Pore-network models for haematocrit transport in disordered porous domains reflecting the human placenta

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The Human Placenta


3D synchrotron micro-CT rendering of placental tissue (Tun et al., 2021).
Aim: Construct reduced-order models for blood flow informed by lattice Boltzmann simulations and microfluidic experiments.
Describing Blood Flow

Quantify behaviour of blood using discharge haematocrit:

\[ H = \frac{\iint_{\Omega} u_{RBC}(x, t)dA}{\iint_{\Omega} u(x, t)dA} \]

Where \( u_{rbc} \) and \( u \) are the RBC and whole blood velocities.

Due to particulate nature of blood, rheological accurate models must incorporate:

1. Variable viscosity
2. Fåhraeus effect
3. Unequal RBC partitioning
Constructing a reduced-order model

Define the incidence matrix $A$ such that

$$A_{ij} = \begin{cases} 
1, & \text{if edge } i \text{ points into vertex } j, \\
-1, & \text{if edge } i \text{ points out of vertex } j, \\
0, & \text{otherwise.}
\end{cases}$$

Define $V$ to be the diagonal matrix containing the volume associated with each vertex, with pressure $P$ and haematocrit $H$ defined on vertices and flux $Q$ on the edges.

$$
\begin{align*}
AP &= -RQ & \text{Pressure-flux equation} \\
ATQ &= 0 & \text{Conservation of flux} \\
V \frac{dH}{dt} &= BH & \text{Conservation of haematocrit}
\end{align*}
$$

Where we define resistance relative to plasma Stokes resistance

$$R = R_p \chi \left( H, \frac{w}{D}, \frac{L}{D}, \ldots \right),$$

and define the transport operator

$$B = B(A, H, Q).$$
Evaluating haematocrit across the network

\[ R = (1 + H)R_p \]
PCA of lattice Boltzmann simulations

- Principal component analysis (PCA) quantifies parameter contribution to variability within data sets.

- Lattice Boltzmann simulations provide data on relative resistance $\chi$, haematocrit $H$, and non-dimensionalised throat width $\frac{w}{D}$ and length $\frac{L}{D}$.

- PCs represent an orthogonal linear transformation of data variables.

- PC2 dominated by non-dimensionalised length $\frac{L}{D}$, suggesting $\frac{L}{D}$ must be one of the key determinants of $\chi$. 
Effect of tissue heterogeneity on flow

(left) 3D synchrotron micro-CT rendering of placental tissue, (centre) associated ball and stick model showing throats and pores, (right) CFD analysis of flow (Tun et al., 2021).

Kruskal-Wallis Null Hypothesis Test:

\[ H_0: \text{Groups have same distribution.} \]

\[ H = 14.0 > 7.2 = H_C \text{ (1\% significance level)} \]

At least one group stochastically dominates.
Summary & Future directions

• New models to characterise blood micro-rheology in porous media are required.

• PCA of lattice Boltzmann data suggests a relationship between relative resistance and geometric variables in the network.

• The discrete network model detects the anisotropy in 3D networks identified by more computationally expensive CFD simulations.

• Work is planned to parameterise relative resistance using regression models using microfluidic experiments and lattice Boltzmann simulation data.

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