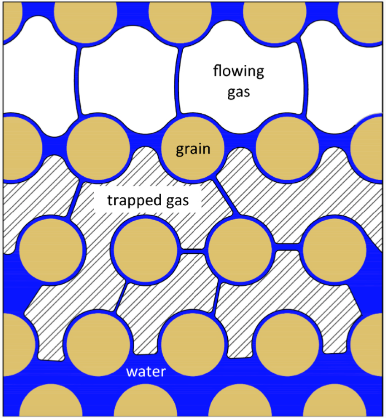
Foam in porous media

What is foam in porous media

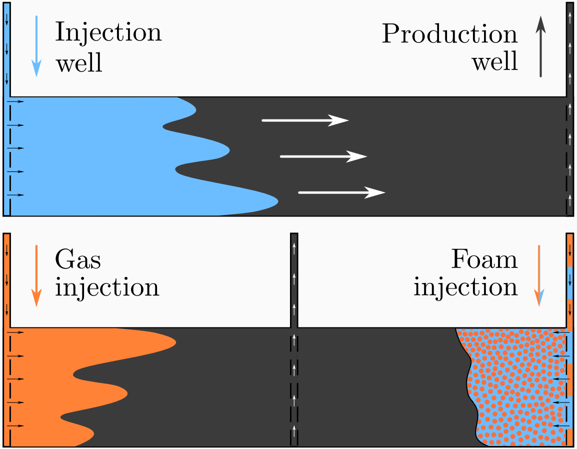
- Lamellae (liquid films) separate gas bubbles.
- Foam reduces the gas mobility.
- Foam texture is modeled as a tracer in the gas phase.
- Foam does not affect water phase relative mobility.



Almajid M.M. and Kovscek A. R., **TiPM**, 2020.

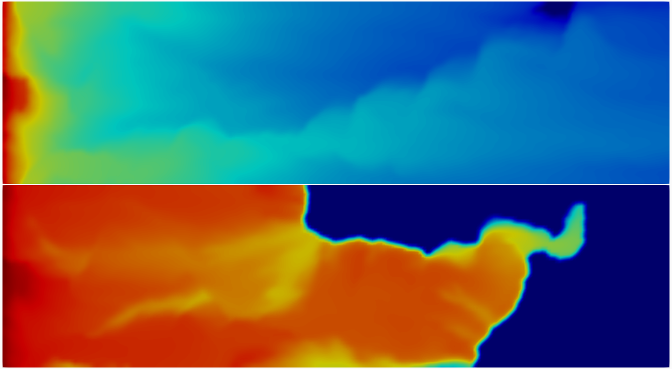
Applications of foam in porous media

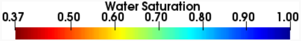
- Soil/aquifer remediation.
- EOR.
- CO_2 sequestration.
- Others.



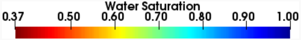
Cedro J. B., Chapiro G., Under review, 2023.

Two-dimensional simulation in heterogeneous porous media with gravity.





No foam.



With foam.

Figure 1: Water saturation in SPE10(36) por. medium at time $t = 10000$s.

F. F. de Paula, T. Quinelato, I. Igreja, G. Chapiro., **LNCS**, 2020.

Mathematical modeling of foam flow in a two-layered porous medium

2D linear kinetic model

The two-dimensional model is:

$$\begin{aligned}
\phi\frac{\partial}{\partial t}S_{\mathrm{w}}+\nabla\cdot\mathbf{u}_{w} &= 0;\\
\phi\frac{\partial}{\partial t}(S_{\mathrm{g}}n_{\mathrm{D}})+\nabla\cdot(\mathbf{u}_{g}n_{\mathrm{D}}) &= \phi S_{\mathrm{g}}\varPhi;\\
\mathbf{u} &= -\lambda\nabla P;\\
\nabla\cdot\mathbf{u} &= 0,
\end{aligned}$$

Assumptions:

- Incompressible and immiscible two-phase
- $S_{\mathrm{w}}+S_{\mathrm{g}}=1$.
- Isotropic medium.
- $n_D = n_f/n_{max}$.

Foamed gas relative permeability is reduced by Mobility Reduction Factor: $k_{\mathrm{rg}}(S_{\mathrm{w}},n_{\mathrm{D}})=\frac{k^0_{\mathrm{rg}}(S_{\mathrm{w}})}{\mathrm{MRF}(n_{\mathrm{D}})}$.

Linear Kinetic Model

Ashoori et al., **TiPM**, 2011.

- $\mathrm{MRF}(n_{\mathrm{D}})=18500n_{\mathrm{D}}+1$.
- Linear generation and coalescence rate: $\varPhi=r_{\mathrm{g}}-r_{\mathrm{c}}=K_c(n_{\mathrm{D}}^{\mathrm{LE}}(S_{\mathrm{w}})-n_{\mathrm{D}})$.
- Local equilibrium foam texture:

$$n_{\mathrm{D}}^{\mathrm{LE}}(S_{\mathrm{w}})=\begin{cases}\tanh(A(S_{\mathrm{w}}-S_{\mathrm{w}}^*)), & S_{\mathrm{w}}>S_{\mathrm{w}}^*,\\ 0, & S_{\mathrm{w}}\leq S_{\mathrm{w}}^*.\end{cases}$$

- S_{w}^* critical value at which foam collapses.

$$\begin{aligned}
u_{\mathrm{w}} &= uf_{\mathrm{w}}+\lambda_{\mathrm{g}}f_{\mathrm{w}}\nabla P_c,\\
P_{\mathrm{c}} &= \sigma\sqrt{\frac{\phi}{k}}\,\frac{0.022\,(1-S_{\mathrm{w}}-S_{\mathrm{gr}})^c}{(S_{\mathrm{w}}-S_{\mathrm{wc}})}.
\end{aligned}$$

Numerical evidences of the traveling waves solution

We use Foam Displament Simulator **FOSSIL**

- No-flow boundary conditions at $z = \pm d$.
- $u = u_i$ constant for each layer.
- $S_\mathrm{w} = S_\mathrm{w}^-$.
- In $x = L$, $\partial S_\mathrm{w} / \partial x = 0$.

de Paula et al., **LNCS**, 2020.

de Paula et al., **AWR**, 2023.

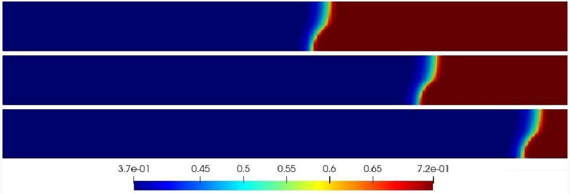


Figure 2: Stable traveling water saturation profile at 3000 s (upper plot), 4000 s (middle plot) and at 5000 s (lower plot).

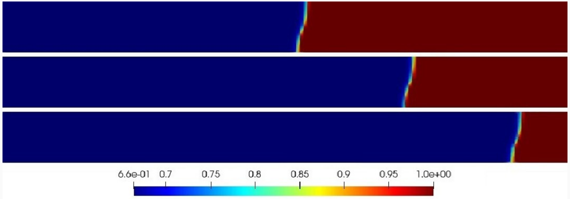


Figure 3: Stable traveling foam texture profile at 3000 s (upper plot), 4000 s (middle plot) and at 5000 s (lower plot).

1D approximation for the foam flow in two layers

Considering different permeabilities with the water saturations S_{w_1}, S_{w_2} and the foam texture n_{D_1}, n_{D_2} in each layer, the one-dimensional model is:

$$\begin{cases} \phi_1 \frac{\partial}{\partial t} S_{w_1} + \frac{\partial}{\partial x} u_{w_1} & = & -\theta_{s_1}(S_{w_1} - S_{w_2}), \\ \phi_1 \frac{\partial}{\partial t}(n_{D_1} S_{g_1}) + \frac{\partial}{\partial x}(u_{g_1} n_{D_1}) & = & \phi_1 S_{g_1} \Phi_1, \\ \phi_2 \frac{\partial}{\partial t} S_{w_2} + \frac{\partial}{\partial x} u_{w_2} & = & \theta_{s_2}(S_{w_1} - S_{w_2}), \\ \phi_2 \frac{\partial}{\partial t}(n_{D_2} S_{g_2}) + \frac{\partial}{\partial x}(u_{g_2} n_{D_2}) & = & \phi_2 S_{g_2} \Phi_2. \end{cases}$$

$$u_{w_i} = u_i f_{w_i} + \lambda_g f_{w_i} \nabla P_{c_i}, \; i = 1, 2.$$

$$(S_{w_1}, n_{D_1}, S_{w_2}, n_{D_2})(x, 0) = \begin{cases} (S^-_{w_1}, n^-_{D_1}, S^-_{w_2}, n^-_{D_2}), & \text{if } x < 0, \\ (S^+_{w_i}, n^+_{D_i}, S^+_{w_2}, n^+_{D_2}), & \text{if } x \geq 0. \end{cases}$$

Castrillon *et al.*, **COMG**, 2022.

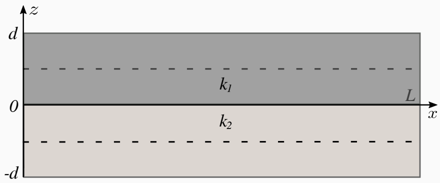


Figure 4: Schematic representation of the two-layer domain.

$$\begin{aligned} S_{w_1} &= \frac{1}{d}\int_{-d}^{0} S_w(z)\,dz, \\ S_{w_2} &= \frac{1}{d}\int_{0}^{d} S_w(z)\,dz. \end{aligned}$$

Results

Estimating the mass exchange between layers

Considering only viscous crossflow ($u|_z = 0$) as in

Zimmerman, R., et al. **TiPM**, 1996.

and quadratic water saturation profile (see Fig. 6 on the right), we can estimate the mass exchange coefficients:

$$\theta_{s_1} = \frac{-3\bar{D}_2\bar{D}_1}{d^2\left(\bar{D}_2 + \bar{D}_1\right)}, \quad \theta_{s_2} = \frac{3\bar{D}_2\bar{D}_1}{d^2\left(\bar{D}_2 + \bar{D}_1\right)}.$$

where

$$\bar{D}_1 = -\lim_{z\to 0^-} \lambda_{g_1} f_{w_1} \frac{\partial P_{c_1}}{\partial S_w}, \quad \bar{D}_2 = -\lim_{z\to 0^+} \lambda_{g_2} f_{w_2} \frac{\partial P_{c_2}}{\partial S_w}.$$

$$R_1 = \frac{\bar{D}_1}{d} \left.\frac{\partial S_w}{\partial z}\right|_{z\to 0^-}, \qquad R_2 = -\frac{\bar{D}_2}{d} \left.\frac{\partial S_w}{\partial z}\right|_{z\to 0^+}.$$

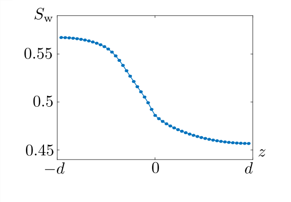


Figure 5: Average saturation inside the wave front.

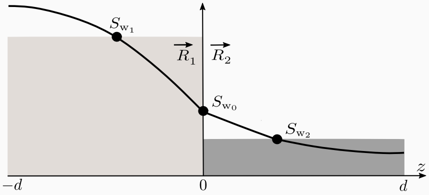


Figure 6: Schematic representation of $S_w(z)$.

What is a traveling wave solution?

Consider the PDE:

$$\frac{\partial u}{\partial t} - \frac{\partial u}{\partial x} + \frac{\partial F(u)}{\partial x} = \epsilon \Delta_{xx} u + G(u), \qquad u, F(u) \in \mathbb{R}^n.$$

Two (three?) steps:

(1) Change of variables $(x, t) \to (\xi, t)$, where $\xi = x - vt$ with v - constant traveling wave velocity, ξ - traveling variable (Euler–Lagrange coordinates)

(2) Search for the stationary solution of the system

$$\frac{\partial u}{\partial t} - V\frac{\partial u}{\partial \xi} + \frac{\partial F(u)}{\partial \xi} = \epsilon \Delta_{\xi\xi} u + G(u), \qquad u, F(u) \in \mathbb{R}^n.$$

(3) If we are dealing with the Riemann problem, the solution must satisfy the corresponding asymptotic boundary conditions.

A. I. Volpert et al., **AMS**, 2000.

The Traveling Wave Velocity

Mathematical formula for the traveling wave velocity:

$$v = \frac{a_1 v_1 + a_2 v_2}{a_1 + a_2},$$

$$a_1 = \phi_1(S_{w_1}^{+} - S_{w_1}^{-}), \; a_2 = \phi_2(S_{w_2}^{+} - S_{w_2}^{-}).$$

where v_1 and v_2 are TW velocities as in isolated layers.

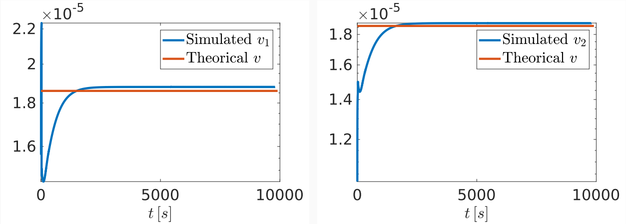


Figure 7: Moving average of the velocities v_1 (left) and v_2 for 1D model.

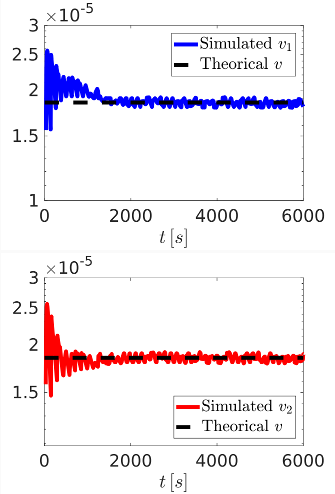


Figure 8: Simulated velocities for 2D model in each layer.

Comparing front profiles in each layer for 1D and 2D solutions

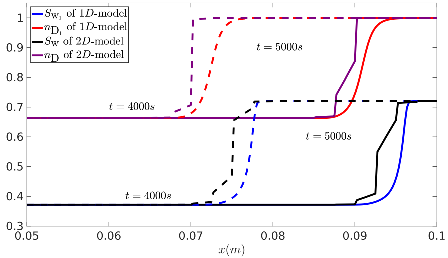


Figure 9: Water saturation S_{w} and foam texture n_{D} profiles in the first layer, at cross section $z = -2.5\,mm$.

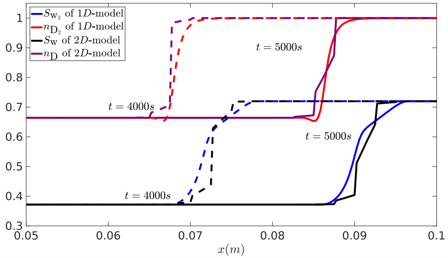


Figure 10: Water saturation S_{w} and foam texture n_{D} profiles in the second layer, at cross section $z = 2.5\,mm$.

Comparing with experimental data

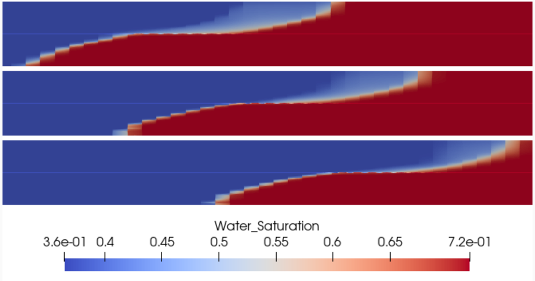


Figure 11: Numerical simulation of LKM at 0.0219 *PV* (upper plot), 0.0274 *PV* (middle plot) and at 0.0328 *PV* (lower plot).

Traveling wave stabilization times are $T_{char}^{FOSSIL} = 0.0219\,PV$ and $T_{char}^{theorical} = 0.0229\,PV$.

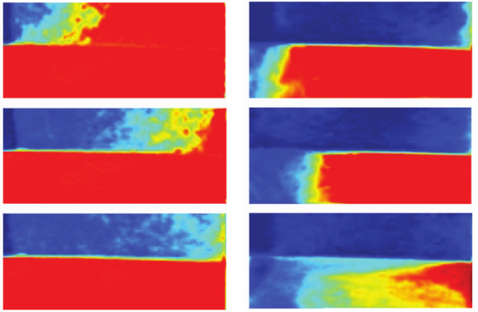


Figure 12: Experimental foam displacement (CT scan) at 0.13 *PV*, 0.30 *PV*, and 0.41 *PV* (left), and at 1.56 *PV*, 4.10 *PV*, and 8.05 *PV* (right).

Quoc P. Nguyen *et al.* (2005). **SPE**.

Discussions & Conclusions

- We show that the displacement of the foam in two parallel layers with different permeabilities forms a single traveling wavefront whose velocity is the weighted average of the velocities of the layers, considering them isolated.
- The mass transfer between layers was estimated Using a simplified 1D model.
- It is natural to expect that the traveling wavefront takes some time to stabilize both in laboratory experiments and in computational simulations. Assuming the main contribution is due to the mass exchange between layers, this time is approximately $1/\theta$. For our simulations: $1/\theta \approx 3367$s, which is the same order of magnitude that the 2D (≈ 2500s) and 1D (≈ 3500s) simulations take to stabilize.

Extended discussion can be seen in

A. J. Castrillón Vásquez, L. F. Lozano, J. B. Cedro, W. S. Pereira and G. Chapiro., **Comp. Geosc.**, 26, p. 1549-1561, 2022

Thank you for attention!

Questions?

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