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A macro-scale elasto-thermo-viscoplastic constitutive model for frozen soils

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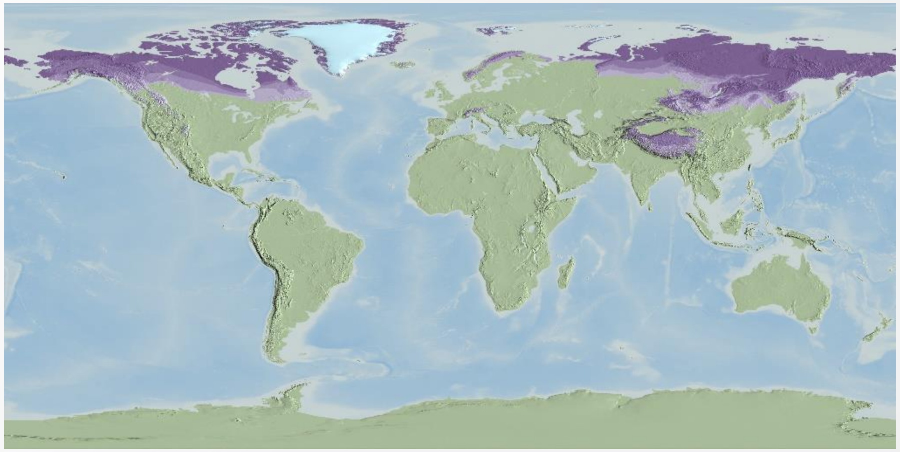
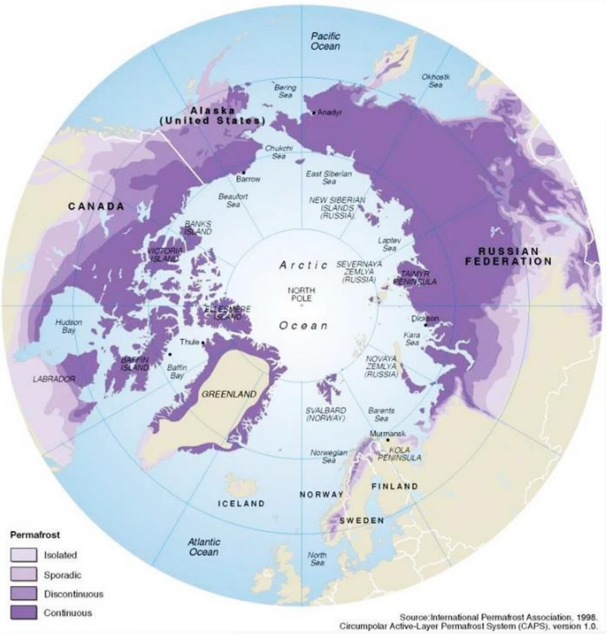
Content

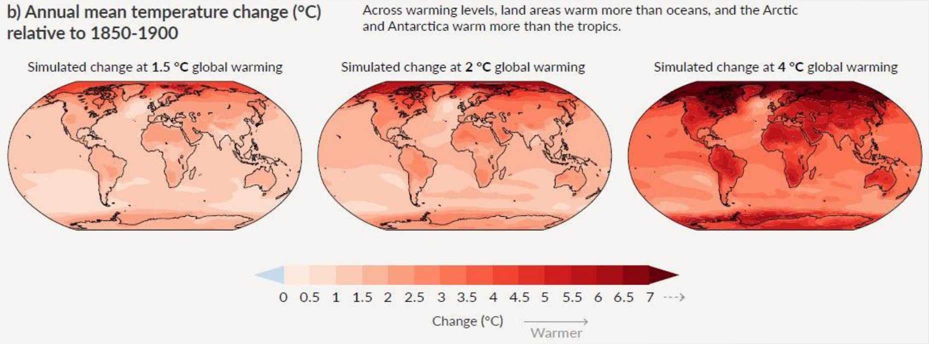
Introduction



Global Permafrost Layers designed for Science On a Sphere (SOS) and WMS
Credit: NASA Scientific Visualization Studio



Map of permafrost in the northern hemisphere. Credit: Map by Philippe Rekacewicz, UNEP/GRID-Arendal; data from International Permafrost Association, 1998. Circumpolar Active-Layer Permafrost System (CAPS), version 1.0.



Changes in annual mean surface temperature (*Masson-Delmotte et al., 2021*).

Introduction

Permafrost damaged roads, Yellowknife, Northwest territories. Credit: Ryerson Clark/iStock

- ☐ Ice melting results in unfrozen water, strength loss, ground surface deformations, and permafrost degradation.
- ☐ Irrecoverable slow-rate time-dependent deformation (i.e., creep) of permafrost. Primary, secondary and tertiary creep deformation (Temperature, confining stress level, strain rate, ice content) (*Andersland and Ladanyi 2003*).
- ☐ Experimental attempts (e.g., *Vyalov 1986; Arenson and Springman 2005; Yao et al. 2018*).
- ☐ Constitutive creep models based on the theory of elastic-visco-plasticity or visco-elastic-plasticity (e.g., *Ghoreishian Amiri et al. 2016; Sun et al. 2021; Li et al. 2022*).

Permafrost degradation is a major threat to Arctic communities. Credit: US National Parks Service Climate Change Response

Frozen Soils - Basic Concepts and Stress State Variables

$$\boxed{\theta_s = 1 - n,\quad \theta_w = nS_w,\quad \theta_i = nS_i,\quad S_w + S_i = 1}$$

Volumetric fractions

Solid Skeleton

Water

Ice

$n = \theta_w + \theta_i$

Degree of saturation

Water *Ice*

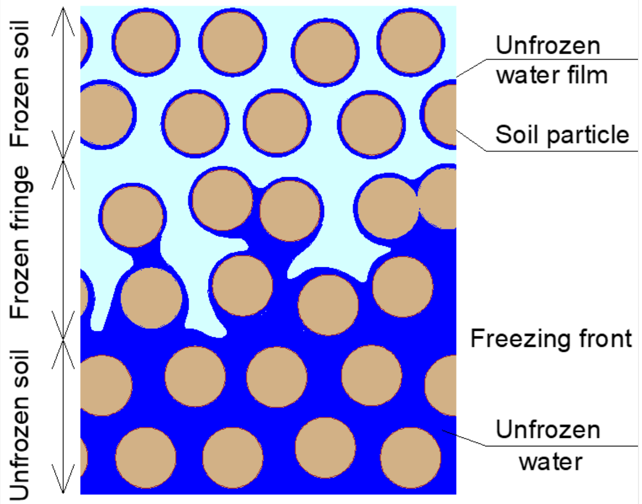


Illustration of frozen soil composition.

❑ Two-stress state variables framework

$$\boxed{S = P_{ice} - P_w}$$

Pore pressures

Ice *Water*

Cryogenic suction

$$\boxed{\boldsymbol{\sigma}^* = \boldsymbol{\sigma} - S_w P_w \boldsymbol{I}}$$

Solid phase stress

Total stress

Unit tensor

$$\boxed{S \approx \rho_w L \ln\left(\frac{T}{273.15}\right)}$$

Temperature

Density

Latent heat of fusion of water

TEVP Model

Total strain rate decomposition:

- Mechanical (solid phase stress-dependent)
  - Elastic (time-independent recoverable)
  - Thermal-viscoplastic (time- and temperature-dependent irrecoverable)
- Cryogenic suction-dependent

$$\boxed{\dot{\boldsymbol{\varepsilon}} = \dot{\boldsymbol{\varepsilon}}^{\sigma} + \dot{\boldsymbol{\varepsilon}}^{suc} = (\dot{\boldsymbol{\varepsilon}}^{\sigma e} + \dot{\boldsymbol{\varepsilon}}^{\sigma Tvp}) + \dot{\boldsymbol{\varepsilon}}^{suc}}$$

$$\boxed{p^* = \sigma^*_{ii}/3 = (\sigma^*_{11} + \sigma^*_{22} + \sigma^*_{33})/3} \rightarrow$$ *Mean solid phase stress*

$$\boxed{q^* = \sqrt{3 s^*_{ij} s^*_{ij}/2}} \rightarrow$$ *Deviatoric stress*

$$\boxed{s^*_{ij} = \sigma^*_{ij} - p^*\delta_{ij}} \rightarrow$$ *Deviatoric solid phase stress tensor*

TEVP Model

Elasticity | Viscoplasticity

- The elastic component of strain rate tensor due to the solid phase stress variation

$$\dot{\varepsilon}_{ij}^{\sigma e} = \frac{\dot{p}^*}{3K_{eq}}\delta_{ij} + \frac{1}{2G_{eq}}\dot{s}_{ij}^*$$

$$G_{eq} = (1 - S_i)G_{uf} + S_i \frac{E_f}{2(1+\nu_f)}$$

$$K_{eq} = (1 - S_i)K_{uf} + S_i \frac{E_f}{3(1-2\nu_f)}$$

$$E_f = E_{uf}(1 + a\theta)$$

Temperature-dependent equivalent elastic modulus

Elastic modulus in an unfrozen state

Poisson's ratio

material parameter

Number of sub-zero temperature

- The strain due to cryogenic suction changes is assumed to be elastic and volumetric:

$$\delta\boldsymbol{\varepsilon}^{suc} = (\boldsymbol{D}^{suc})^{-1}\delta S$$

$$(\boldsymbol{D}^{suc})^{-1} = \frac{1}{3V}\frac{\kappa_s}{(S+P_{atm})}\boldsymbol{I}$$

Elastic stiffness parameter for changes in cryogenic suction

specific volume

TEVP Model

Elasticity

Viscoplasticity

A suction-dependent criterion is required to capture the essential features of frozen and unfrozen behavior:

$$F = {q^*}^2 - M^2(p^* - p_t^*)(p_f^* - p^*) = 0$$

Apparent cohesion of the frozen soil due to cryogenic suction

$$p_t^* = -k_t S$$

Slope of the CSL

$$p_f^* = p_r^* \left(\frac{p_o^*}{p_r^*}\right)^{\frac{\lambda_o - \kappa_o}{\lambda_f - \kappa_f}}$$

Pre-consolidation stress in an unfrozen state

Compressibility coefficients within the elastic region

Reference stress

Elastoplastic compressibility coefficient

$$\lambda_f = \lambda_o\,[(1-\alpha)\exp(-\beta S) + \alpha]$$

Model parameter controlling the maximum stiffness

Model parameter controlling the rate of change in stiffness with cryogenic suction

Plastic potential and yield surfaces should be described at the current stress state ($q^* - p^* - S_1$)

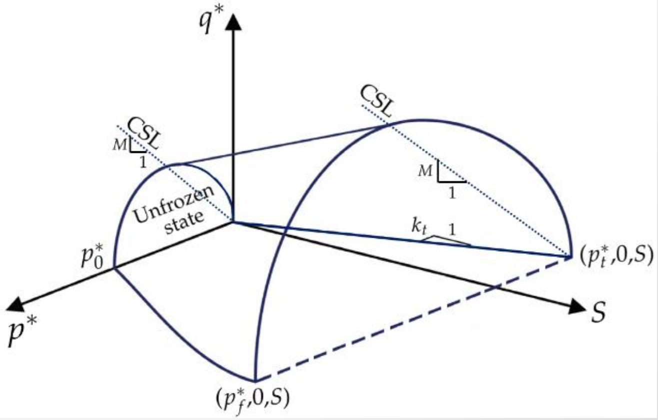


Illustration of the surface adopted for the TEVP constitutive modeling of frozen soils in $q^* - p^* - S$ space.

TEVP Model

Elasticity

Viscoplasticity

TVP deformation is formulated by considering the response of frozen soil under the corresponding isotropic stress state ($q^* = 0 - p^* = p_m^* - S_1$):

$$\boxed{V_{NCL} = N_f - \lambda_f \ln p_m^*}$$

Specific volume of the frozen soil under p_m^* (at the end of primary volumetric compression)

$$\boxed{N_f = N_o + \kappa_s \ln \frac{S + P_{atm}}{P_{atm}}}$$

Specific volume at unit pressure in an unfrozen state

Adopting a logarithmic creep function for viscoplastic volume changes: $\boxed{\delta V_m^{\sigma Tvp} = -\psi_T \ln\left(\frac{t_o + t}{t_o}\right)}$

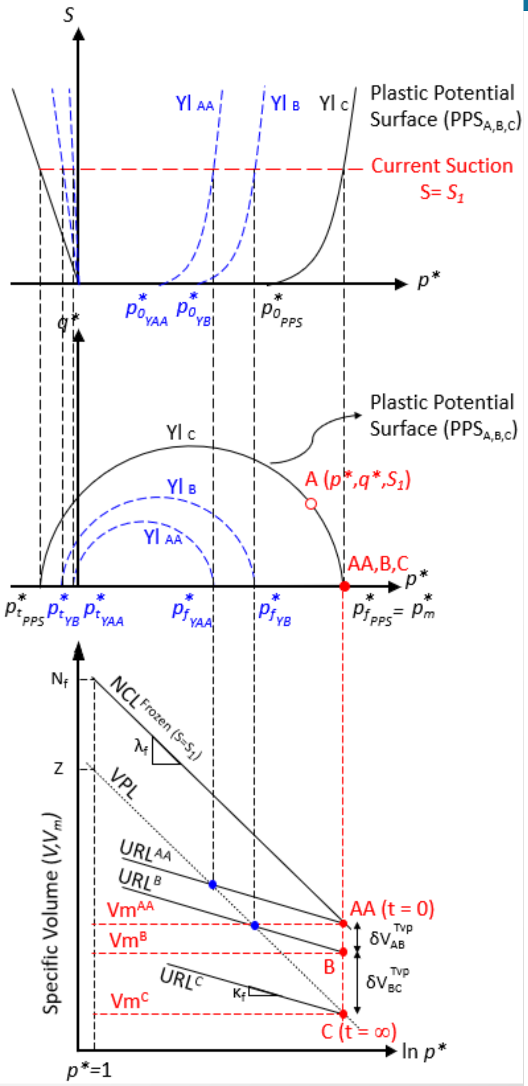
Temperature-dependent creep parameter

Material paramete denoting the initiation of secondary creep

Material parameter

$$\boxed{\psi_T = \psi_o \left(1 + \frac{\theta}{\theta_0}\right)^b}$$

TVP volumetric strain rate $\boxed{\dot{\varepsilon}_p^{\sigma Tvp} = \frac{\delta \varepsilon_p^{\sigma Tvp}}{\delta t} = \frac{\delta V_m^{vp}/V_m}{\delta t} = \frac{\psi_T}{V_m\,(t_o + t)}}$



Description of the TEVP model.

TEVP Model

Elasticity

Viscoplasticity

At time t after primary compression, isotropically compressed states $p_m^* - V_m$ of the frozen soil can be defined in $\ln p^* - V$ plane by a line that is parallel to and at constant vertical separation from the NCL for the current frozen state. As the elapsed time for thermal viscoplastic straining approaches infinity, these states are defined by a line called the viscoplastic limit line (VPL).

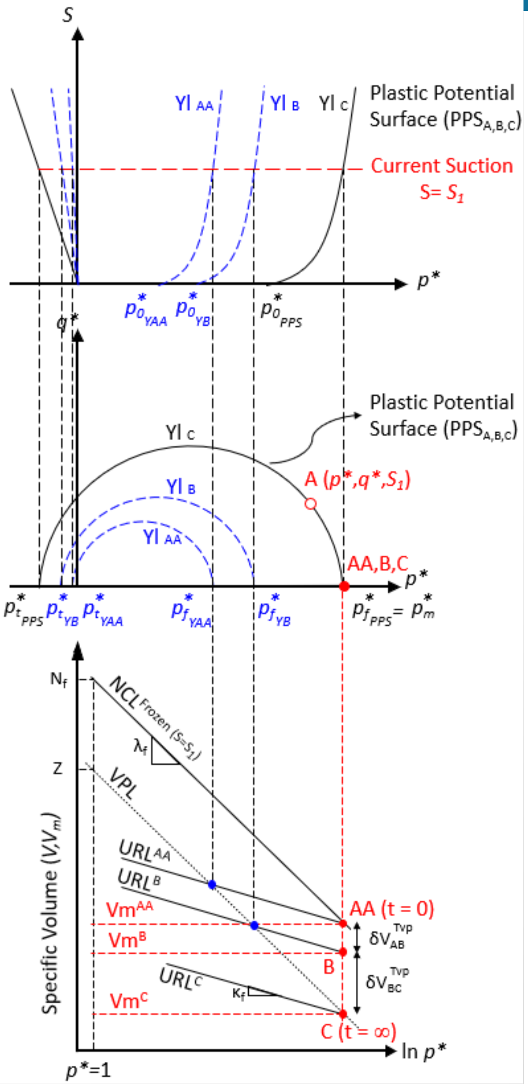
$$\boxed{V_{VPL} = \underbrace{Z}_{} - \lambda_f \ln p^*}$$

Vertical intercept of the VPL at unit pressure in the current frozen state

At a specific time:

$$\boxed{V_m = N_f - \lambda_f \ln p_m^* - \psi_T \ln\left(\frac{t_o+t}{t_o}\right)}$$

$$\boxed{\dot{\varepsilon}_p^{\sigma Tvp} = \frac{\psi_T}{V_m\, t_o} \exp\left(\frac{V_m - N_f}{\psi_T}\right) (p_m^*)^{\frac{\lambda_f}{\psi_T}}}$$



Description of the TEVP model.

TEVP Model

Elasticity

Viscoplasticity

Hardening Law: $F_Y = q^{*2} - M^2(p^* - \chi_t)(\chi - p^*) = 0$

Yielding criterion in isotropic compression and in isotropic tension

TVP volumetric straining is associated with hardening of the soil and the expansion of the yield locus:

$$p^*_{f_Y} = \exp\left[\left(\frac{1}{\lambda_f - \kappa_f}\right)(Z - V - \kappa_f \ln p^*)\right] \qquad \delta p^*_{f_Y} = \left(\frac{V}{\lambda_f - \kappa_f}\, p^*_{f_Y}\right)\delta\varepsilon_p^{\sigma Tvp}$$

Flow Rule:

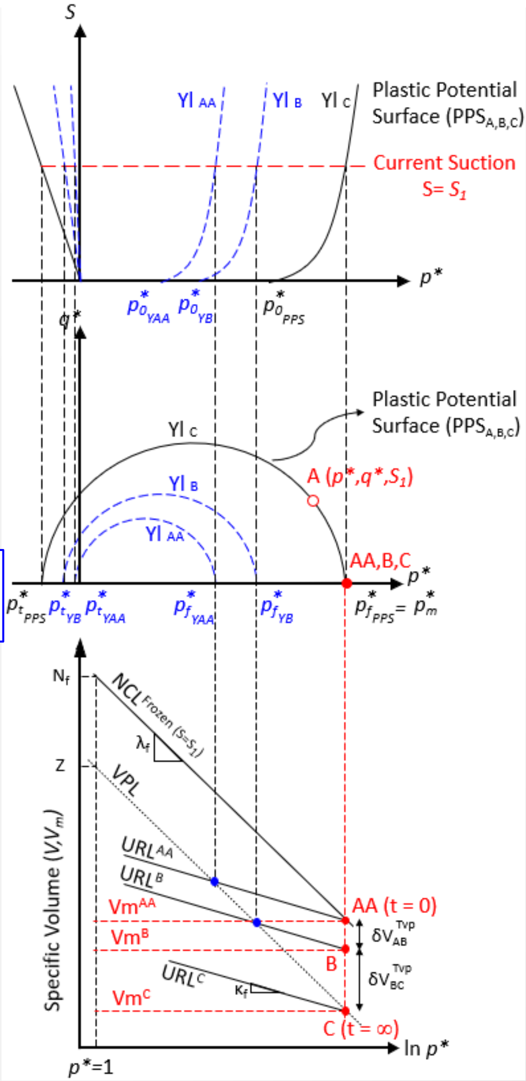
A non-associated flow rule is adopted to generalize the model to any loading path and stress state:

$$\dot{\varepsilon}_{ij}^{\sigma Tvp} = \Lambda \frac{\partial F_{PPS}}{\partial \sigma^*_{ij}}$$

Scaler multiplier

$$\Lambda = \frac{\psi_T}{V_m\, t_o} \exp\left(\frac{V_m - N_f}{\psi_T}\right)(p^*_m)^{\frac{\lambda_f}{\psi_T}} \frac{1}{|\partial F_{PPS}/\partial p^*|}$$

$$\dot{\varepsilon}_{ij}^{\sigma Tvp} = \frac{\psi_T}{V_m\, t_o} \exp\left(\frac{V_m - N_f}{\psi_T}\right)(p^*_m)^{\frac{\lambda_f}{\psi_T}} \frac{1}{|\partial F_{PPS}/\partial p^*|} \frac{\partial F_{PPS}}{\partial \sigma^*_{ij}}$$

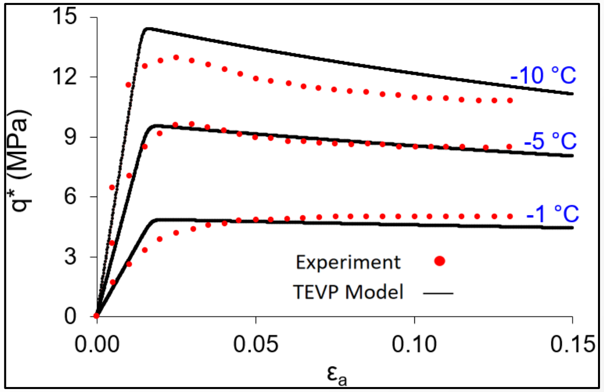


Description of the TEVP model.

Model Performance

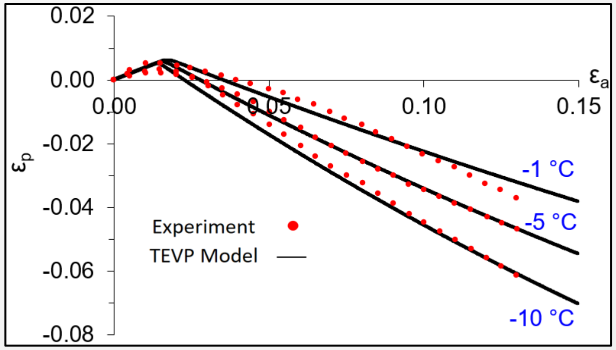
Triaxial Compression Tests

Xu (2014) conducted several triaxial compression tests on frozen sand samples at different temperatures.



Deviatoric stress-axial strain
$(q^* - \varepsilon_a)$ plot.

Constant axial strain rate of 1.67×10^{-4} s^{-1} and initial confinement of 1 Mpa



Volumetric strain-axial strain
$(\varepsilon_p - \varepsilon_a)$ plot.

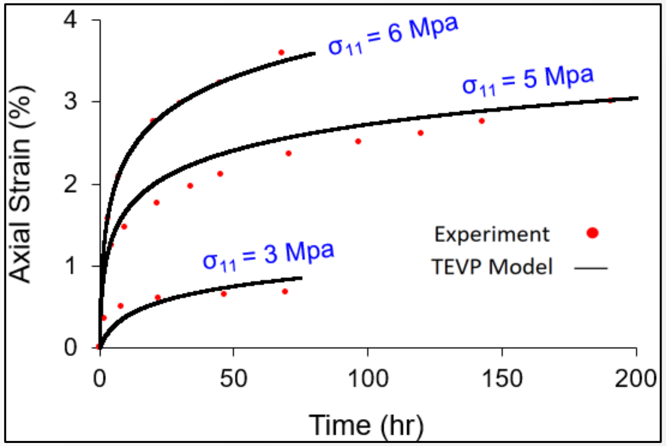
Model parameters used in this simulation

Parameter	Unit	Value
G_{uf}	kPa	3500
E_{uf}	kPa	200×10^3
κ_o	---------	0.08
λ_o	---------	0.85
N_o	---------	8.6
p_o^*	kPa	5550
p_r^*	kPa	100
M	---------	1.5
a	---------	0.4
ν_f	---------	0.31
α	---------	0.66
β	kPa^{-1}	0.00011
k_t	---------	0.1
κ_s	---------	0.008
ψ_o	---------	0.02
b	---------	0.3
t_o	min	1440
Z	---------	1

Model Performance

Uniaxial Creep Tests

Eckardt (1979) investigated the stress-strain behavior of frozen sand samples by conducting uniaxial creep tests:



Uniaxial compression tests on frozen sand at T = - 15 °C: axial strain-time plot.

Model parameters used in this simulation.

Parameter	Unit	Value
G_{uf}	kPa	5000
E_{uf}	kPa	140×10^3
κ_o	---------	0.01
λ_o	---------	0.02
N_o	---------	1.62
p_o^*	kPa	280
p_r^*	kPa	50
M	---------	0.85
a	---------	0.07
ν_f	---------	0.48
α	---------	0.49
β	kPa^{-1}	0.00015
k_t	---------	0.45
κ_s	---------	0.008
ψ_o	---------	0.001
b	---------	0.4
t_o	min	1
Z	---------	1

Conclusions

- A TEVP constitutive model based on the framework of CSSM was proposed to examine the rate-dependent behavior of frozen soils.
- The model was formulated within the two stress-state variables framework.
- Plastic potential and yield surfaces were defined based on the current stress state of the soil. The hardening (softening) of the soil was formulated based on the definition of the VPL.
- The capability of the model was examined by reproducing the conventional triaxial compression and creep tests results.
- The model can be used to investigate the behavior of the frozen ground under extreme short-term as well as long-term climatic events in permafrost regions.

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Thank you for your attention!