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A macro-scale elasto-thermo-viscoplastic constitutive model for frozen soils

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Introduction


Global Permafrost Layers designed for Science On a Sphere (SOS) and WMS Credit: NASA Scientific Visualization Studio

Changes in annual mean surface temperature (Masson-Delmotte et al., 2021).
Introduction

- Ice melting results in unfrozen water, strength loss, ground surface deformations, and permafrost degradation.
- Irrecoverable slow-rate time-dependent deformation (i.e., creep) of permafrost. Primary, secondary and tertiary creep deformation (Temperature, confining stress level, strain rate, ice content) (Andersland and Ladanyi 2003).
- Experimental attempts (e.g., Vyalov 1986; Arenson and Springman 2005; Yao et al. 2018).
- Constitutive creep models based on the theory of elastic-visco-plasticity or visco-elastic-plasticity (e.g., Ghoreishian Amiri et al. 2016; Sun et al. 2021; Li et al. 2022).
Frozen Soils - Basic Concepts and Stress State Variables

\[ \theta_s = 1 - n, \quad \theta_w = nS_w, \quad \theta_i = nS_i, \quad S_w + S_i = 1 \]

- **Volumetric fractions**
  - Solid Skeleton
  - Water
  - Ice
  \[ n = \theta_w + \theta_i \]

- **Degree of saturation**
  - Water
  - Ice

- **Two-stress state variables framework**

\[ S = P_{\text{ice}} - P_w \]

- **Pore pressures**
  - Ice
  - Water

\[ \sigma^* = \sigma - S_w P_w I \]

- **Solid phase stress**
  - Total stress
  - Unit tensor

\[ S \approx \rho_w L \ln \left( \frac{T}{273.15} \right) \]

- **Cryogenic suction**
  - Density
  - Latent heat of fusion of water

Illustration of frozen soil composition.
TEVP Model

Total strain rate decomposition:

- Mechanical (solid phase stress-dependent)
  - Elastic (time-independent recoverable)
  - Thermal-viscoplastic (time- and temperature-dependent irrecoverable)
- Cryogenic suction-dependent

\[
\dot{\varepsilon} = \dot{\varepsilon}^\sigma + \dot{\varepsilon}^{suc} = (\dot{\varepsilon}^{\sigma e} + \dot{\varepsilon}^{\sigma_Tvp}) + \dot{\varepsilon}^{suc}
\]

\[
p^* = \frac{\sigma_{ii}^*}{3} = (\sigma_{11}^* + \sigma_{22}^* + \sigma_{33}^*)/3 \quad \text{Mean solid phase stress}
\]

\[
q^* = \sqrt{3s_{ij}^*s_{ij}^*}/2 \quad \text{Deviatoric stress}
\]

\[
s_{ij}^* = \sigma_{ij}^* - p^*\delta_{ij} \quad \text{Deviatoric solid phase stress tensor}
\]
TEVP Model

**Elasticity**

- The elastic component of strain rate tensor due to the solid phase stress variation

\[
\dot{\epsilon}_{ij}^e = \frac{\dot{\nu}^*}{3K_{eq}} \delta_{ij} + \frac{1}{2G_{eq}} \dot{S}_{ij}^*
\]

\[
G_{eq} = (1 - S_i)G_{uf} + S_i \frac{E_f}{2(1+\nu_f)}
\]

\[
K_{eq} = (1 - S_i)K_{uf} + S_i \frac{E_f}{3(1-2\nu_f)}
\]

- Temperature-dependent equivalent elastic modulus

- Elastic modulus in an unfrozen state

- Poisson’s ratio

**Viscoplasticity**

- Elastic stiffness parameter for changes in cryogenic suction

\[
\delta \varepsilon^{suc} = (D^{suc})^{-1} \delta S
\]

\[
(D^{suc})^{-1} = \frac{1}{3V} \frac{\kappa_5}{(S+P_{atm})} I
\]

- Specific volume

- Elastic stiffness parameter for changes in cryogenic suction

- Number of sub-zero temperature

- Material parameter
TEVP Model

A suction-dependent criterion is required to capture the essential features of frozen and unfrozen behavior:

\[
F = q^* - M^2 (p^* - p_t^*) (p_f^* - p^*) = 0
\]

Slope of the CSL

\[
\lambda_f = \lambda_o \left[ (1 - \alpha) \exp(-\beta S) + \alpha \right]
\]

Elastoplastic compressibility coefficient

Model parameter controlling the maximum stiffness

Model parameter controlling the rate of change in stiffness with cryogenic suction

Apparent cohesion of the frozen soil due to cryogenic suction

\[
p_t^* = -k_t S
\]

Pre-consolidation stress in an unfrozen state

Compressibility coefficients within the elastic region

Reference stress

Plastic potential and yield surfaces should be described at the current stress state \((q^* - p^* - S_1)\)

Illustration of the surface adopted for the TEVP constitutive modeling of frozen soils in \(q^* - p^* - S\) space.
TEVP Model

**Elasticity**

Viscoplasticity

TVP deformation is formulated by considering the response of frozen soil under the corresponding isotropic stress state \((q^* = 0 - p^* = p_m^* - S_1)\):

\[
V_{NCL} = N_f - \lambda_f \ln p_m^* \quad \text{Specific volume of the frozen soil under } p_m^* \text{ (at the end of primary volumetric compression)}
\]

\[
N_f = N_0 + \kappa_s \ln \frac{S + p_{atm}}{p_{atm}} \quad \text{Specific volume at unit pressure in an unfrozen state}
\]

Adopting a logarithmic creep function for viscoplastic volume changes:

\[
\delta V_{m}^{\sigmaTv} = - \psi_T \ln \left( \frac{t_0 + t}{t_0} \right) \quad \text{Material parameter denoting the initiation of secondary creep}
\]

\[
\psi_T = \psi_0 \left( 1 + \frac{\theta}{\theta_0} \right)^b \quad \text{Material parameter}
\]

\[
\sigma^{\sigmaTv} = \frac{\delta \varepsilon_{\\text{vp}}}{\delta t} = \frac{\delta V_{m}^{\text{vp}}/V_m}{\delta t} = \frac{\psi_T}{V_m (t_0 + t)}
\]

TVP volumetric strain rate

**Description of the TEVP model.**
**TEVP Model**

### Elasticity

At time $t$ after primary compression, isotropically compressed states $p_m^* - V_m$ of the frozen soil can be defined in $\ln p^* - V$ plane by a line that is parallel to and at constant vertical separation from the NCL for the current frozen state. As the elapsed time for thermal viscoplastic straining approaches infinity, these states are defined by a line called the viscoplastic limit line (VPL).

$$V_{VPL} = Z - \lambda_f \ln p^*$$

**Vertical intercept of the VPL at unit pressure in the current frozen state**

At a specific time:

$$V_m = N_f - \lambda_f \ln p_m^* - \psi_T \ln \left(\frac{t_o + t}{t_o}\right)$$

$$\varepsilon_p^\sigma_{TVP} = \frac{\psi_T}{V_m t_o} \exp \left(\frac{V_m - N_f}{\psi_T}\right) \left(p_m^*\right)^{\frac{\lambda_f}{\psi_T}}$$

**Description of the TEVP model.**
**TEVP Model**

### Elasticity

#### Hardening Law:

\[
F_Y = q^* - M^2 (\lambda^* - \chi^t) (\chi - \lambda^*) = 0
\]

Yielding criterion in isotropic compression and in isotropic tension

TVP volumetric straining is associated with hardening of the soil and the expansion of the yield locus:

\[
p_f^* = \exp \left( \frac{1}{\lambda_f - \kappa_f} \right) (Z - V - \kappa_f \ln p^*)
\]

\[
\delta p_f^* = \left( \frac{\lambda_p}{\lambda_f - \kappa_f} p_f^* \right) \delta \varepsilon_T \text{VTP}
\]

### Viscoplasticity

#### Flow Rule:

A non-associated flow rule is adopted to generalize the model to any loading path and stress state:

\[
\dot{\varepsilon}_{ij}^{\text{TVP}} = \Lambda \frac{\partial F_{\text{VPS}}}{\partial \sigma_{ij}^*}
\]

\[
\Lambda = \frac{\psi_V}{V_m t_o} \exp \left( \frac{V_m - N_f}{\psi_T} \right) \left( p_m^* \right)^{\lambda_f} \frac{1}{\partial F_{\text{VPS}} / \partial p^*}
\]

Scaler multiplier

\[
\dot{\sigma}_{ij}^{\text{TVP}} = \frac{\psi_T}{V_m t_o} \exp \left( \frac{V_m - N_f}{\psi_T} \right) \left( p_m^* \right)^{\lambda_f} \frac{1}{\partial F_{\text{VPS}} / \partial p^*} \frac{1}{\partial \sigma_{ij}^*}
\]

Description of the TEVP model.
Model Performance

Triaxial Compression Tests

Xu (2014) conducted several triaxial compression tests on frozen sand samples at different temperatures.

Model parameters used in this simulation

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Constant axial strain rate of $1.67 \times 10^{-4}$ s$^{-1}$ and initial confinement of 1 Mpa

- Deviatoric stress-axial strain ($q^*-\varepsilon_a$) plot.
- Volumetric strain-axial strain ($\varepsilon_p-\varepsilon_a$) plot.
Model Performance

Uniaxial Creep Tests

Eckardt (1979) investigated the stress-strain behavior of frozen sand samples by conducting uniaxial creep tests:

Model parameters used in this simulation.

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</table>

Uniaxial compression tests on frozen sand at $T = -15^\circ$C: axial strain-time plot.
Conclusions

- A TEVP constitutive model based on the framework of CSSM was proposed to examine the rate-dependent behavior of frozen soils.

- The model was formulated within the two stress-state variables framework.

- Plastic potential and yield surfaces were defined based on the current stress state of the soil. The hardening (softening) of the soil was formulated based on the definition of the VPL.

- The capability of the model was examined by reproducing the conventional triaxial compression and creep tests results.

- The model can be used to investigate the behavior of the frozen ground under extreme short-term as well as long-term climatic events in permafrost regions.
References

Thank you for your attention!