

# Digital twin of a laboratory-scale porous medium

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E. Fonn, K. Johannessen  
*SINTEF Digital, Norway*

InterPore2022, Abu Dhabi, 30 May – 2 June 2022







# Conference on Mathematical & Computational Issues in the Geosciences

June 19 - 22, 2023  
Bergen, Norway

## Important Deadlines

**November 21, 2022**

Submission of Minisymposium Proposals

**December 6, 2022**

SIAM Student Travel Award Applications and  
Early Career Travel Award Applications

**December 19, 2022**

Contributed Lecture, Poster, and  
Minisymposium Presentation Abstracts

## Organizing Committee Co-Chairs

**Jan M. Nordbotten**

University of Bergen, Norway

**Maša Prodanović**

The University of Texas at Austin, U.S.

*Learn more: [go.siam.org/gs23](https://go.siam.org/gs23)*





# PhD positions (mathematics) in Bergen

1. Solvers for coupled problems involving ML and PBM; deadline June 9
2. Dynamic poromechanics; deadline June 1
3. Data assimilation and optimization; deadline June 15



Or visit [jobbnorge.no](http://jobbnorge.no), or get in touch with me

# Digital twin of a laboratory-scale porous medium

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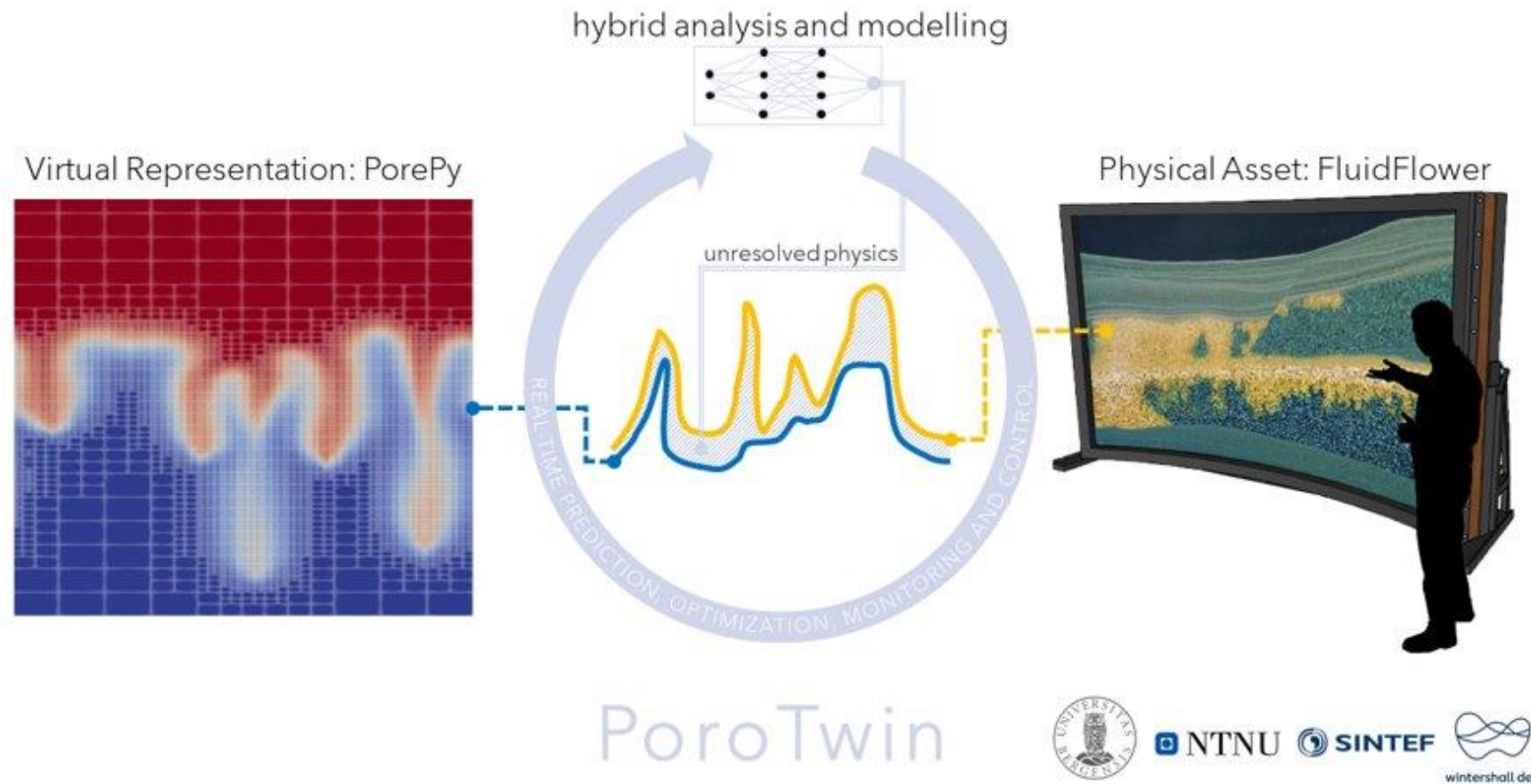
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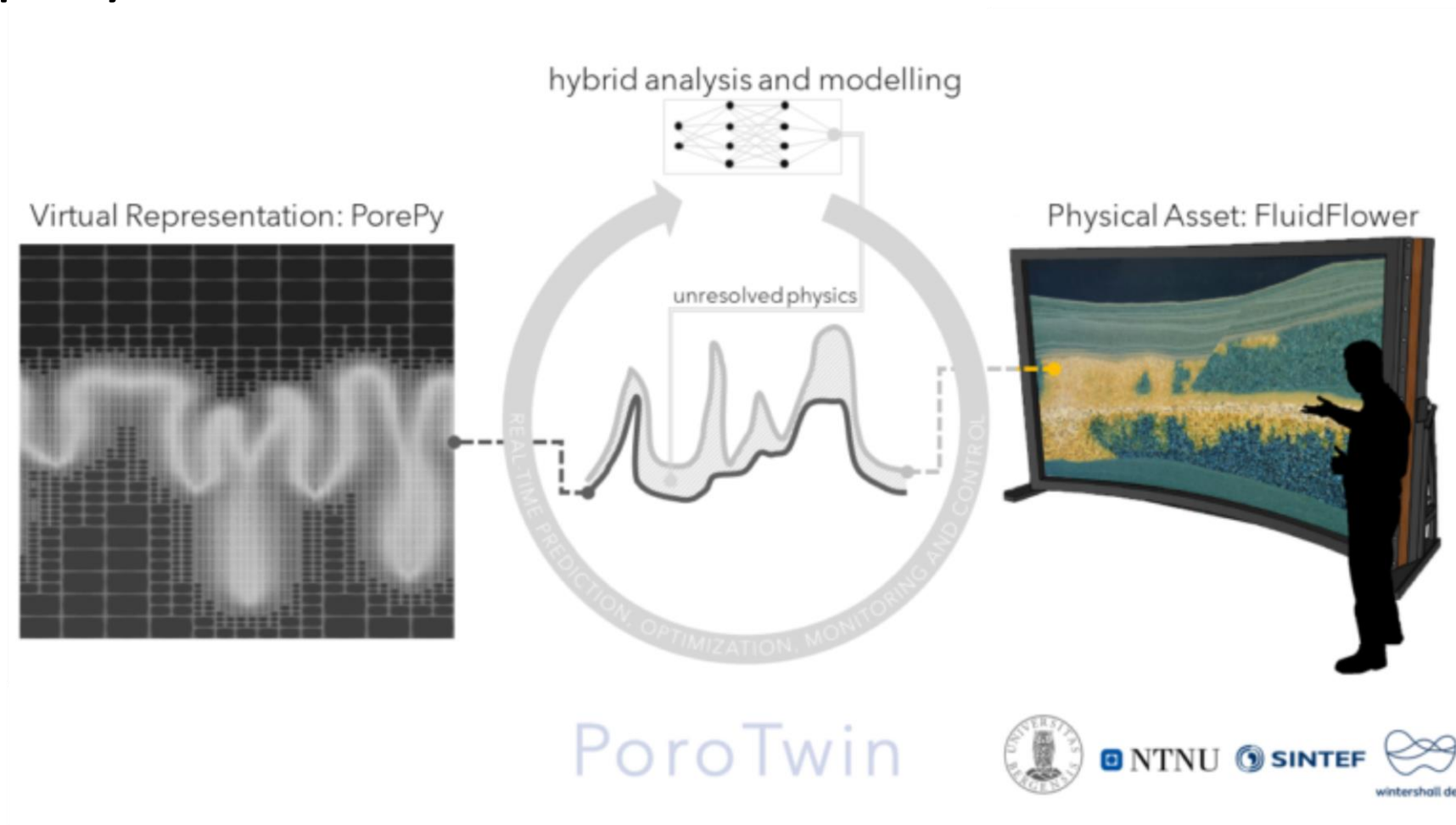


# Digital twin concept



**Long-term goal:** Digital twin of a reservoir

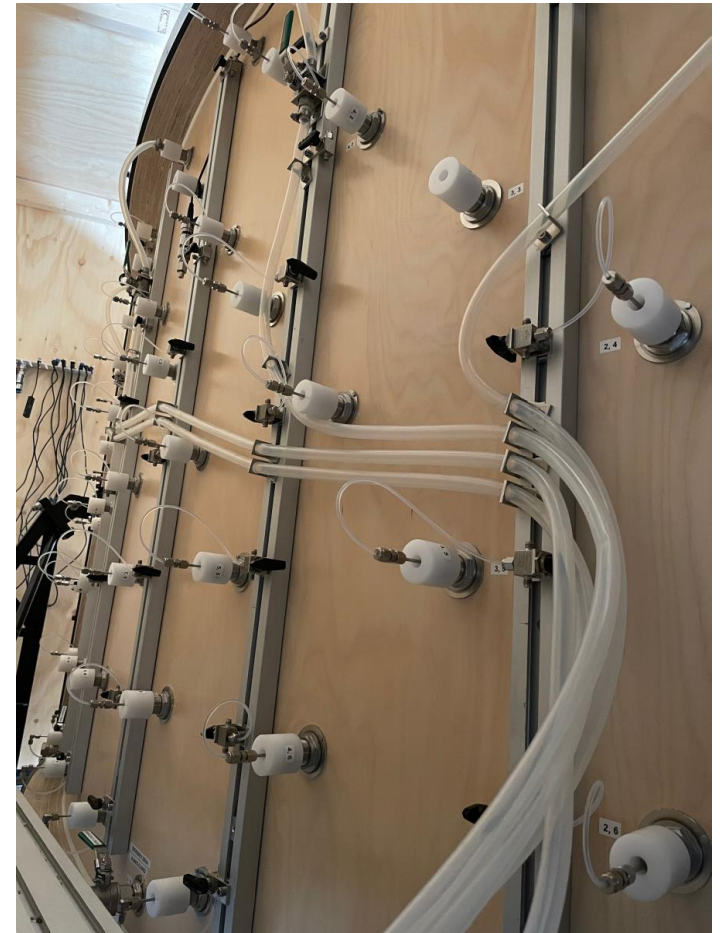
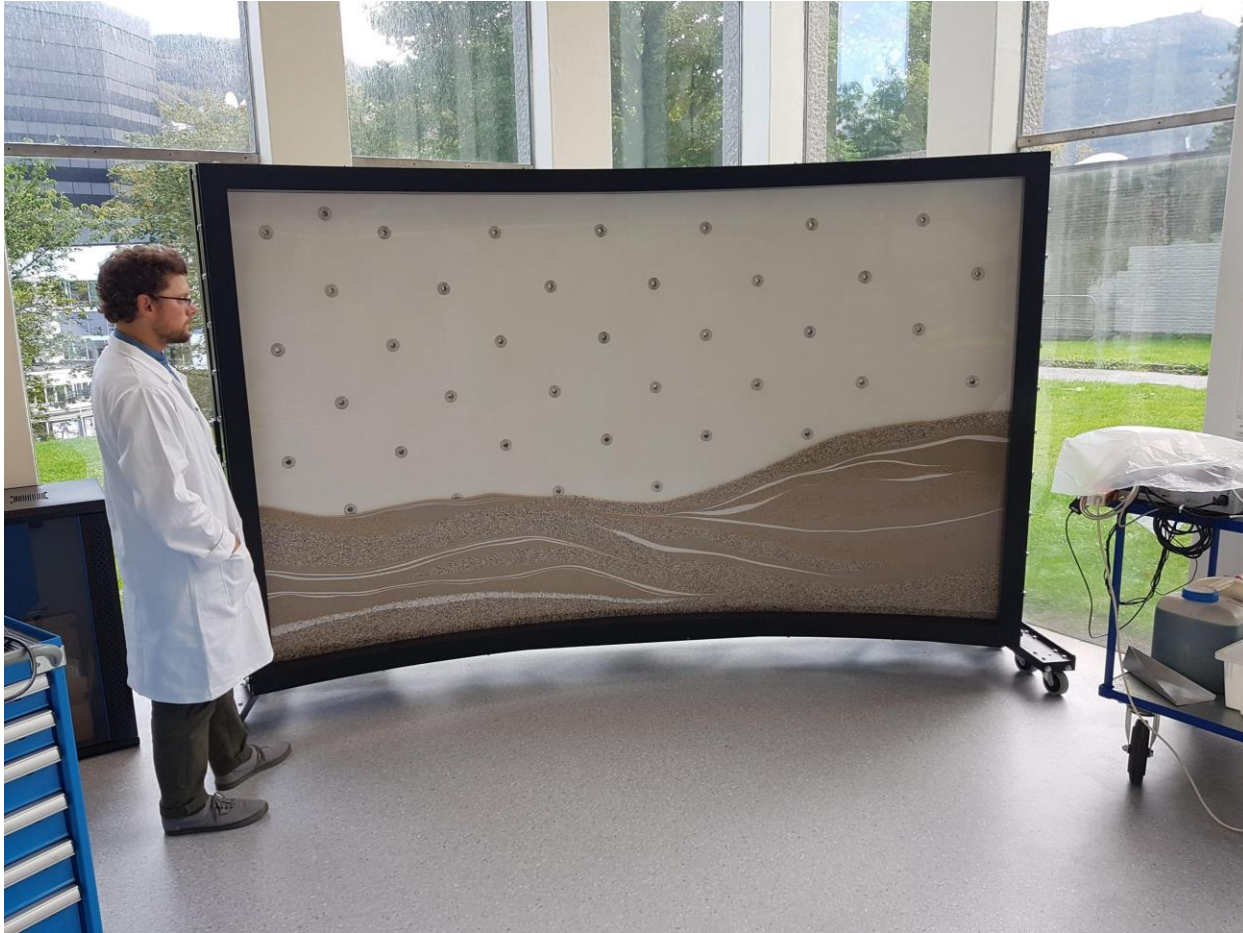
# The physical asset



**Long-term goal:** Digital twin of a reservoir

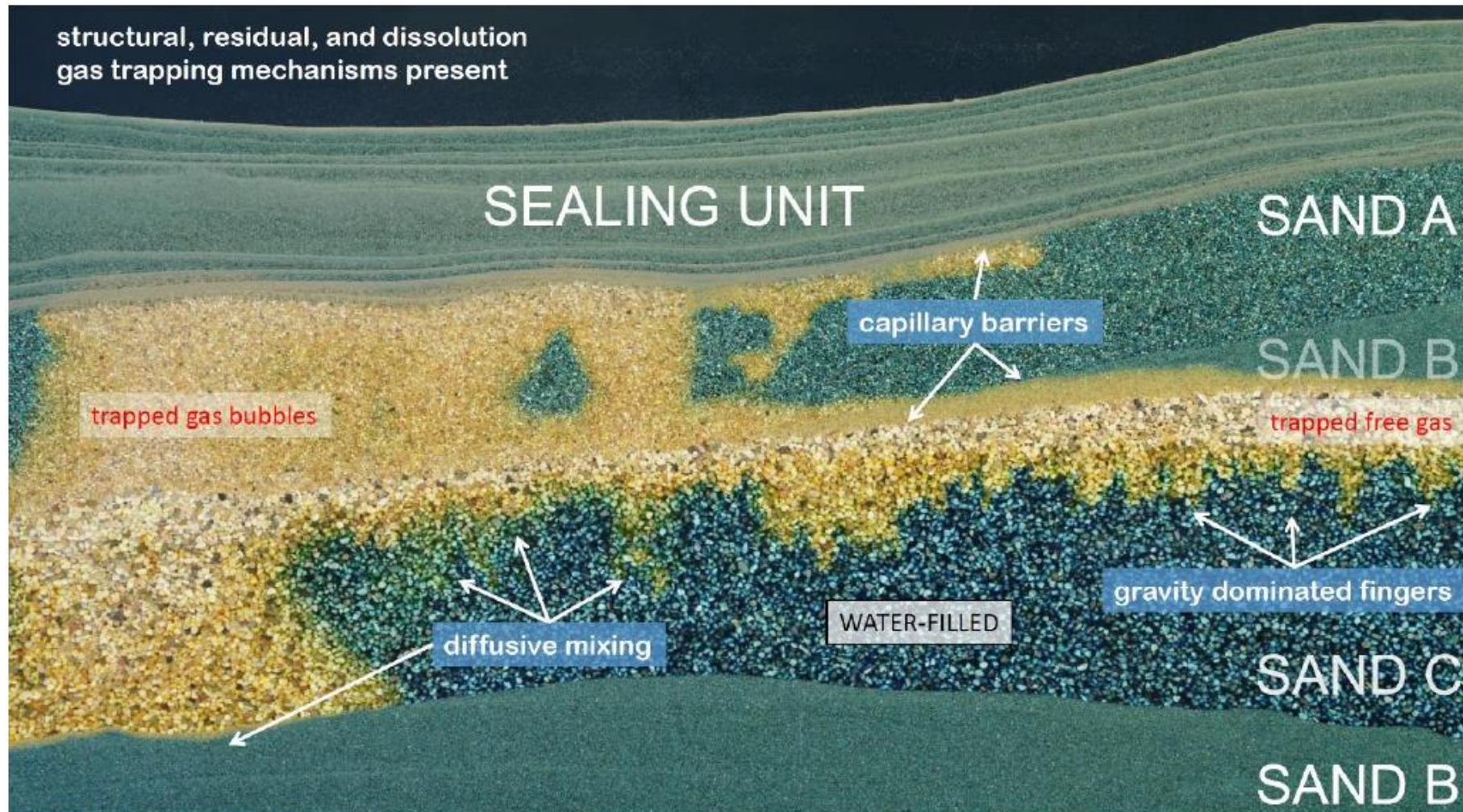


# FluidFlower



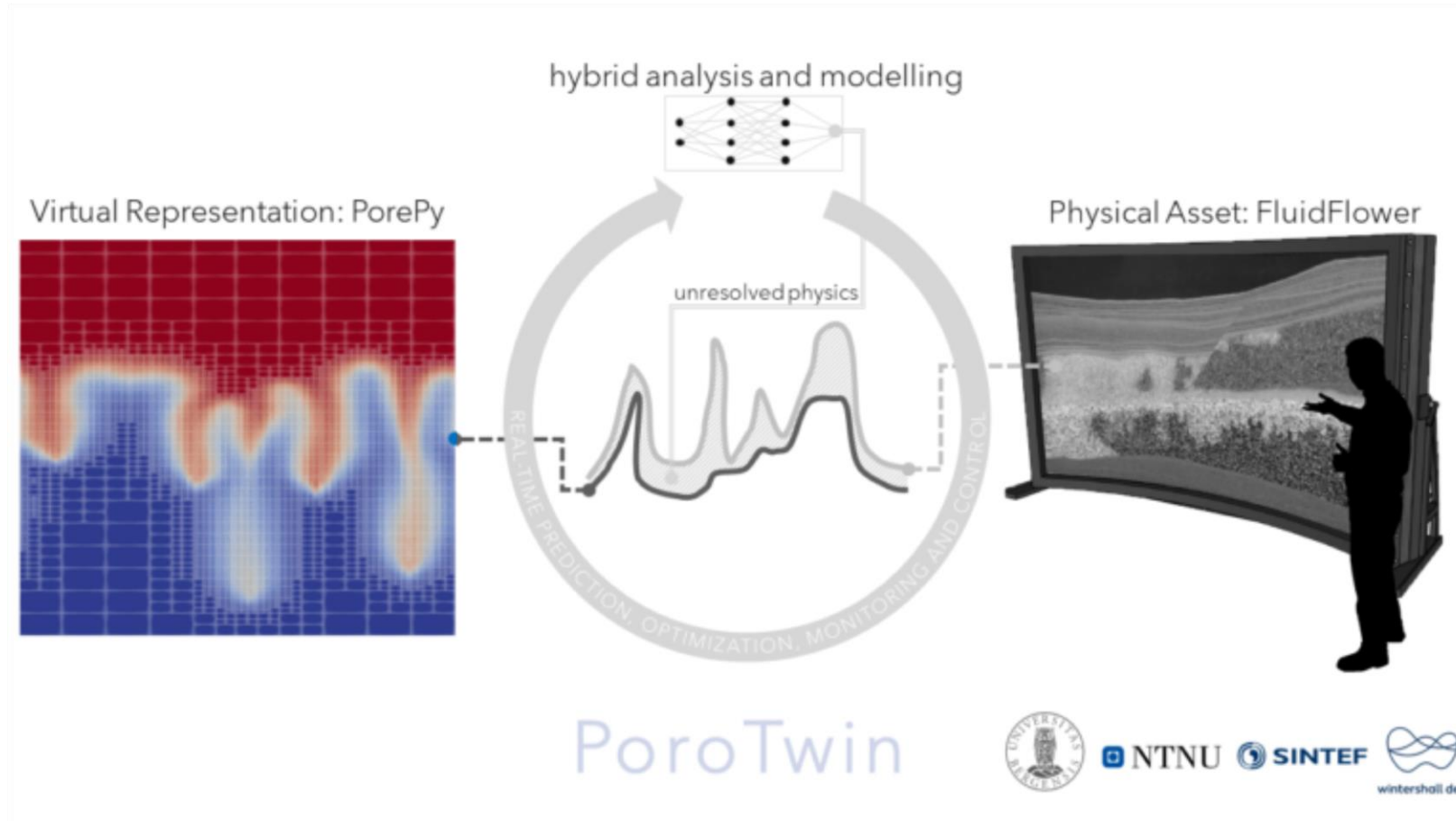


# CO2 storage experiments in the FluidFlower



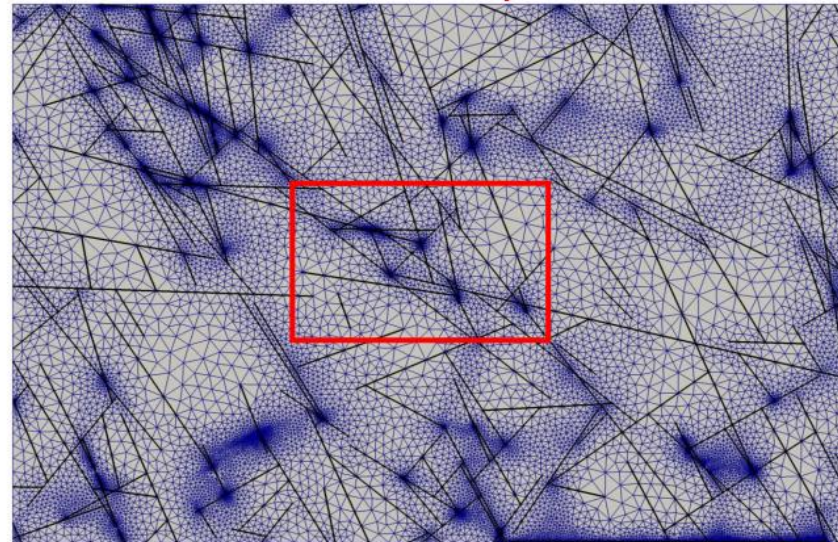
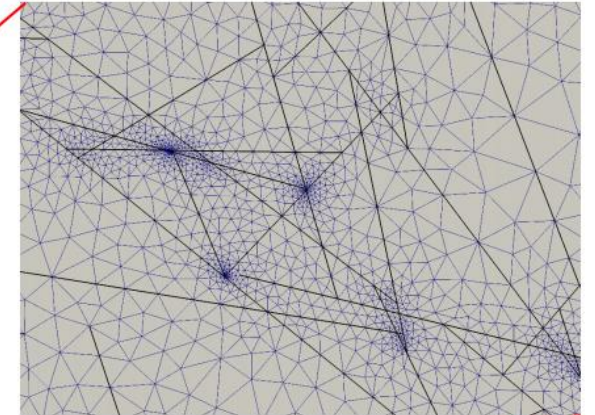
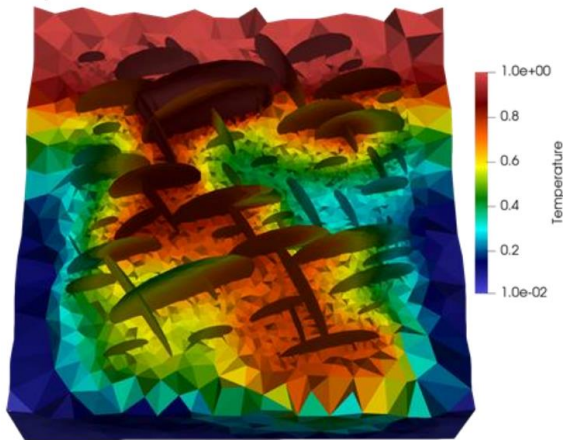


# The virtual representation



# PorePy: Physics based simulator

- Multi-physics FV simulator
- Flow, transport, geomechanics
- Tailored to fractured media
- Open-source



Keilegavlen *et al.* PorePy: an open-source software for simulation of multiphysics processes in fractured porous media. *Comput Geosci* (2021)

PorePy: [github.com/pmgbergen/porepy](https://github.com/pmgbergen/porepy)



# PBM: Flow and passive transport

Incompressible, single-phase flow:

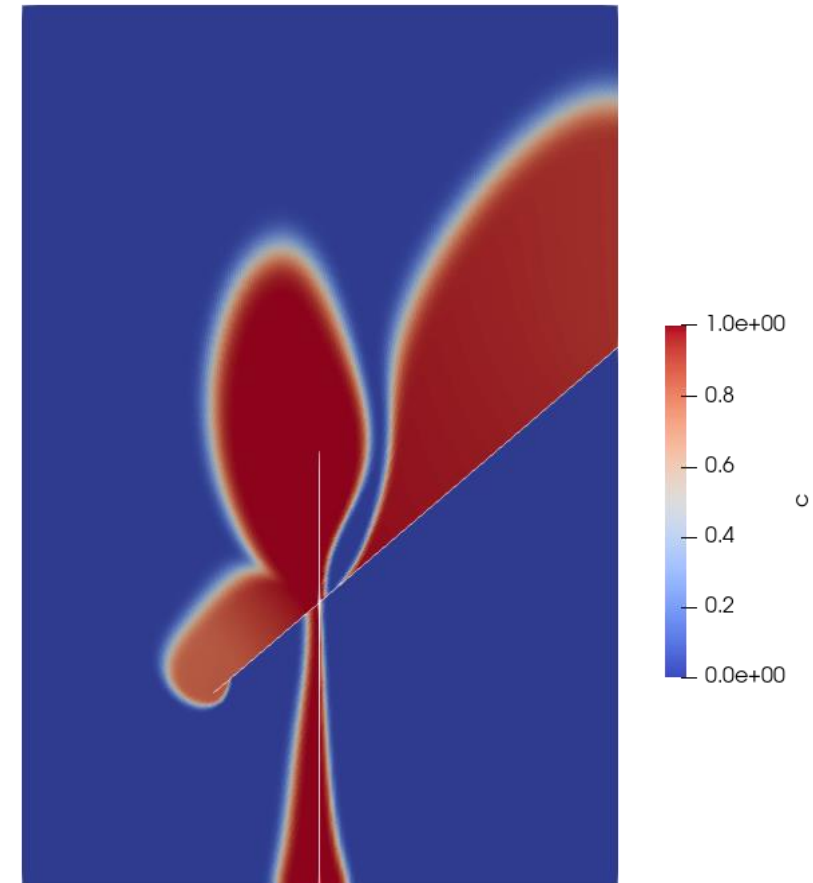
$$\nabla \cdot \mathbf{q} = h$$

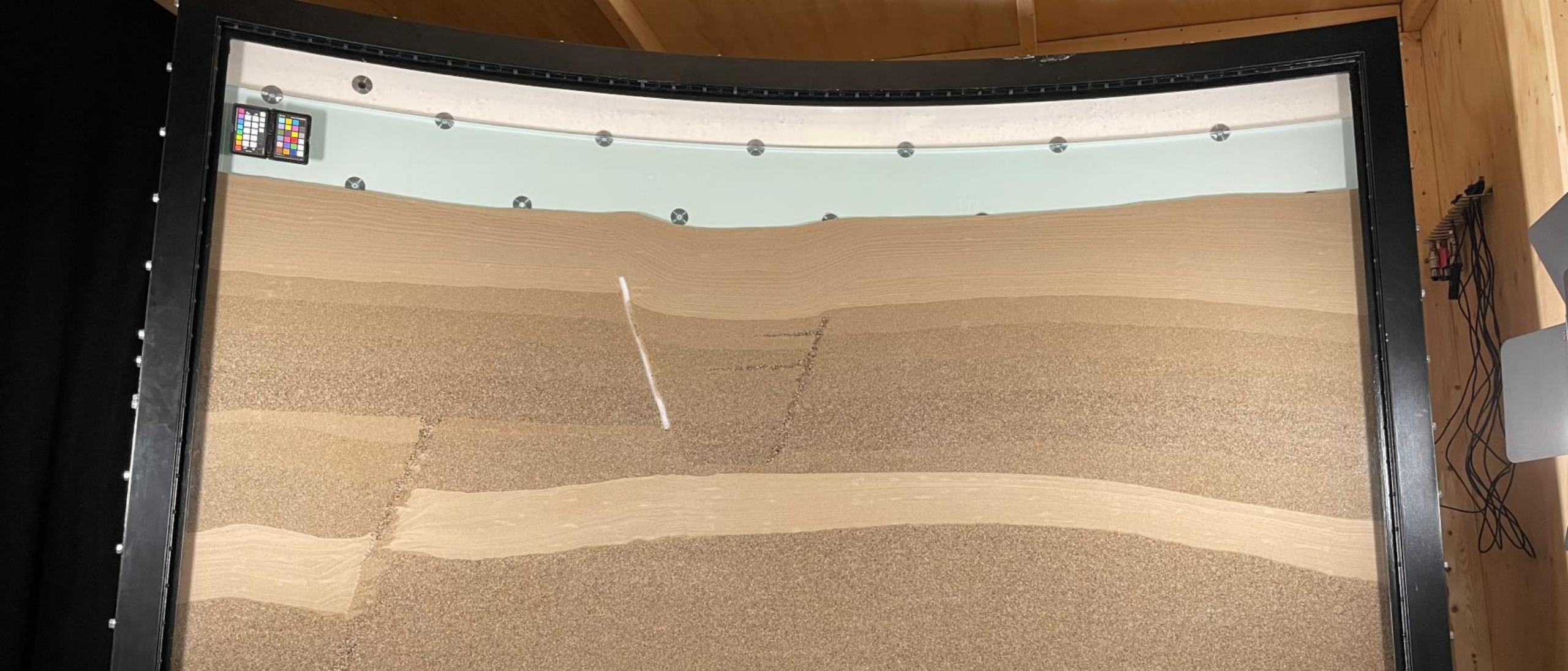
$$\mathbf{q} = -\kappa \nabla p$$

Transport with dispersion:

$$\phi \partial_t c + \nabla \cdot (c\mathbf{q} - D\nabla c) = 0$$

Example: Injection in fracture



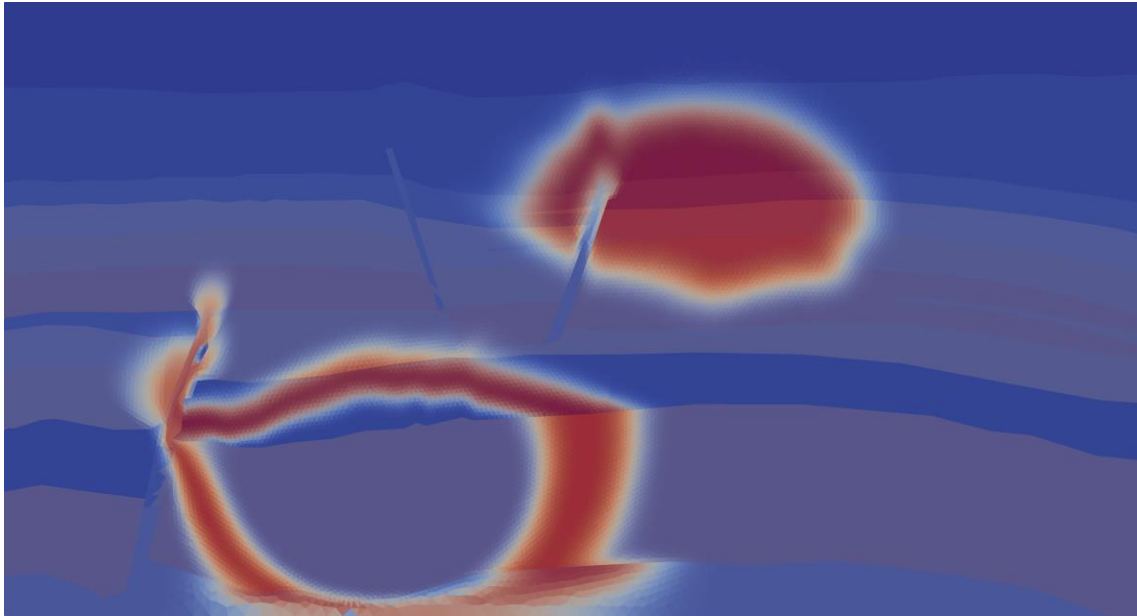


Multi-layered sand with fault-like structures

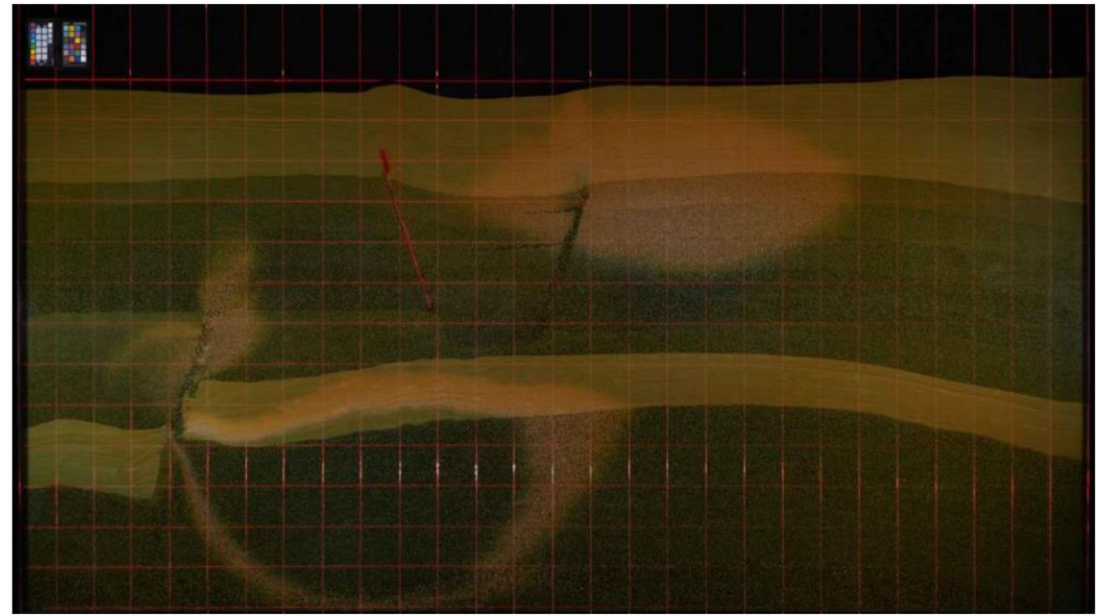


# Tracer test in multi-layered sand

PorePy Simulation

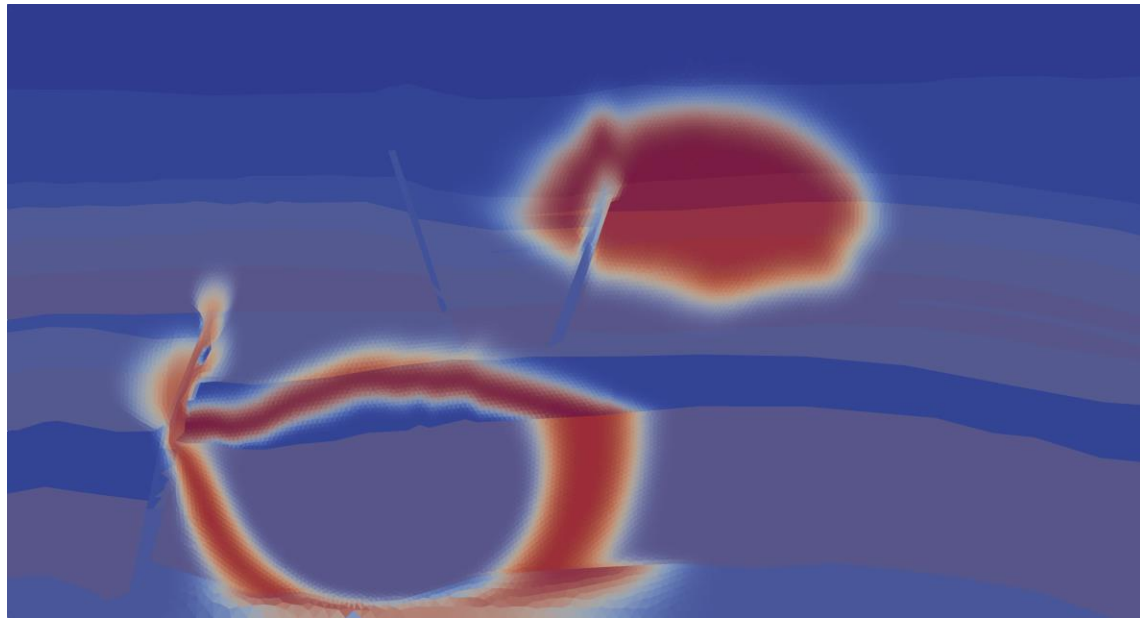


FluidFlow experiment



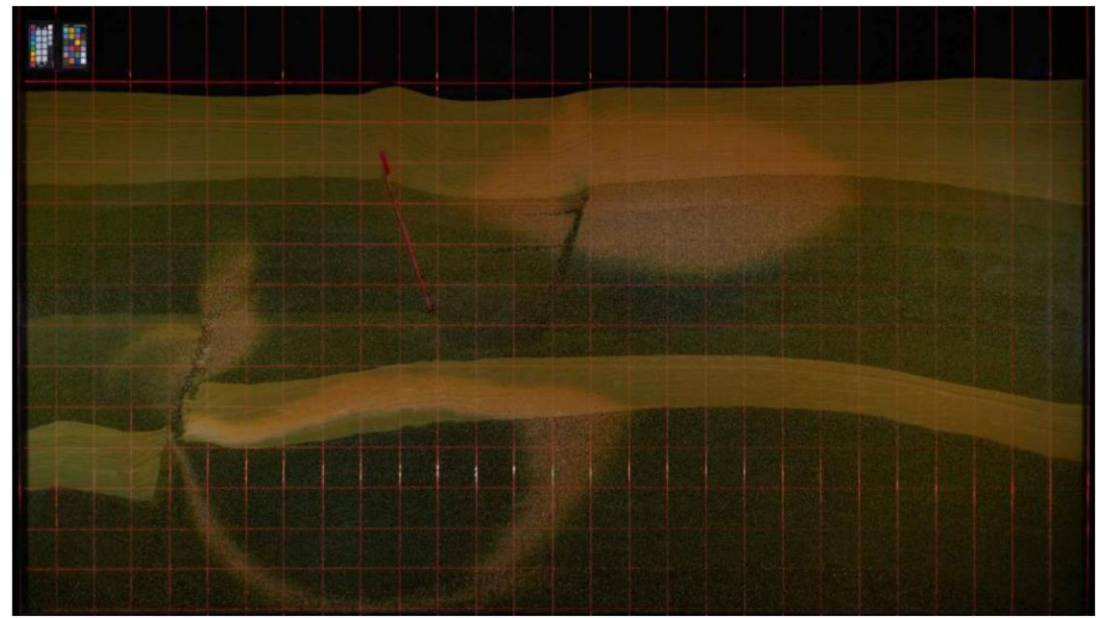
# Tracer test in multi-layered sand

PorePy Simulation



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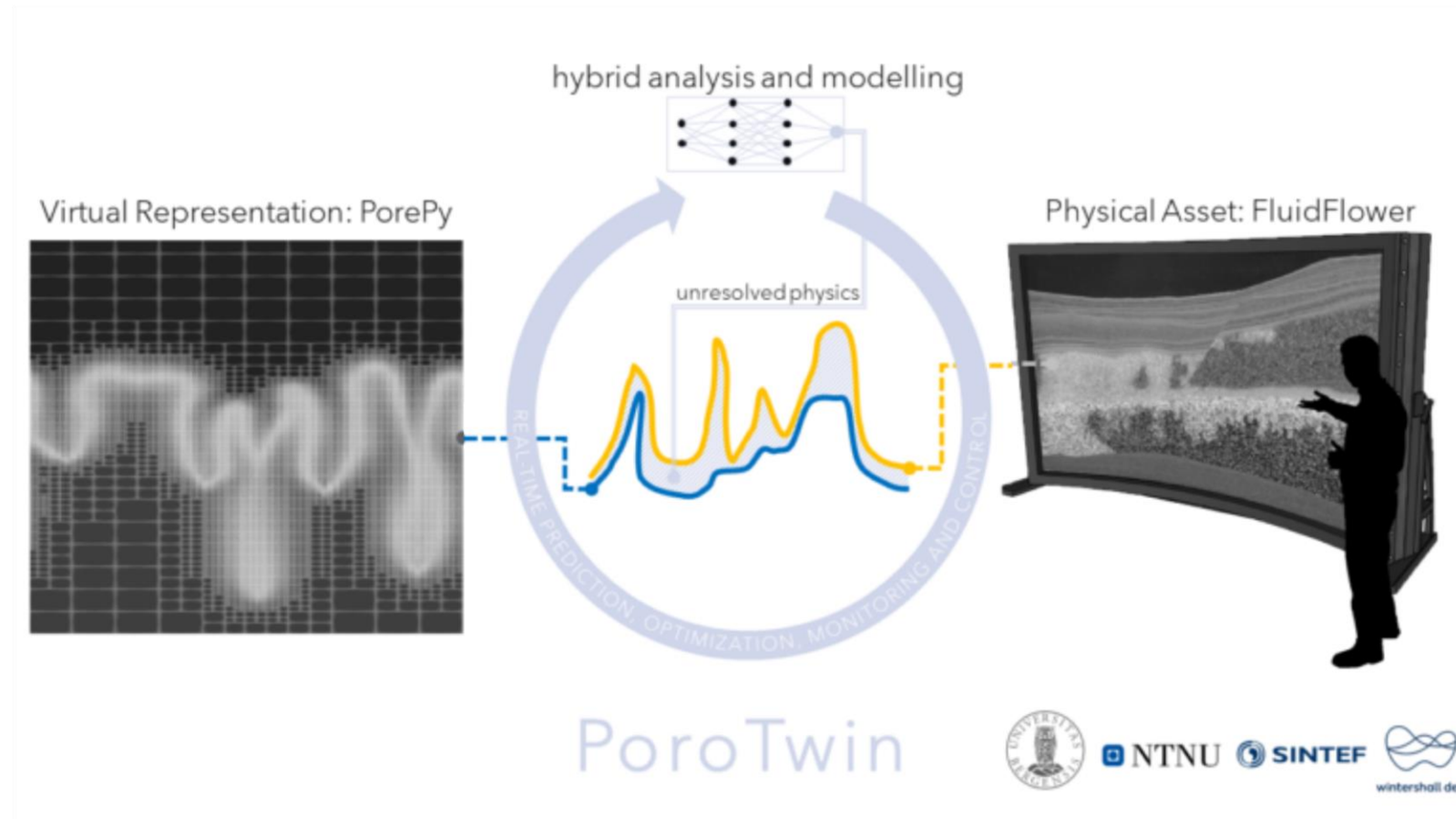
FluidFlow experiment



**Missing physics:** Inaccurate material parameters, density variation (tracer vs. water), ...



# Hybrid analysis and modeling



# Hybrid modeling approach

True model:

$$\mathcal{L}u_{\text{TRUE}} = f$$



# Hybrid modeling approach

True model:

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True solution

# Hybrid modeling approach

True model:

$$\mathcal{L}u_{\text{TRUE}} = f$$

True solution

PBM: Assumed physics



# Hybrid modeling approach

True model:

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True solution

Resulting RHS

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True model:

$$\mathcal{L}u_{\text{TRUE}} = f$$

Resulting RHS

True solution

PBM: Assumed physics

PBM model:

$$\mathcal{L}u_{\text{PBM}} = f_{\text{PBM}}$$



# Hybrid modeling approach

True model:

$$\mathcal{L}u_{\text{TRUE}} = f$$

Resulting RHS

True solution

PBM: Assumed physics

PBM model:

$$\mathcal{L}u_{\text{PBM}} = f_{\text{PBM}}$$

PBM: Assumed physics

# Hybrid modeling approach

True model:

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Resulting RHS

True solution

PBM: Assumed physics

PBM model:

$$\mathcal{L}u_{\text{PBM}} = f_{\text{PBM}}$$

RHS: Assumed physics

PBM: Assumed physics



# Hybrid modeling approach

True model:

$$\mathcal{L}u_{\text{TRUE}} = f$$

Resulting RHS

True solution

PBM: Assumed physics

PBM model:

$$\mathcal{L}u_{\text{PBM}} = f_{\text{PBM}}$$

RHS: Assumed physics

PBM solution

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# Hybrid modeling approach

True model:

$$\mathcal{L}u_{\text{TRUE}} = f$$

Resulting RHS

True solution

PBM: Assumed physics

PBM model:

$$\mathcal{L}u_{\text{PBM}} = f_{\text{PBM}}$$

RHS: Assumed physics

PBM solution

PBM: Assumed physics

Corrective source term approach:

$$\mathcal{L}u_{\text{COSTA}} \stackrel{!}{=} \mathcal{L}u_{\text{TRUE}}$$

Remark: Theoretically no modelling error.

# Hybrid modeling approach

True model:

$$\mathcal{L}u_{\text{TRUE}} = f$$

The diagram shows the equation  $\mathcal{L}u_{\text{TRUE}} = f$ . A blue curved line connects the operator  $\mathcal{L}$  to the text 'PBM: Assumed physics' below. A purple line connects the variable  $u_{\text{TRUE}}$  to the text 'True solution' below. A red line connects the function  $f$  to the text 'Resulting RHS' below.

PBM model:

$$\mathcal{L}u_{\text{PBM}} = f_{\text{PBM}}$$

The diagram shows the equation  $\mathcal{L}u_{\text{PBM}} = f_{\text{PBM}}$ . A blue curved line connects the operator  $\mathcal{L}$  to the text 'PBM: Assumed physics' below. A purple line connects the variable  $u_{\text{PBM}}$  to the text 'PBM solution' below. A red line connects the function  $f_{\text{PBM}}$  to the text 'RHS: Assumed physics' below.

Corrective source term approach:

$$\begin{aligned}\mathcal{L}u_{\text{COSTA}} &\stackrel{!}{=} \mathcal{L}u_{\text{TRUE}} \\ &= \mathcal{L}u_{\text{PBM}} + \mathcal{L}(u_{\text{TRUE}} - u_{\text{PBM}}) \\ &= f_{\text{PBM}} + r\end{aligned}$$

Remark: Theoretically no modelling error.



# Hybrid modeling approach

True model:

$$\mathcal{L}u_{\text{TRUE}} = f$$

The diagram shows the equation  $\mathcal{L}u_{\text{TRUE}} = f$ . A blue curved line connects the operator  $\mathcal{L}$  to the text "PBM: Assumed physics". A purple line connects the variable  $u_{\text{TRUE}}$  to the text "True solution". A red line connects the function  $f$  to the text "Resulting RHS".

PBM model:

$$\mathcal{L}u_{\text{PBM}} = f_{\text{PBM}}$$

The diagram shows the equation  $\mathcal{L}u_{\text{PBM}} = f_{\text{PBM}}$ . A blue curved line connects the operator  $\mathcal{L}$  to the text "PBM: Assumed physics". A purple line connects the variable  $u_{\text{PBM}}$  to the text "PBM solution". A red line connects the function  $f_{\text{PBM}}$  to the text "RHS: Assumed physics".

Corrective source term approach:

$$\begin{aligned}\mathcal{L}u_{\text{COSTA}} &\stackrel{!}{=} \mathcal{L}u_{\text{TRUE}} \\ &= \mathcal{L}u_{\text{PBM}} + \mathcal{L}(u_{\text{TRUE}} - u_{\text{PBM}}) \\ &= f_{\text{PBM}} + r\end{aligned}$$

The diagram shows the derivation of the corrective source term approach. A red line connects the residual term  $r$  to the text "Unknown residual".

Remark: Theoretically no modelling error.

Hence, learn the residual from incorrect solution using deep neural networks.

# Hybrid modeling approach

1. Compute PBM:

$$\mathcal{L}u_{\text{PBM}} = f_{\text{PBM}}$$

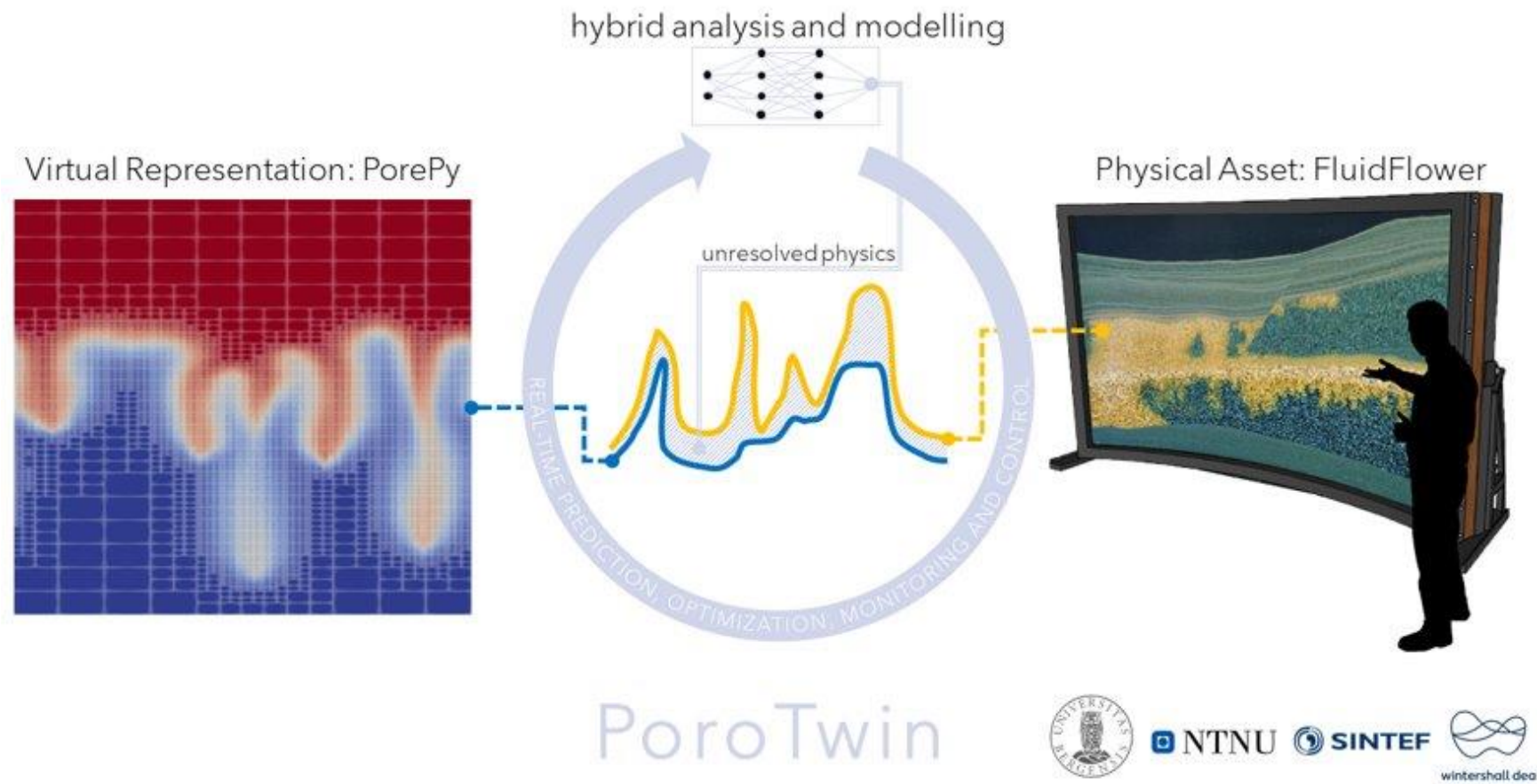
2. Determine the corrective source:

$$r = \text{DDM}(u_{\text{PBM}})$$

3. Compute the PBM with corrected physics:

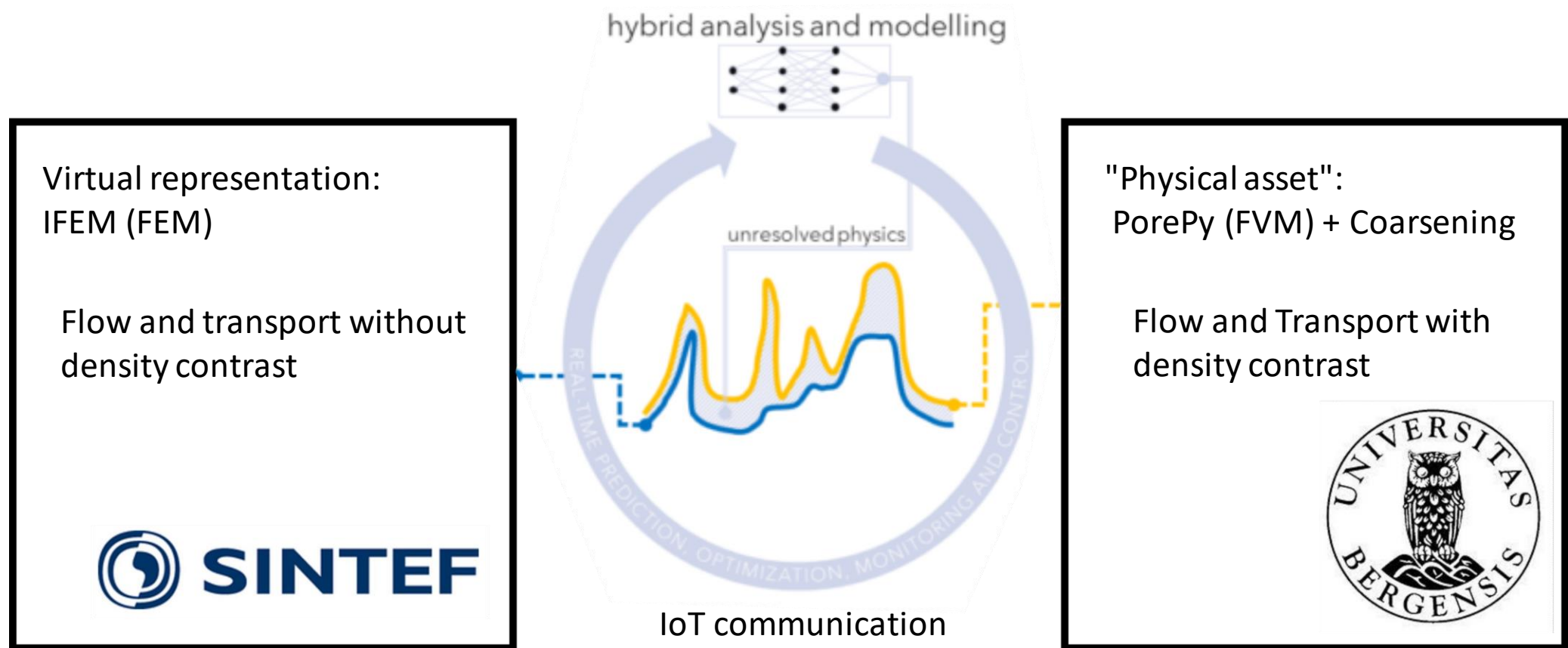
$$\mathcal{L}u_{\text{COSTA}} = f_{\text{PBM}} + r$$

# Digital twin concept



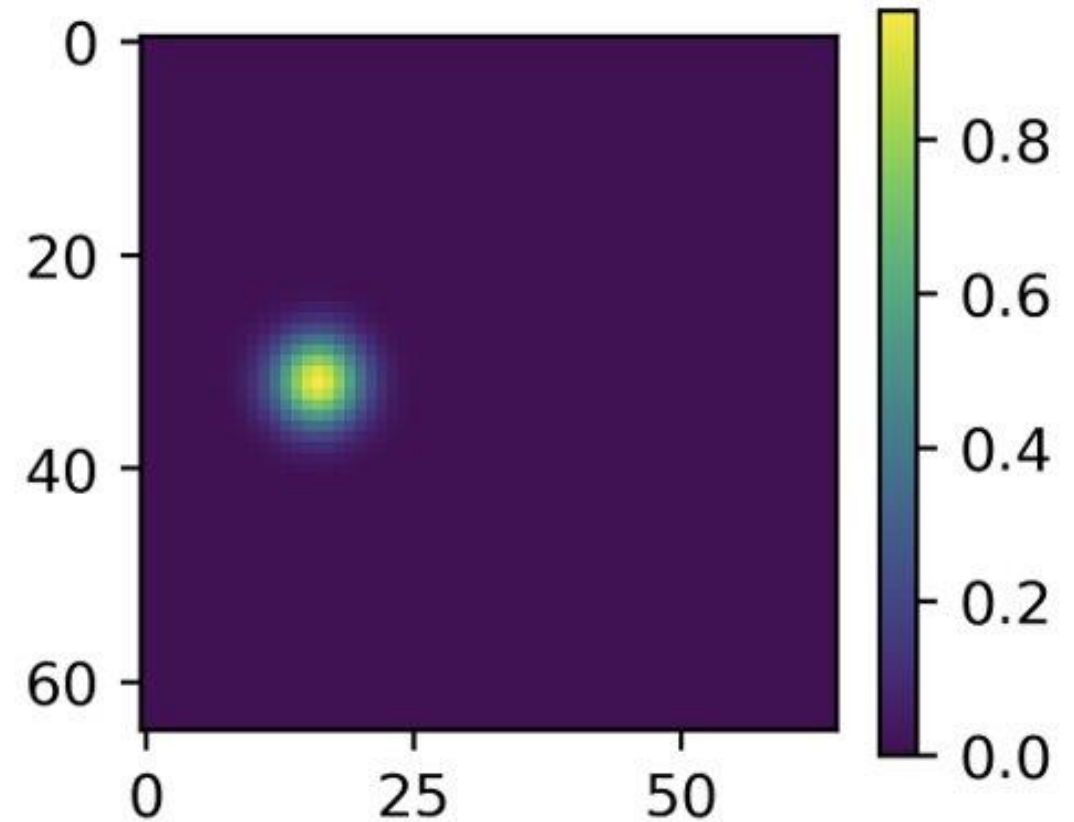


# Proof-of-concept

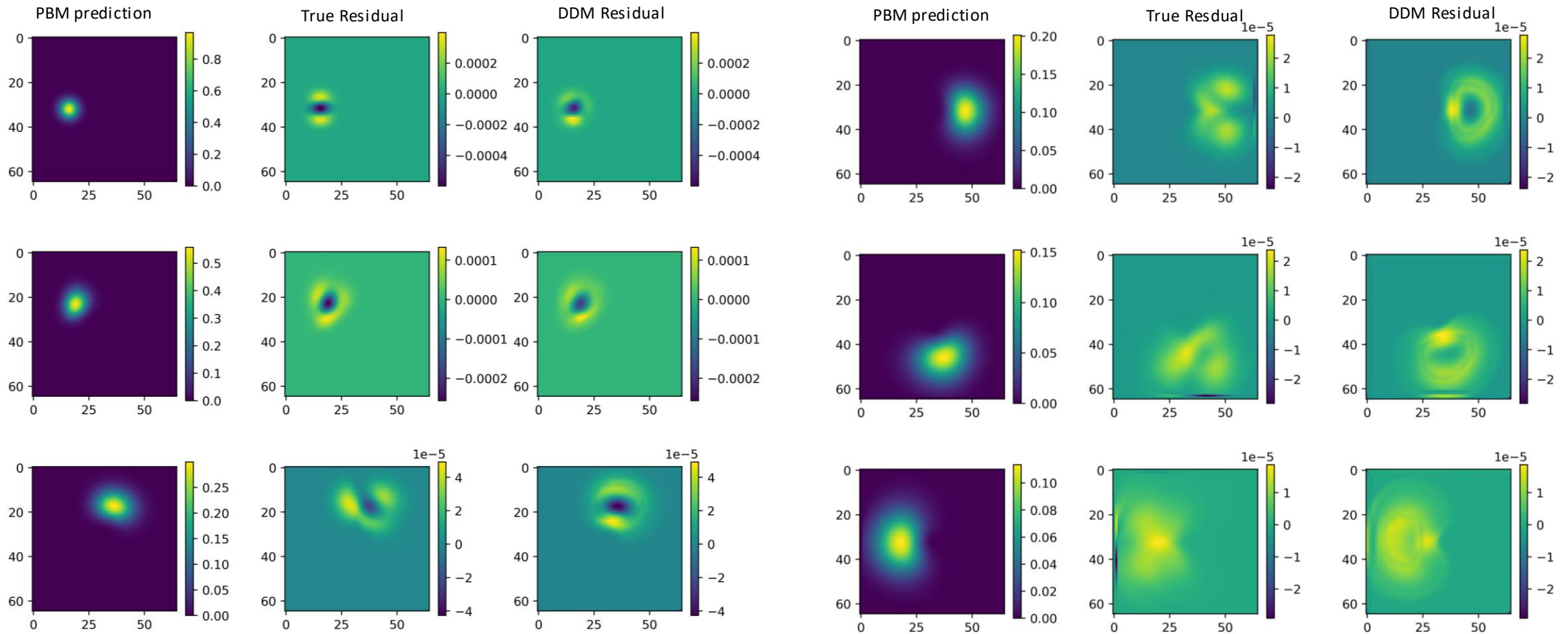


# Test case setup

- Transport problem without dispersion
$$\partial_t c + \nabla \cdot (cq) = 0$$
- Flux prescribed as circular
- Initial tracer concentration to the right
- Simulate one rotation
- (Density contrast effects only in the "physical asset")

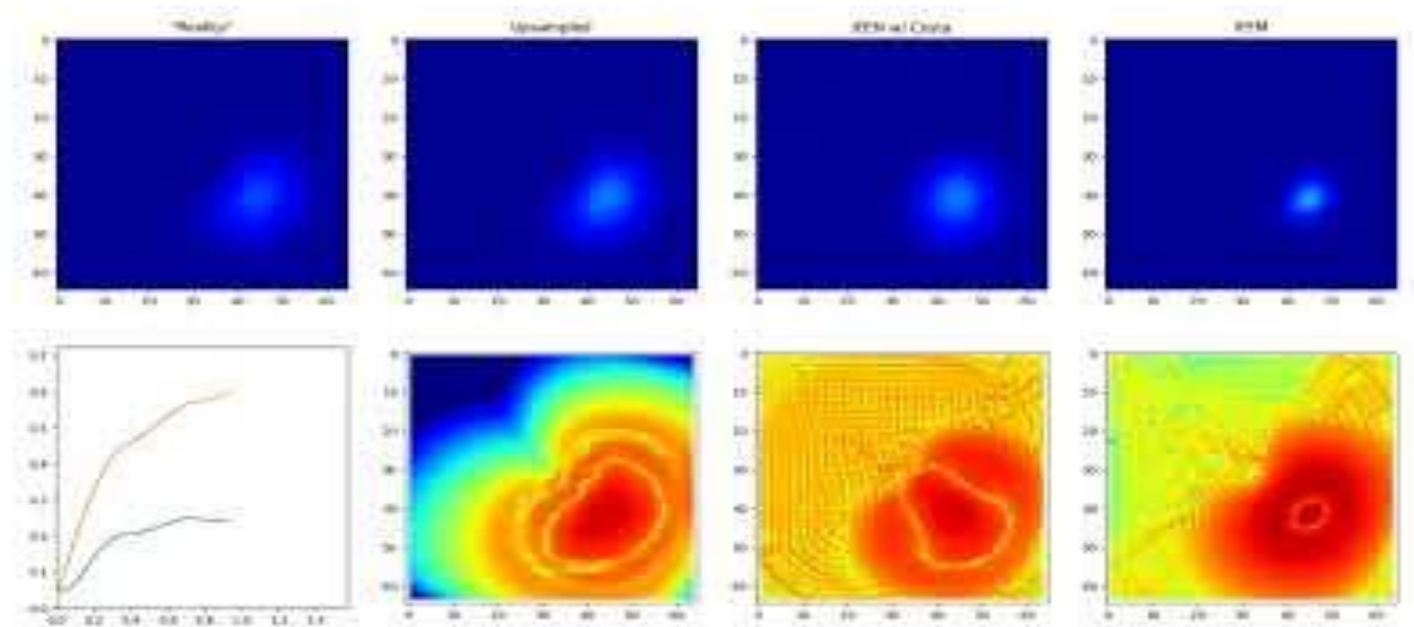


# Test case I without hidden physics



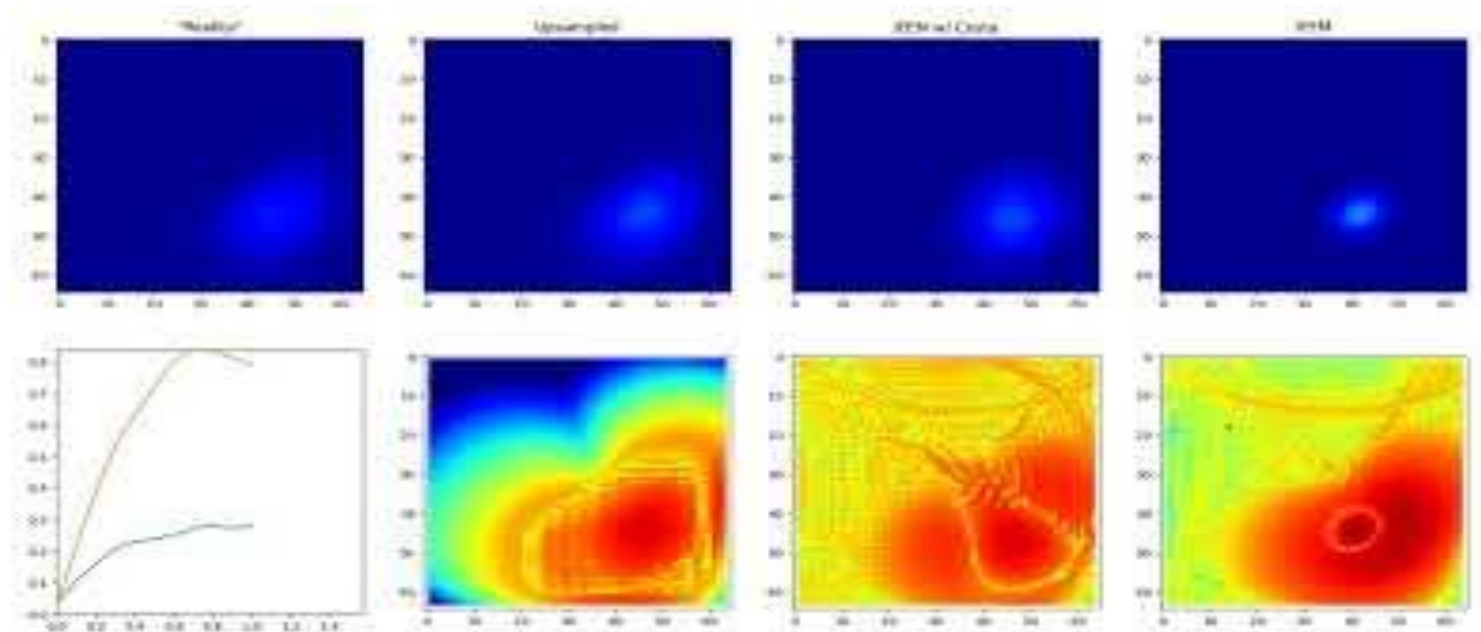
DDM makes FE-code learn FV.

# Test case I without hidden physics

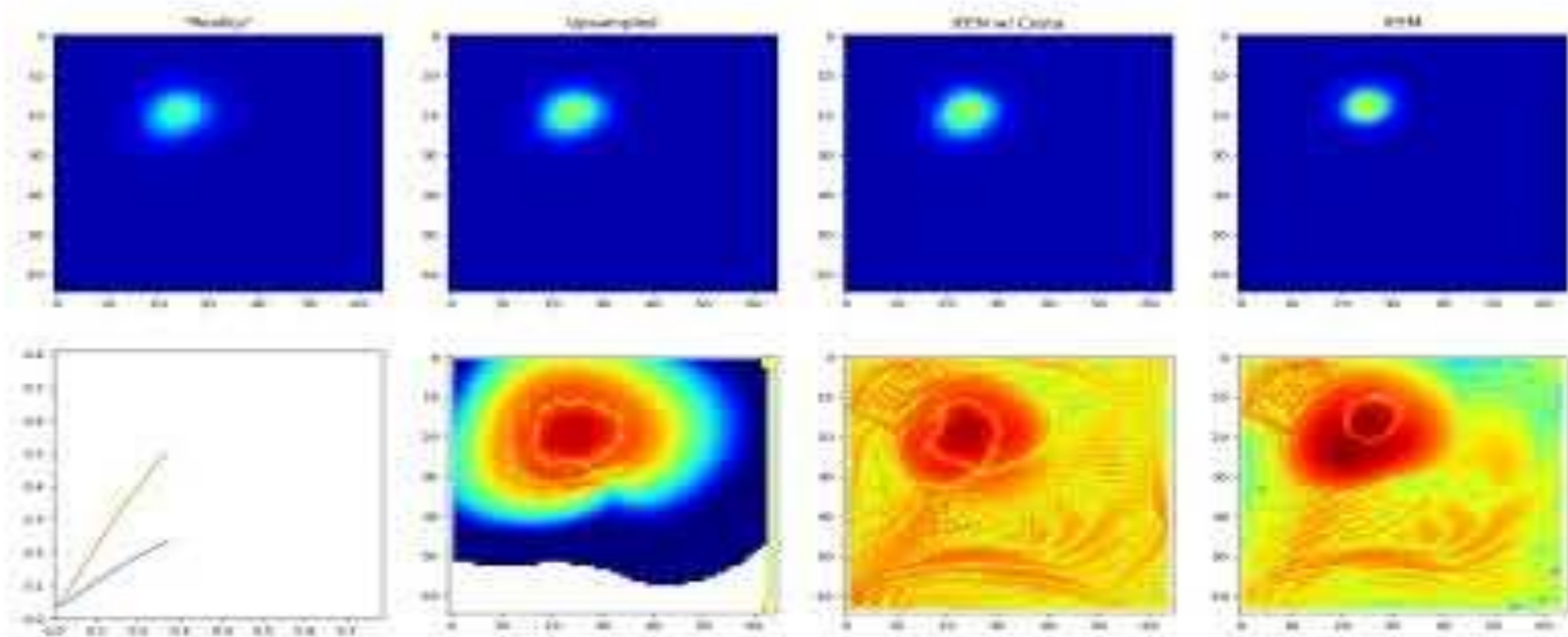




# Test case II with hidden physics (lighter tracer)



# Test case III with hidden physics (heavier tracer)



# Outlook

## Vision: Hybrid Analysis and Modeling (HAM)

