Inverse and Forward Uncertainty Quantification of Relative Permeability and Foam Model Parameters for EOR Processes


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Enhanced Oil Recovery (EOR)

- Water-alternating gas (WAG) injection process can increase the sweep efficiency in Enhanced Oil Recovery (EOR).
- However, this technique may be hampered by gas fingering, channeling, and gravity override.
- The injection of foam in EOR processes can help reducing the gas mobility, which in turn results in increased recovery factor.
- In this context the use of mathematical models and computer simulations is of utmost importance to provide insight and predictions of the production.
- Quantification of uncertainties: essential for developing robust simulators.
In a series of previous works, we have focused on UQ and SA of relative permeability models and of foam models (CMG-STARS).

- *Uncertainty quantification and sensitivity analysis for relative permeability models of two-phase flow in porous media*, A. R. Valdez et al., JPSE, 2020.¹
- *Assessing uncertainties and identifiability of foam displacement models employing different objective functions for parameter estimation*, A. R. Valdez et al., JPSE, 2022.³

However, these works did not consider both components (rel. perm. and foam parameters) of two-phase flow in porous media together during the UQ and SA studies.

¹ doi: 10.1016/j.petrol.2020.107297  
² doi: 10.1007/s11242-021-01550-0  
³ doi: 10.1016/j.petrol.2022.110551
What are the main goals of this work?

- Present a more comprehensive approach for uncertainty quantification of two-phase flow models with foam injection for EOR processes.
- The framework for inverse and forward UQ and SA considers both the relative permeability model and the foam model.
- For relative permeability, we consider the Corey model.
- For foam flow, we consider the CMG-STARS apparent viscosity model.
F. de Paula, T. Quinelato, I. Igreja, G. Chapiro
A Numerical Algorithm to Solve the Two-Phase Flow in Porous Media Including Foam Displacement -
Lecture Notes in Computer Science, 2020
The mathematical model for two-phase flow in porous media

Two-Phase Water and Gas Flow with the presence of foam

Fully saturated porous medium, i.e. $S_w + S_g = 1$.

$$\frac{\partial}{\partial t} (\phi S_w) + \frac{\partial}{\partial x} (u_w) = 0, \quad \text{in } \Omega \times [0, T],$$

$$\frac{\partial}{\partial t} (\phi S_g n_D) + \frac{\partial}{\partial x} (u_g n_D) = \frac{\phi}{n_{\text{max}}} S_g \Phi, \quad \text{in } \Omega \times [0, T],$$

- $S_w$: water phase saturation;
- $u_w$: water phase velocity;
- $\phi$: effective porosity of the medium;
- $n_D$: foam texture;
- $S_g$: gas phase saturation;
- $u_g$: gas phase velocity;
- $\Phi$: foam generation and destruction;
- $n_{\text{max}}$: maximum foam texture.
Corey relative permeability model and Foam model

Foam Model

\[ \mu_{app} = \frac{1}{\left( \lambda_w + \frac{\lambda_q}{\text{MRF}} \right)} , \]  

(STARS model:)

\[ \text{MRF} = 1 + f_{mmob} F_2 , \]  

\[ F_2 = \frac{1}{2} + \frac{1}{\pi} \arctan \left( s f_{bet} (S_w - SF) \right) . \]

Corey relative permeability model

\[ k_{rw} = k_{rw}^0 \left( \frac{S_w - S_{wc}}{1 - S_{wc} - S_{gr}} \right)^{n_w} , \quad \text{and} \quad k_{rg} = k_{rg}^0 \left( \frac{S_g - S_{gr}}{1 - S_{wc} - S_{gr}} \right)^{n_g} . \]
The high-level framework for UQ & SA

Experimental Data

Mathematical Model

\[ f(x, t) := f(x, t, p_1, p_2, \ldots) \]
Uncertainty Quantification (UQ) and Sensitivity Analysis (SA)

Likelihood based on the following objective function, $F(\theta)$

$$\theta = \{k_{rw}^0, n_w, k_{rg}^0, n_g, f_{mmob}, sf_{bet}, SF\},$$

(5)

$$F = \sum_{k=1}^{N_p} (\mu_{app}^{exp} - \mu_{app}^{model}(\theta))^2 + (k_{rw}^{exp} - k_{rw}^{model}(\theta))^2 + (k_{rg}^{exp} - k_{rg}^{model}(\theta))^2$$

(6)

Bayesian inference (via Markov Chain Monte Carlo)

$$\mathbb{P}(\theta|F) \propto \mathbb{P}(F|\theta)\mathbb{P}(\theta)$$

(7)

The main Sobol indices and the total Sobol indices are given by

$$S_i = \frac{\mathbb{V}[\mathbb{E}[\mathcal{Y}|\theta_i]]}{\mathbb{V}[\mathcal{Y}]} \quad \text{and} \quad S_{T_i} = 1 - \frac{\mathbb{V}[\mathbb{E}[\mathcal{Y}|\theta_{-i}]]}{\mathbb{V}[\mathcal{Y}]},$$

(8)

with $\mathcal{Y}$ the QoIs = $\{\mu_{app}, k_{rw}, k_{rg}\}$. 
Chosen prior distributions for MCMC

Corey relative permeability parameters:

\[ k_{rw}^0 \sim \mathcal{U}[0.01, 1.0], \]
\[ k_{rg}^0 \sim \mathcal{U}[0.01, 1.0], \]
\[ n_w \sim \mathcal{U}[0.7, 3.0], \]
\[ n_g \sim \mathcal{U}[0.7, 3.0], \]

CMG-STARS Foam parameters:

\[ fmmob \sim \mathcal{U}[0.0, 1000], \]
\[ SF \sim \mathcal{U}[S_{wc}, 1 - S_{gr}], \]
\[ sfbet \sim \mathcal{U}[10, 1000]. \]
Numerical Experiments

- Experiment I: uses experimental data reported in Valdez; 2021


- Experiment II: uses Dataset I and introduces a new point for \((S_w, k_{rw})\) and \((S_w, k_{rg})\)

- Experiment III: uses Dataset I and introduces a new point in \((f_g, \mu_{app}) = (0.95, 0.05)\)
Experimental data obtained from the literature

Valdez, A. et al., Transport in Porous Media (2021)
Experiment I: Estimated posterior distribution of the parameters
Augmenting the data set of Relative Permeability, Experiment II
Experiment II: Estimated posterior distribution of the parameters

- An additional synthetic data point is included for \((S_w, k_{rw})\) and \((S_w, k_{rg})\).
- The new posteriors of relative permeability parameters reflect reduced uncertainty.
- No changes were observed in the new posteriors of apparent viscosity parameters (same uncertainties)
Augmenting the data set of Apparent Viscosity, Experiment III
Experiment III: Estimated posterior distribution of the parameters

- An additional data point for the apparent viscosity is considered for this case.
- Improved estimates for the posterior distributions of all the parameters.
  - WHY???
Experiment I (and II): Propagation of uncertainties

- Propagated uncertainties within the confidence interval (CI) of 80% with mean value (solid line) for $k_{rw}$, $k_{rg}$ and $\mu_{app}$
- Almost no uncertainty in the $k_{rw}$ relative permeability
- Few uncertainty for the $k_{rg}$ relative permeability
- More uncertainty for the $\mu_{app}$ apparent viscosity of foam
Experiment III: Propagation of uncertainties in $k_{rw}$, $k_{rg}$ and $\mu_{app}$

- An additional data point in the high-quality regime **significantly reduces** the uncertainty for $\mu_{app}$, as demonstrated in a previous work \(^4\).

\(^4\)Valdez et al., TIPM, 2021.
Experiment III: Sensitivity analysis based on Sobol indices

Rel. permeabilities $k_{rw}$ and $k_{rg}$

Apparent viscosity $\mu_{app}$
Inverse and Forward UQ analyses were performed for foam-assisted EOR model considering the parameters of both relative permeability model and of the CMG-STAR foam model.

Augmenting the experimental data set where the model has low-uncertainties may be useless both in terms of parameter estimation and model reliability (uncertainties).

Augmenting the experimental data set where the model has high-uncertainties ($\mu_{app}$) improves both parameter estimation, sensitivity analysis and model reliability (for both $\mu_{app}$ and relative permeability).

The combined calibration of foam and relative permeability models is highly beneficial.

Forward UQ highlights were we need more data to improve model reliability. The presented framework can be used to guide core-flooding experiments.
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Thank you for your attention!