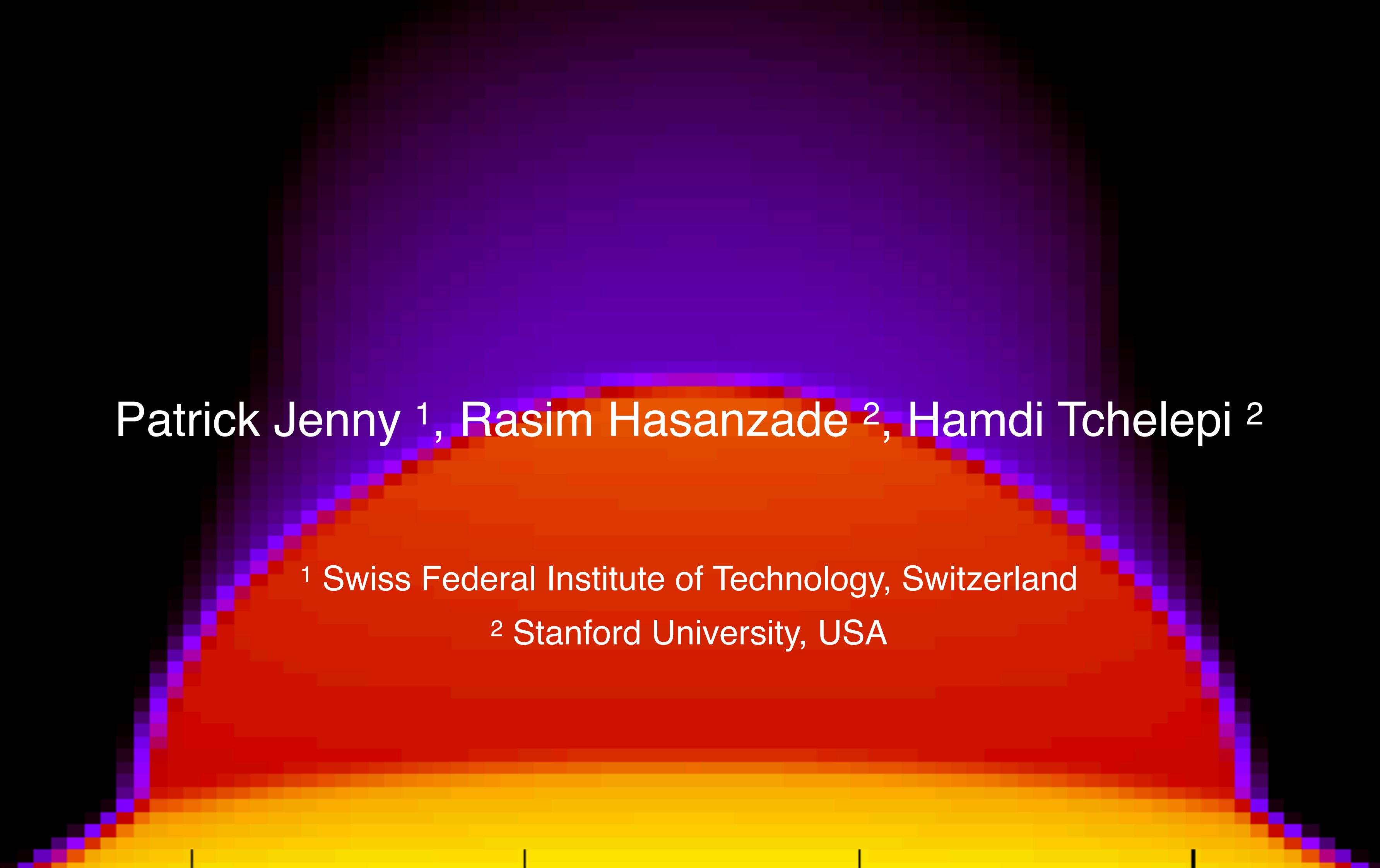
Tightly Coupled Hyperbolic Treatment of Buoyant Two-Phase Flow and Transport in Porous Media



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Goal:

tightly coupled, explicit flow and transport scheme for natural convection of multiple phases which is suited for computations on GPUs

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Problem and Motivation:

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coupled hyperbolic system for flow and transport

based on augmented isothermal Euler equations with source terms

explicit time integration using an approximate Riemann solver

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Problem and Motivation:

fully implicit methods are expensive and sequential methods require small time steps due to tight coupling between flow and transport

Method:

coupled hyperbolic system for flow and transport
based on augmented isothermal Euler equations with source terms
explicit time integration using an approximate Riemann solver
high order in space and time

Results:

demonstration of accuracy and computational efficiency
for 1D and 2D test cases with natural convection

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Swiss Federal Institute of Technology Zurich

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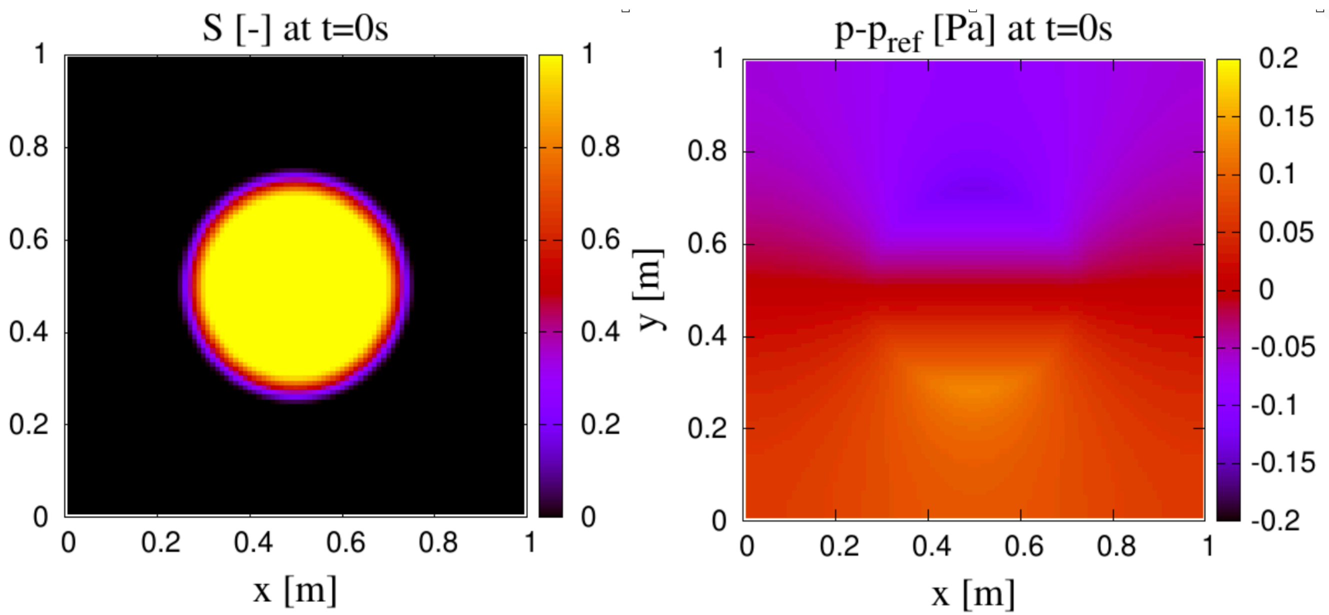
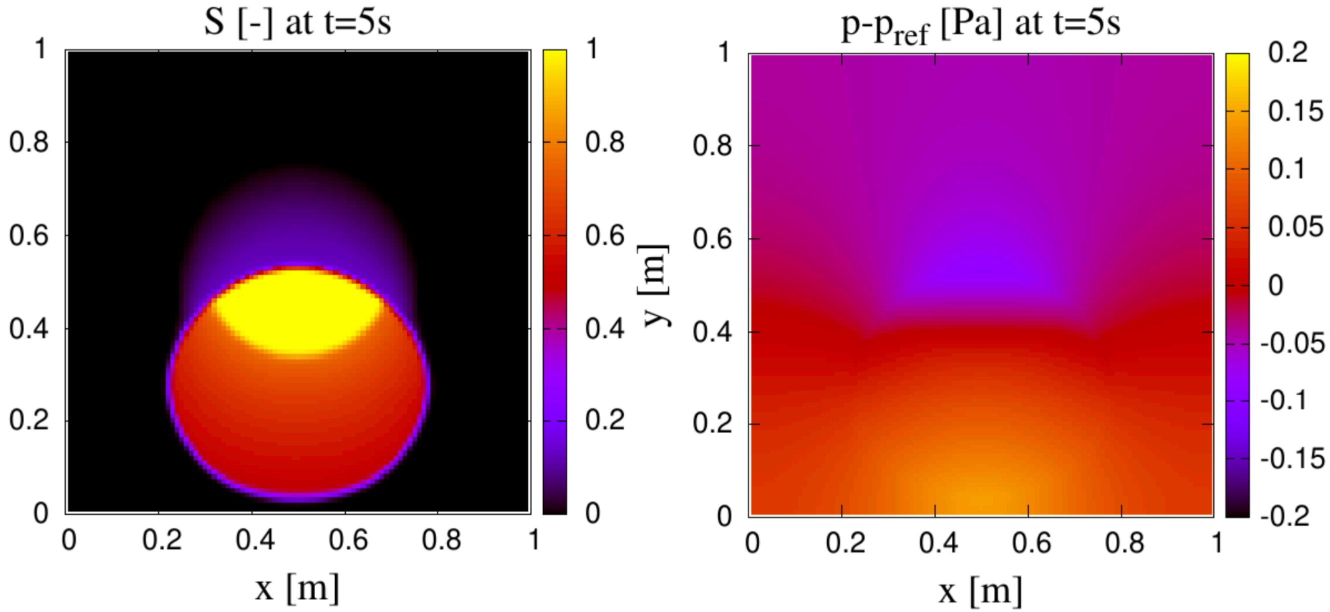
Motivation

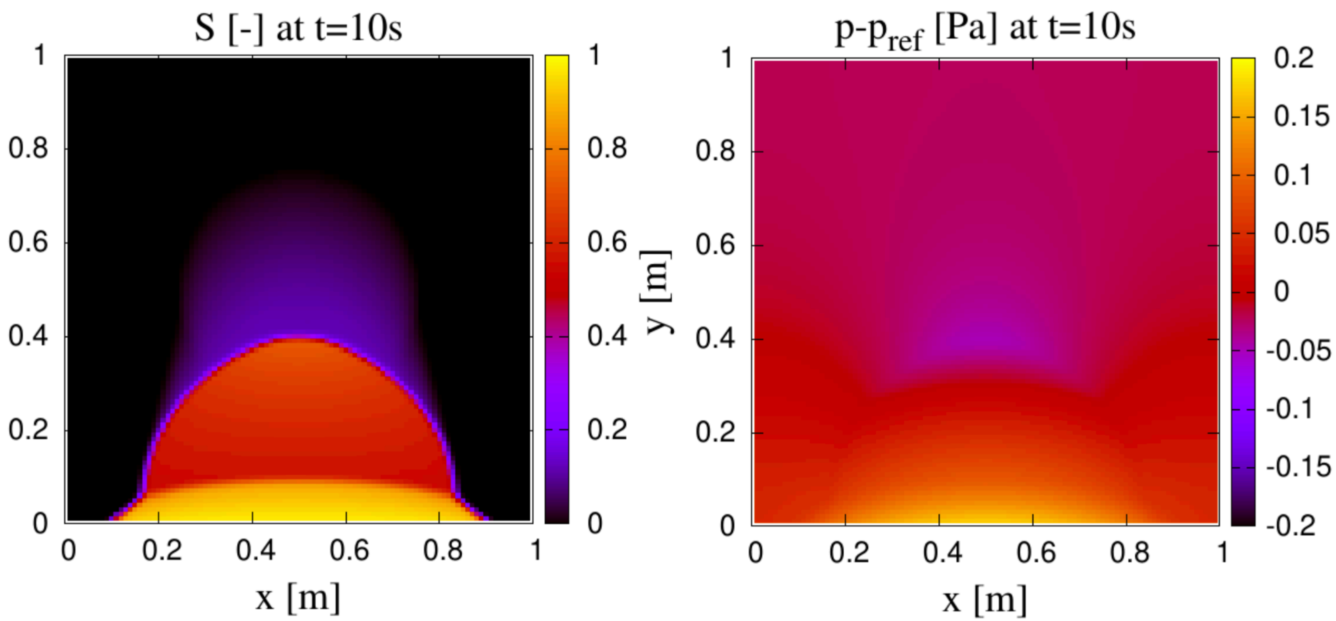


Motivation



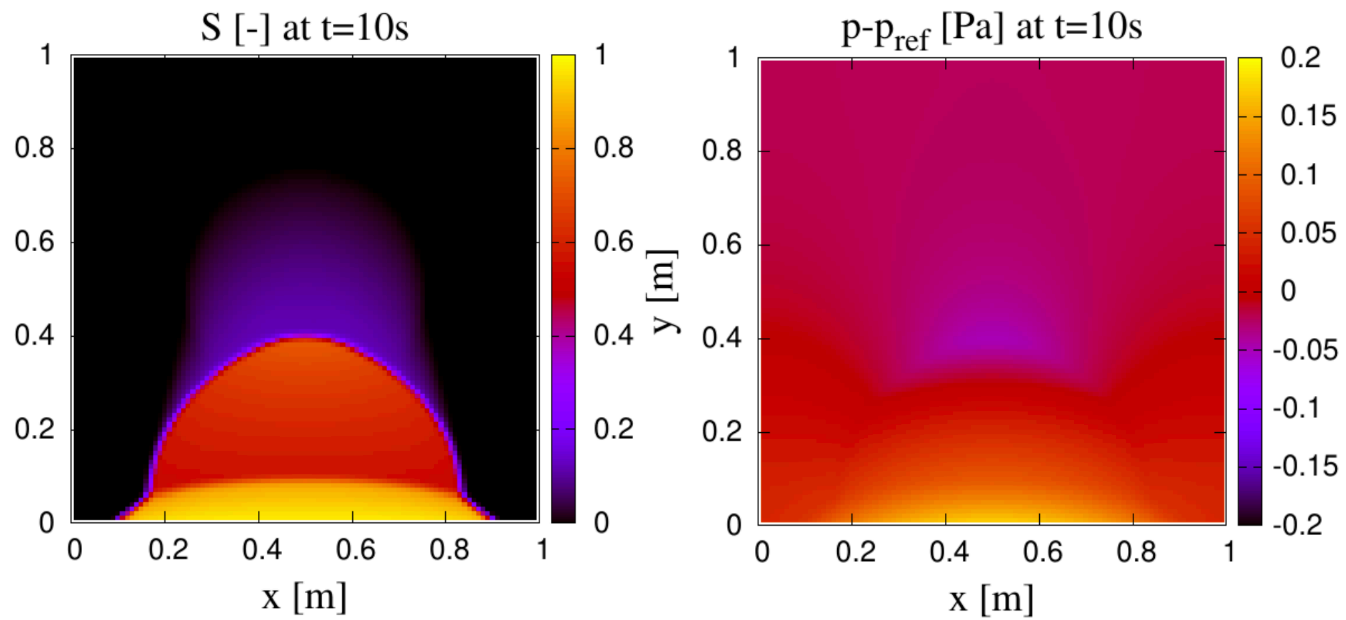
S [-] at t=5s
p-p_ref [Pa] at t=5s
x [m]
y [m]
0
0.2
0.4
0.6
0.8
1
0.15
0.1
0.05
-0.05
-0.1
-0.15
-0.2

Motivation



S [-] at t=10s
x [m]
y [m]
p-p_ref [Pa] at t=10s
x [m]
0
0.2
0.4
0.6
0.8
1
0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2

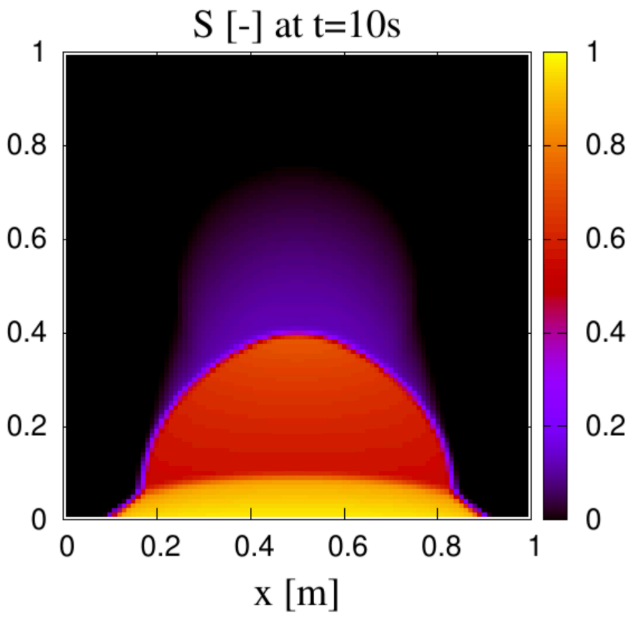
Classical: Elliptic Flow and Hyperbolic Transport

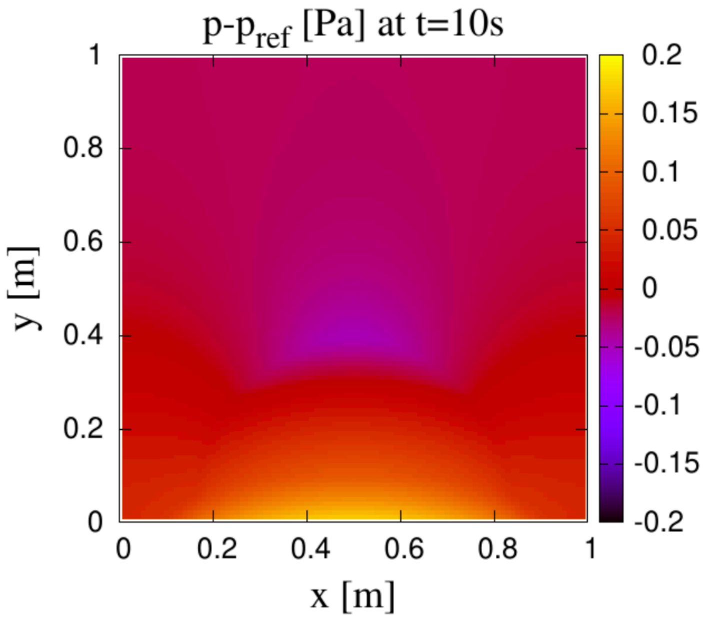


$$\nabla \cdot \boldsymbol{u} = 0$$

$$\boldsymbol{u} = -\lambda k \nabla p + k\gamma \boldsymbol{g}$$

Classical: Elliptic Flow and Hyperbolic Transport



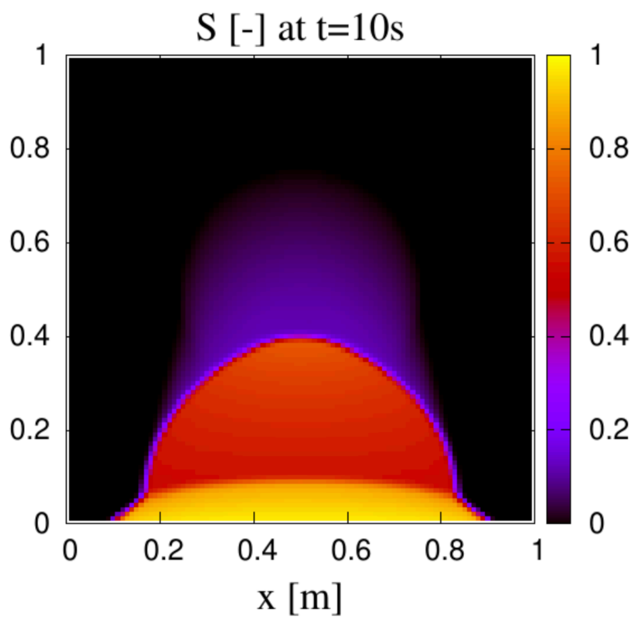


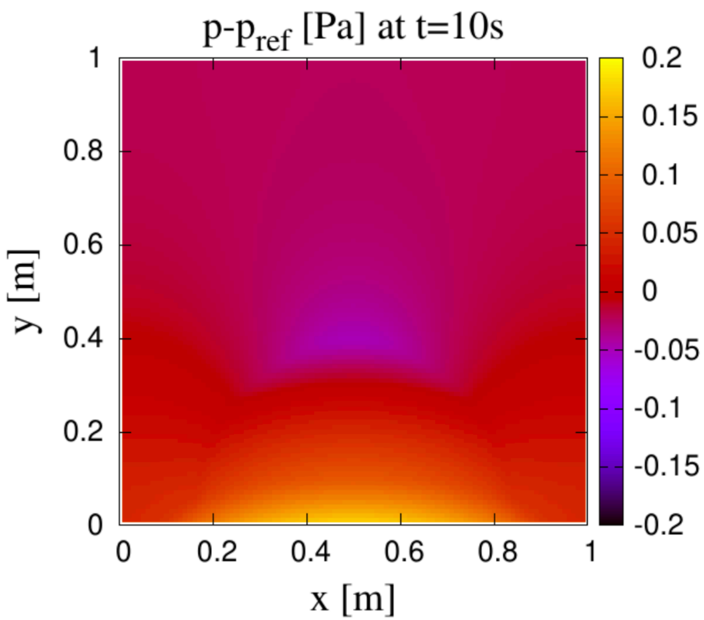
$$\nabla \cdot \boldsymbol{u} = 0$$

$$\boldsymbol{u} = -\lambda k \nabla p + k\gamma \boldsymbol{g}$$

$$\lambda = \overbrace{\frac{k_{r_{p1}}}{\mu_{p1}}}^{\lambda_{p1}} + \overbrace{\frac{k_{r_{p2}}}{\mu_{p2}}}^{\lambda_{p2}} \quad \text{and} \quad \gamma = \lambda_{p1}\rho_{p1} + \lambda_{p2}\rho_{p2}$$

Classical: Elliptic Flow and Hyperbolic Transport





$$\frac{\partial \phi S}{\partial t} + \nabla \cdot \boldsymbol{u}_{p1} = 0$$

$$\boldsymbol{u}_{p1} = f_1 \boldsymbol{u} + f_{12} \boldsymbol{g}$$

$$\nabla \cdot \boldsymbol{u} = 0$$

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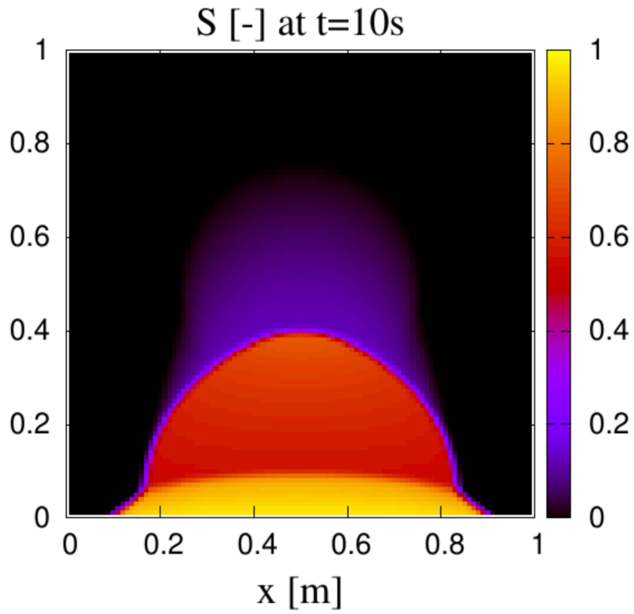
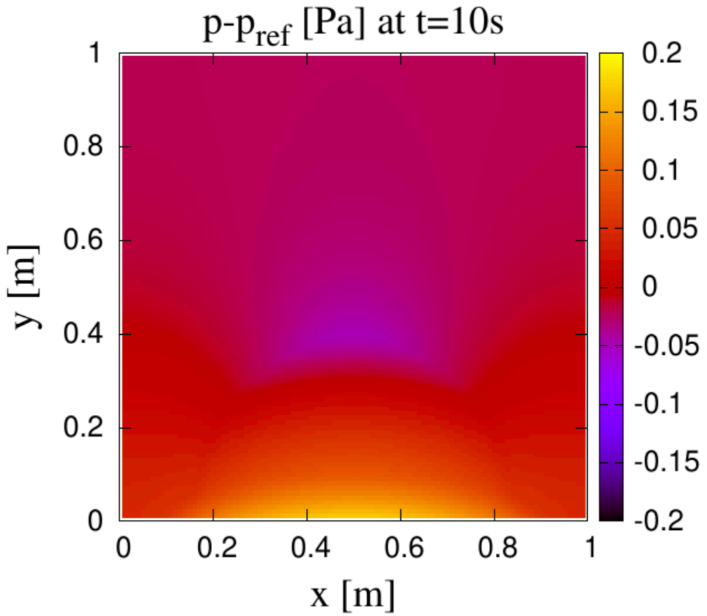
Classical: Elliptic Flow and Hyperbolic Transport





$$\frac{\partial \phi S}{\partial t} + \nabla \cdot \boldsymbol{u}_{p1} = 0$$

$$\boldsymbol{u}_{p1} = f_1 \boldsymbol{u} + f_{12} \boldsymbol{g}$$

$$f_1(S) = \frac{\lambda_{p1}(S)}{\lambda(S)} \quad \text{and} \quad f_{12}(S) = \frac{\lambda_{p1}(S)\lambda_{p2}(S)}{\lambda(S)}(\rho_{p1} - \rho_{p2})k$$

$$\nabla \cdot \boldsymbol{u} = 0$$

$$\boldsymbol{u} = -\lambda k \nabla p + k\gamma \boldsymbol{g}$$

$$\lambda = \overbrace{\frac{k_{r_{p1}}}{\mu_{p1}}}^{\lambda_{p1}} + \overbrace{\frac{k_{r_{p2}}}{\mu_{p2}}}^{\lambda_{p2}} \quad \text{and} \quad \gamma = \lambda_{p1}\rho_{p1} + \lambda_{p2}\rho_{p2}$$

New: Hyperbolic System for Flow and Transport

$$\frac{\partial}{\partial t}\underbrace{\begin{pmatrix}\rho\\ \rho u\\ \rho v\\ \rho Y\end{pmatrix}}_{\boldsymbol{U}}+\frac{\partial}{\partial x}\underbrace{\begin{pmatrix}\rho u\\ \rho u^2+p\\ \rho uv\\ F_x^Y\end{pmatrix}}_{\boldsymbol{F}_x}+\frac{\partial}{\partial y}\underbrace{\begin{pmatrix}\rho v\\ \rho vu\\ \rho v^2+p\\ F_y^Y\end{pmatrix}}_{\boldsymbol{F}_y}=\underbrace{\begin{pmatrix}0\\ R_x\\ R_y\\ 0\end{pmatrix}}_{\boldsymbol{Q}}$$

$$\frac{\partial \phi S}{\partial t}+\nabla\cdot\boldsymbol{u}_{p1}=0 \qquad \nabla\cdot\boldsymbol{u} = 0$$

$$\boldsymbol{u}_{p1} = f_1\boldsymbol{u} + f_{12}\boldsymbol{g} \qquad \boldsymbol{u} = -\lambda k\nabla p + k\gamma\boldsymbol{g}$$

New: Hyperbolic System for Flow and Transport

$$p = c^2\rho$$

$$\frac{\partial}{\partial t}\underbrace{\begin{pmatrix}\rho\\ \rho u\\ \rho v\\ \rho Y\end{pmatrix}}_{\boldsymbol{U}} + \frac{\partial}{\partial x}\underbrace{\begin{pmatrix}\rho u\\ \rho u^2 + p\\ \rho uv\\ F_x^Y\end{pmatrix}}_{\boldsymbol{F}_x} + \frac{\partial}{\partial y}\underbrace{\begin{pmatrix}\rho v\\ \rho vu\\ \rho v^2 + p\\ F_y^Y\end{pmatrix}}_{\boldsymbol{F}_y} = \underbrace{\begin{pmatrix}0\\ R_x\\ R_y\\ 0\end{pmatrix}}_{\boldsymbol{Q}}$$

$$\frac{\partial \phi S}{\partial t} + \nabla\cdot\boldsymbol{u}_{p1} = 0 \qquad\qquad \nabla\cdot\boldsymbol{u} = 0$$

$$\boldsymbol{u}_{p1} = f_1\boldsymbol{u} + f_{12}\boldsymbol{g} \qquad\qquad \boldsymbol{u} = -\lambda k\nabla p + k\gamma\boldsymbol{g}$$

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$$\boxed{\boldsymbol{R} \;=\; -\frac{1}{\lambda k}\boldsymbol{u} \;+\; \frac{\gamma}{\lambda}\boldsymbol{g}}$$

$$\frac{\partial \phi S}{\partial t} + \nabla\cdot\boldsymbol{u}_{p1} = 0$$

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$$\boxed{\boldsymbol{R} = -\frac{1}{\lambda k}\boldsymbol{u} + \frac{\gamma}{\lambda}\boldsymbol{g}}$$

$$\boxed{\begin{aligned} Y &= \phi S \\ \boldsymbol{F}^Y &= \rho \boldsymbol{u}_{p1}\end{aligned}}$$

$$\frac{\partial \phi S}{\partial t} + \nabla \cdot \boldsymbol{u}_{p1} = 0$$

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New: Hyperbolic System for Flow and Transport

$$\frac{\partial}{\partial t}\underbrace{\begin{pmatrix}\rho\\ \rho u\\ \rho v\\ \rho Y\end{pmatrix}}_{\boldsymbol{U}}+\frac{\partial}{\partial x}\underbrace{\begin{pmatrix}\rho u\\ \rho u^2+\overbrace{p}^{p=c^2\rho}\\ \rho uv\\ F_x^Y\end{pmatrix}}_{\boldsymbol{F}_x}+\frac{\partial}{\partial y}\underbrace{\begin{pmatrix}\rho v\\ \rho vu\\ \rho v^2+p\\ F_y^Y\end{pmatrix}}_{\boldsymbol{F}_y}=\underbrace{\begin{pmatrix}0\\ R_x\\ R_y\\ 0\end{pmatrix}}_{\boldsymbol{Q}}$$

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$$\boxed{\begin{aligned} Y &= \phi S \\ \boldsymbol{F}^Y &= \rho \boldsymbol{u}_{p1}\end{aligned}}$$

<u>if the following numbers are small:</u>

- the maximum relative density variation $\Delta\hat{\rho}_{max} = (\rho_{max} - \rho_{min})/\rho_{mean}$
- the dimensionless number $\mathcal{R} = \rho\lambda k\,|\boldsymbol{u}|/L_{flow}$, and
- the Mach number $Ma = |\boldsymbol{u}|/c$

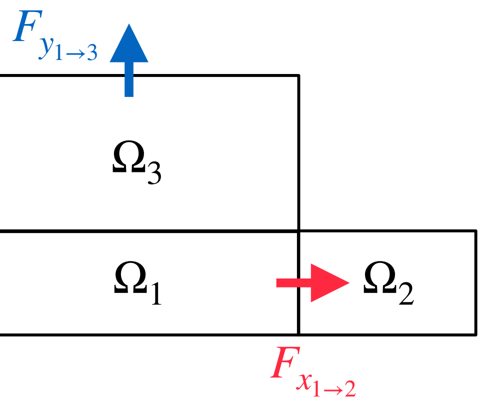
$$\frac{\partial \phi S}{\partial t} + \nabla \cdot \boldsymbol{u}_{p1} = 0 \qquad\qquad \nabla \cdot \boldsymbol{u} = 0$$

$$\boldsymbol{u}_{p1} = f_1\boldsymbol{u} + f_{12}\boldsymbol{g} \qquad\qquad \boldsymbol{u} = -\lambda k \nabla p + k\gamma \boldsymbol{g}$$

Numerical Scheme

$p = c^2\rho$

$$\frac{\partial}{\partial t}\underbrace{\begin{pmatrix}\rho\\ \rho u\\ \rho v\\ \rho Y\end{pmatrix}}_{\boldsymbol{U}} + \frac{\partial}{\partial x}\underbrace{\begin{pmatrix}\rho u\\ \rho u^2 + p\\ \rho u v\\ F_x^Y\end{pmatrix}}_{\boldsymbol{F}_x} + \frac{\partial}{\partial y}\underbrace{\begin{pmatrix}\rho v\\ \rho v u\\ \rho v^2 + p\\ F_y^Y\end{pmatrix}}_{\boldsymbol{F}_y} = \underbrace{\begin{pmatrix}0\\ R_x\\ R_y\\ 0\end{pmatrix}}_{\boldsymbol{Q}}$$



$$\boldsymbol{U}_I^{n+1} = \boldsymbol{U}_I^n - \frac{\Delta t}{V_I}\sum_{J\in\mathcal{A}(I)}\left(\left(\boldsymbol{F}_x n_x + \boldsymbol{F}_y n_y\right) A\right)_{I\to J}^{n\to n+1} + \Delta t\, \boldsymbol{Q}_I^{n\to n+1} \quad \forall I \in \{1, \ldots, N\},$$

$$\boldsymbol{F}_{\tilde{x}}(\rho_F, \tilde{u}_F, \tilde{v}_F, S_F) = \begin{pmatrix}\rho_F\tilde{u}_F\\ \rho_F\tilde{u}_F^2 + \rho_F c^2\\ \rho_F\tilde{u}_F\tilde{v}_F\\ f_1(S_f)\rho_F\tilde{u}_F + f_{12}(S_F)\rho_F g_{\tilde{x}}\end{pmatrix}$$

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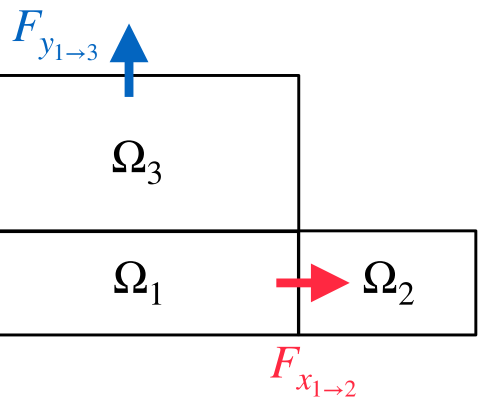
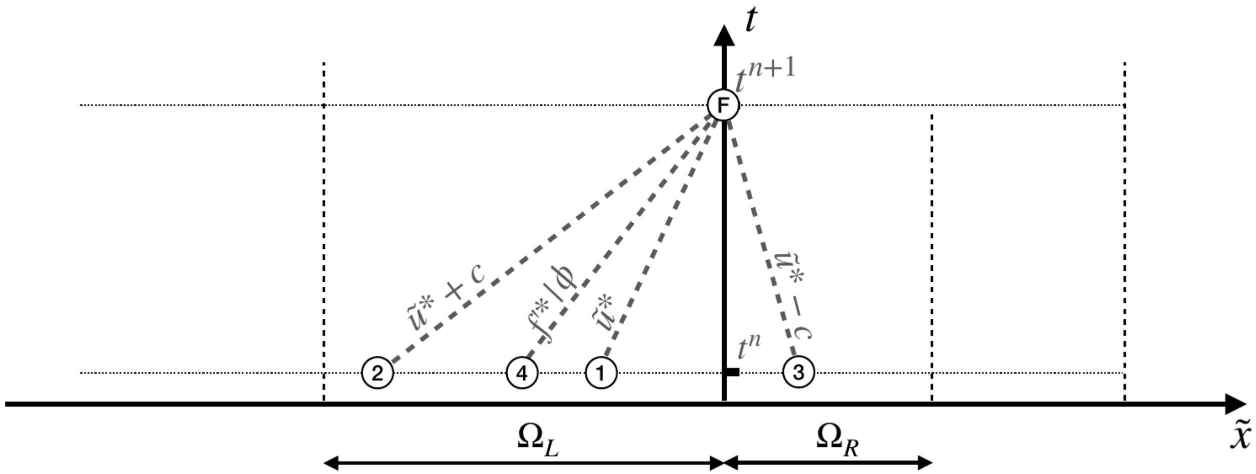
Numerical Scheme

$$\boldsymbol{U}_I^{n+1} = \boldsymbol{U}_I^n - \frac{\Delta t}{V_I} \sum_{J \in \mathcal{A}(I)} \Big((\boldsymbol{F}_x n_x + \boldsymbol{F}_y n_y)\, A \Big)_{I \to J}^{n \to n+1} + \Delta t\, \boldsymbol{Q}_I^{n \to n+1} \quad \forall I \in \{1, \ldots, N\},$$

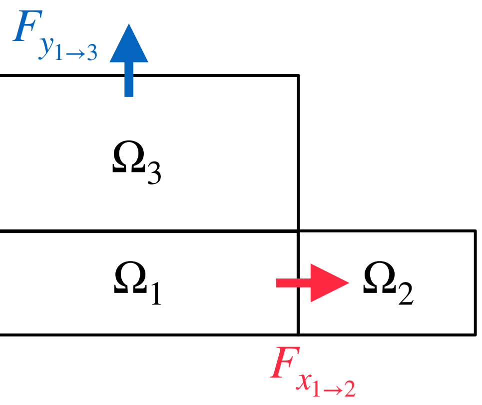


$$\boldsymbol{F}_{\tilde{x}}(\rho_F, \tilde{u}_F, \tilde{v}_F, S_F) = \begin{pmatrix} \rho_F \tilde{u}_F \\ \rho_F \tilde{u}_F^2 + \rho_F c^2 \\ \rho_F \tilde{u}_F \tilde{v}_F \\ f_1(S_f)\rho_F \tilde{u}_F + f_{12}(S_F)\rho_F g_{\tilde{x}} \end{pmatrix}$$

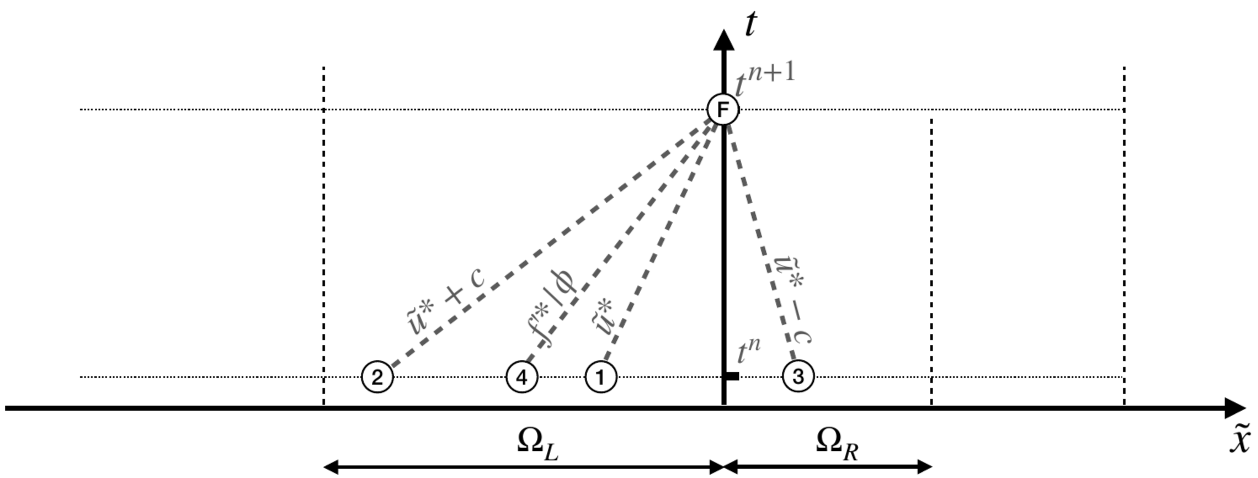


Numerical Scheme

$$\boldsymbol{U}_I^{n+1} = \boldsymbol{U}_I^n - \frac{\Delta t}{V_I} \sum_{J \in \mathcal{A}(I)} \left((\boldsymbol{F}_x n_x + \boldsymbol{F}_y n_y)\, A \right)_{I \to J}^{n \to n+1} + \Delta t\, \boldsymbol{Q}_I^{n \to n+1} \quad \forall I \in \{1, \dots, N\},$$



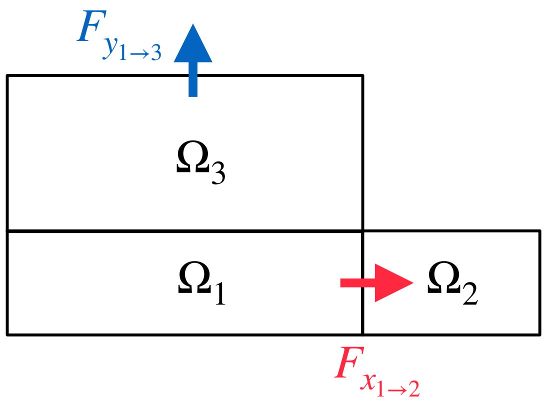
$$\boldsymbol{F}_{\tilde{x}}(\rho_F, \tilde{u}_F, \tilde{v}_F, S_F) = \begin{pmatrix} \rho_F \tilde{u}_F \\ \rho_F \tilde{u}_F^2 + \rho_F c^2 \\ \rho_F \tilde{u}_F \tilde{v}_F \\ f_1(S_f)\rho_F \tilde{u}_F + f_{12}(S_F)\rho_F g_{\tilde{x}} \end{pmatrix}$$



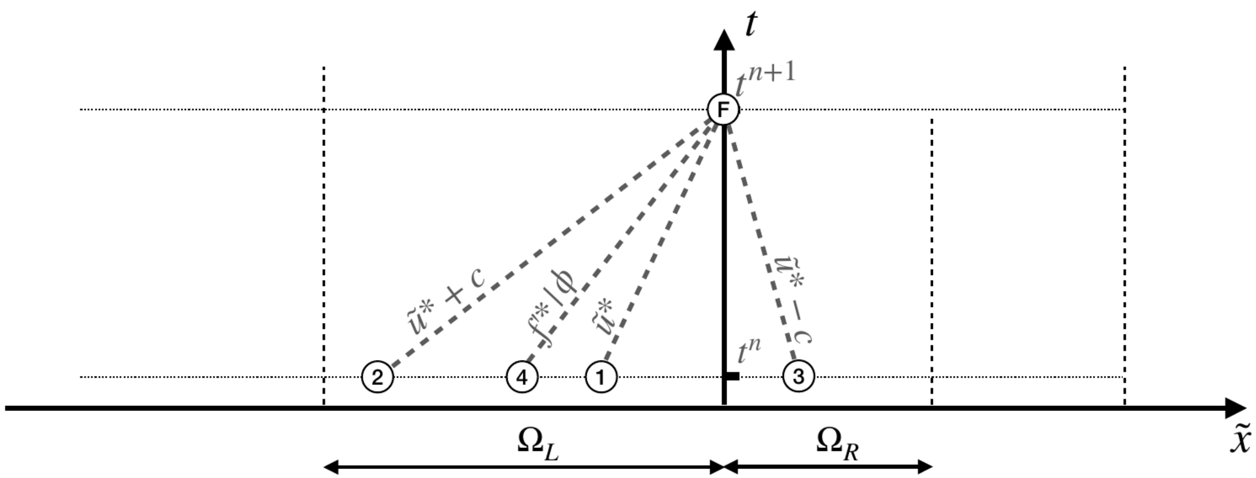
$$\begin{aligned}
d\tilde{v} &= 0 \quad \text{along the characteristic wave with speed } \tilde{u}^*, \\
\rho^*\, d\tilde{u} + c\, d\rho &= 0 \quad \text{along the characteristic wave with speed } \tilde{u}^* + c, \\
\rho^*\, d\tilde{u} - c\, d\rho &= 0 \quad \text{along the characteristic wave with speed } \tilde{u}^* - c \quad \text{and} \\
\mathcal{A} d\rho + \mathcal{B} d\tilde{u} + \mathcal{C} dS &= 0 \quad \text{along the characteristic wave with speed } f'^*/\phi,
\end{aligned}$$

Numerical Scheme

$$\boldsymbol{U}_I^{n+1} = \boldsymbol{U}_I^n - \frac{\Delta t}{V_I} \sum_{J\in\mathcal{A}(I)} \left((\boldsymbol{F}_x n_x + \boldsymbol{F}_y n_y)\, A\right)_{I\to J}^{n\to n+1} + \Delta t\, \boldsymbol{Q}_I^{n\to n+1} \quad \forall I \in \{1,\dots,N\},$$

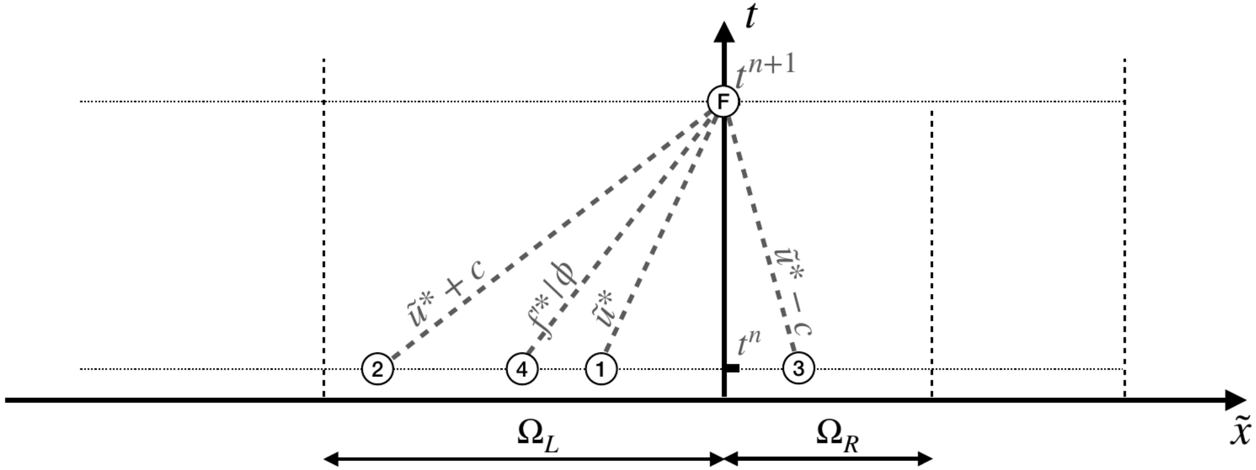


$$\boldsymbol{F}_{\tilde{x}}(\rho_F, \tilde{u}_F, \tilde{v}_F, S_F) = \begin{pmatrix} \rho_F \tilde{u}_F \\ \rho_F \tilde{u}_F^2 + \rho_F c^2 \\ \rho_F \tilde{u}_F \tilde{v}_F \\ f_1(S_f)\rho_F \tilde{u}_F + f_{12}(S_F)\rho_F g_{\tilde{x}} \end{pmatrix}$$



$$\begin{aligned}
(\tilde{v}_F - \tilde{v}_1) &= 0, && \text{along the characteristic wave with speed } \tilde{u}^*, \\
\rho^* (\tilde{u}_F - \tilde{u}_2) + c\,(\rho_F - \rho_2) &= 0, && \text{along the characteristic wave with speed } \tilde{u}^* + c, \\
\rho^* (\tilde{u}_F - \tilde{u}_3) - c\,(\rho_F - \rho_3) &= 0 \text{ and} && \text{along the characteristic wave with speed } \tilde{u}^* - c \text{ and} \\
\mathcal{A}^*(\rho_F - \rho_4) + \mathcal{B}^*(\tilde{u}_F - \tilde{u}_4) + C^*(S_F - S_4) &= 0, && \text{along the characteristic wave with speed } f'^*/\phi,
\end{aligned}$$

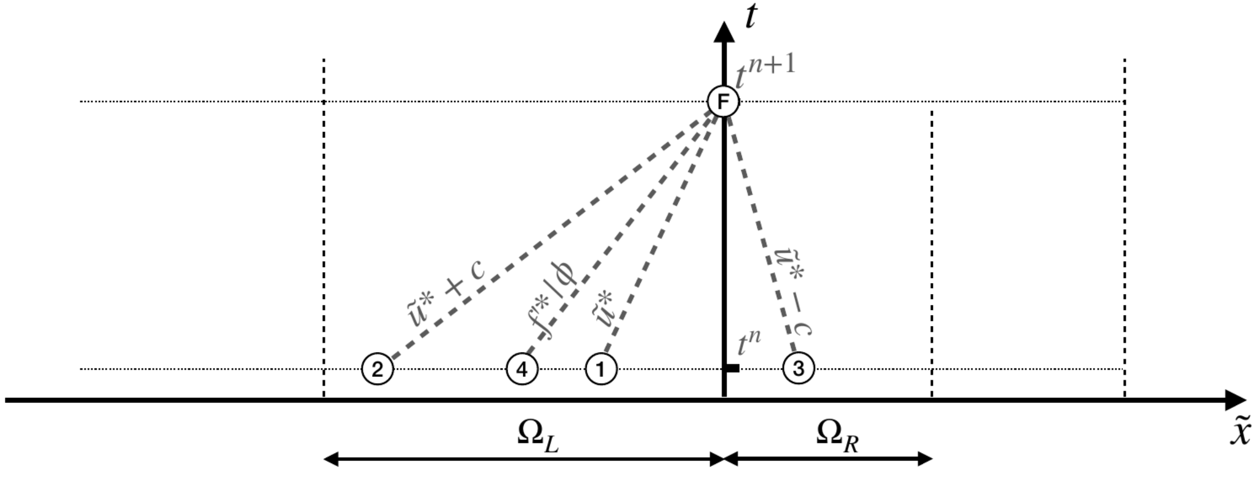
Numerical Scheme



$$
\begin{aligned}
(\tilde{v}_F - \tilde{v}_1) &= 0, && \text{along the characteristic wave with speed } \tilde{u}^*,\\
\rho^*\,(\tilde{u}_F - \tilde{u}_2) + c\,(\rho_F - \rho_2) &= 0, && \text{along the characteristic wave with speed } \tilde{u}^* + c,\\
\rho^*\,(\tilde{u}_F - \tilde{u}_3) - c\,(\rho_F - \rho_3) &= 0 \quad \text{and} && \text{along the characteristic wave with speed } \tilde{u}^* - c \quad \text{and}\\
\mathcal{A}^*(\rho_F - \rho_4) + \mathcal{B}^*(\tilde{u}_F - \tilde{u}_4) + C^*(S_F - S_4) &= 0, && \text{along the characteristic wave with speed } f'^*/\phi,
\end{aligned}
$$

$$
\boxed{
\begin{aligned}
\tilde{v}_F &= \tilde{v}_1,\\
\tilde{u}_F &= (\tilde{u}_2 + \tilde{u}_3)/2 + c(\rho_2 - \rho_3)/(2\rho^*),\\
\rho_F &= \rho_2 - (\tilde{u}_F - \tilde{u}_2)\rho^*/c \quad \text{and}\\
S_F &= S_4 - (\rho_F - \rho_4)\mathcal{A}/C - (\tilde{u}_F - \tilde{u}_4)\mathcal{B}/C
\end{aligned}
}
$$

Numerical Scheme



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(\tilde{v}_F - \tilde{v}_1) &= 0, && \text{along the characteristic wave with speed } \tilde{u}^*,\\
\rho^* (\tilde{u}_F - \tilde{u}_2) + c(\rho_F - \rho_2) &= 0, && \text{along the characteristic wave with speed } \tilde{u}^* + c,\\
\rho^* (\tilde{u}_F - \tilde{u}_3) - c(\rho_F - \rho_3) &= 0 \text{ and} && \text{along the characteristic wave with speed } \tilde{u}^* - c \text{ and}\\
\mathcal{A}^*(\rho_F - \rho_4) + \mathcal{B}^*(\tilde{u}_F - \tilde{u}_4) + \mathcal{C}^*(S_F - S_4) &= 0, && \text{along the characteristic wave with speed } f'^*/\phi,
\end{aligned}
$$

$$
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\begin{aligned}
\tilde{v}_F &= \tilde{v}_1,\\
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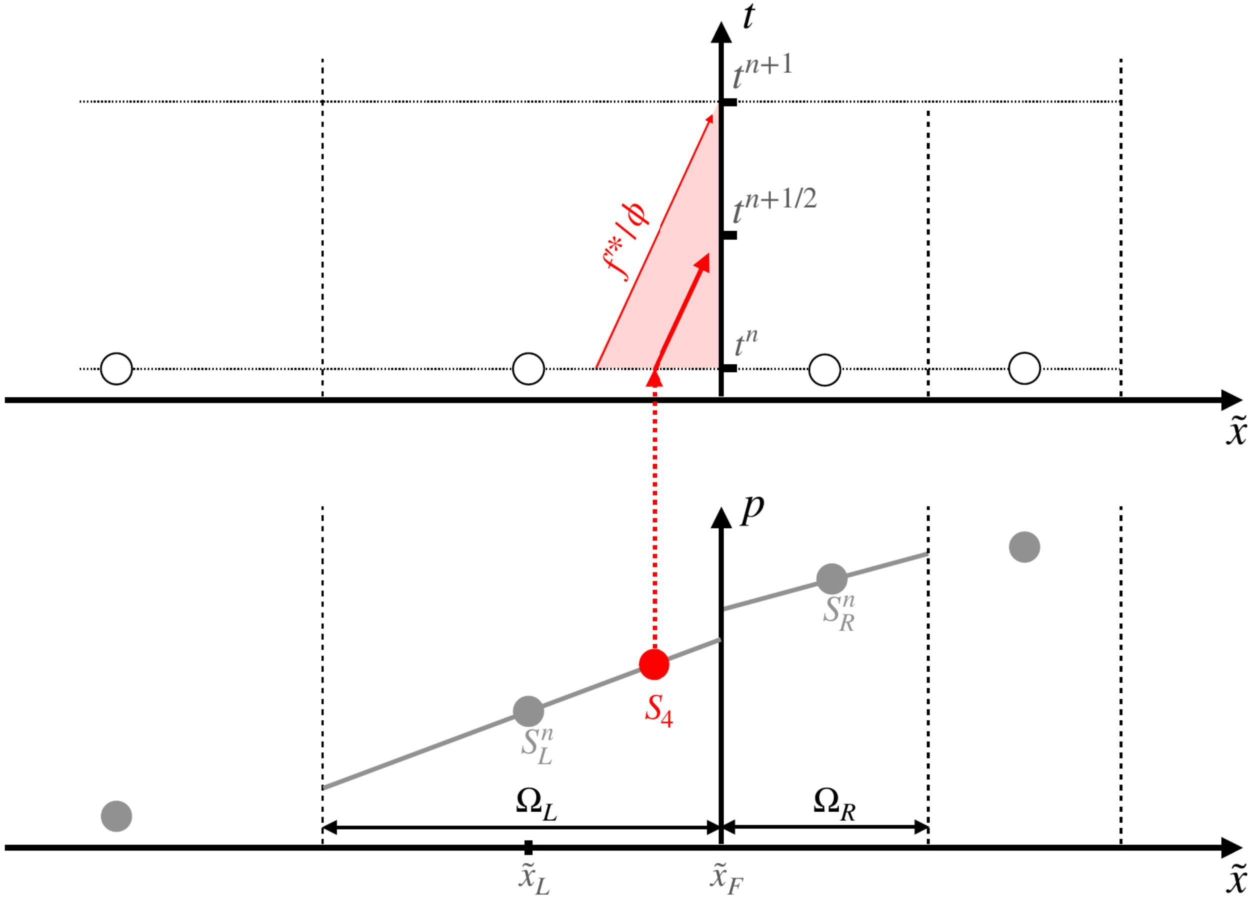
\rho_F &= \rho_2 - (\tilde{u}_F - \tilde{u}_2)\rho^*/c \quad \text{and}\\
S_F &= S_4 - (\rho_F - \rho_4)\mathcal{A}/\mathcal{C} - (\tilde{u}_F - \tilde{u}_4)\mathcal{B}/\mathcal{C}
\end{aligned}
}
$$

$$
\begin{aligned}
\mathcal{A} &= (c - \tilde{u}^*)(c + \tilde{u}^*)S^*\phi^2 + (S^* f'^* \tilde{u}^* - c^2 f_1^* + f^* \tilde{u}^*)\phi + f^* f'^*,\\
\mathcal{B} &= (f_{12}^* g_x - S^* f'^*)\rho^*\phi - f'^* f_1{}^* \rho^* \quad \text{and}\\
\mathcal{C} &= (c - \tilde{u}^*)(c + \tilde{u}^*)\rho^*\phi^2 + 2f'^*\phi\rho^*\tilde{u}^* - f'^* f'^* \rho^*.
\end{aligned}
$$

Numerical Scheme



$$
\begin{aligned}
\tilde{v}_F &= \tilde{v}_1, \\
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S_F &= S_4 - (\rho_F - \rho_4)\mathcal{A}/C - (\tilde{u}_F - \tilde{u}_4)\mathcal{B}/C
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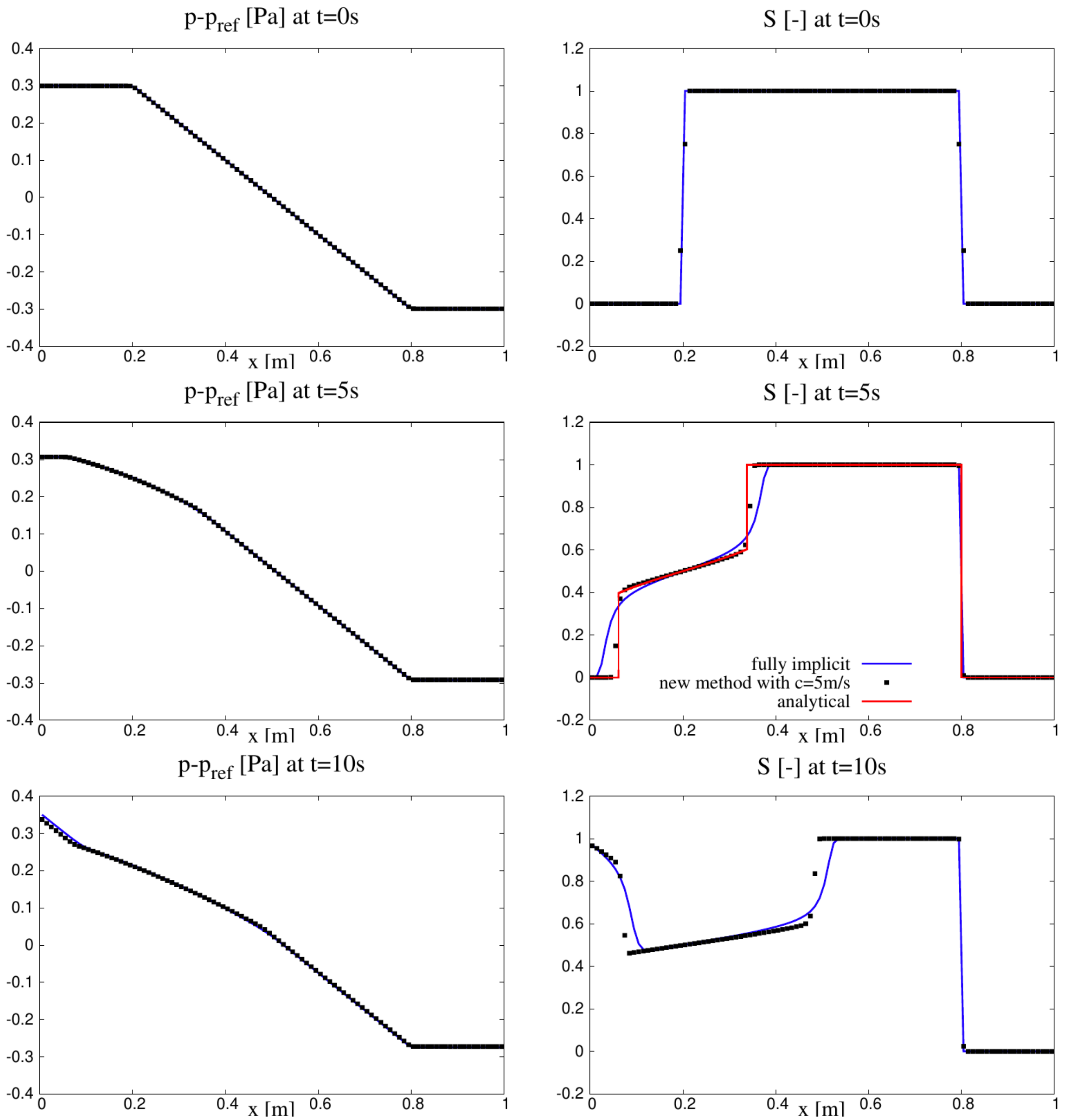
\mathcal{A} &= (c - \tilde{u}^*)(c + \tilde{u}^*)S^*\phi^2 + (S^* f'^* \tilde{u}^* - c^2 f_1^* + f^* \tilde{u}^*)\phi + f^* f'^*, \\
\mathcal{B} &= (f_{12}^* g_x - S^* f'^*)\rho^*\phi - f'^* f_1{}^* \rho^* \quad \text{and} \\
C &= (c - \tilde{u}^*)(c + \tilde{u}^*)\rho^*\phi^2 + 2f'^*\phi\rho^*\tilde{u}^* - f'^* f'^* \rho^*.
\end{aligned}
$$

Case 1: 1D Plume



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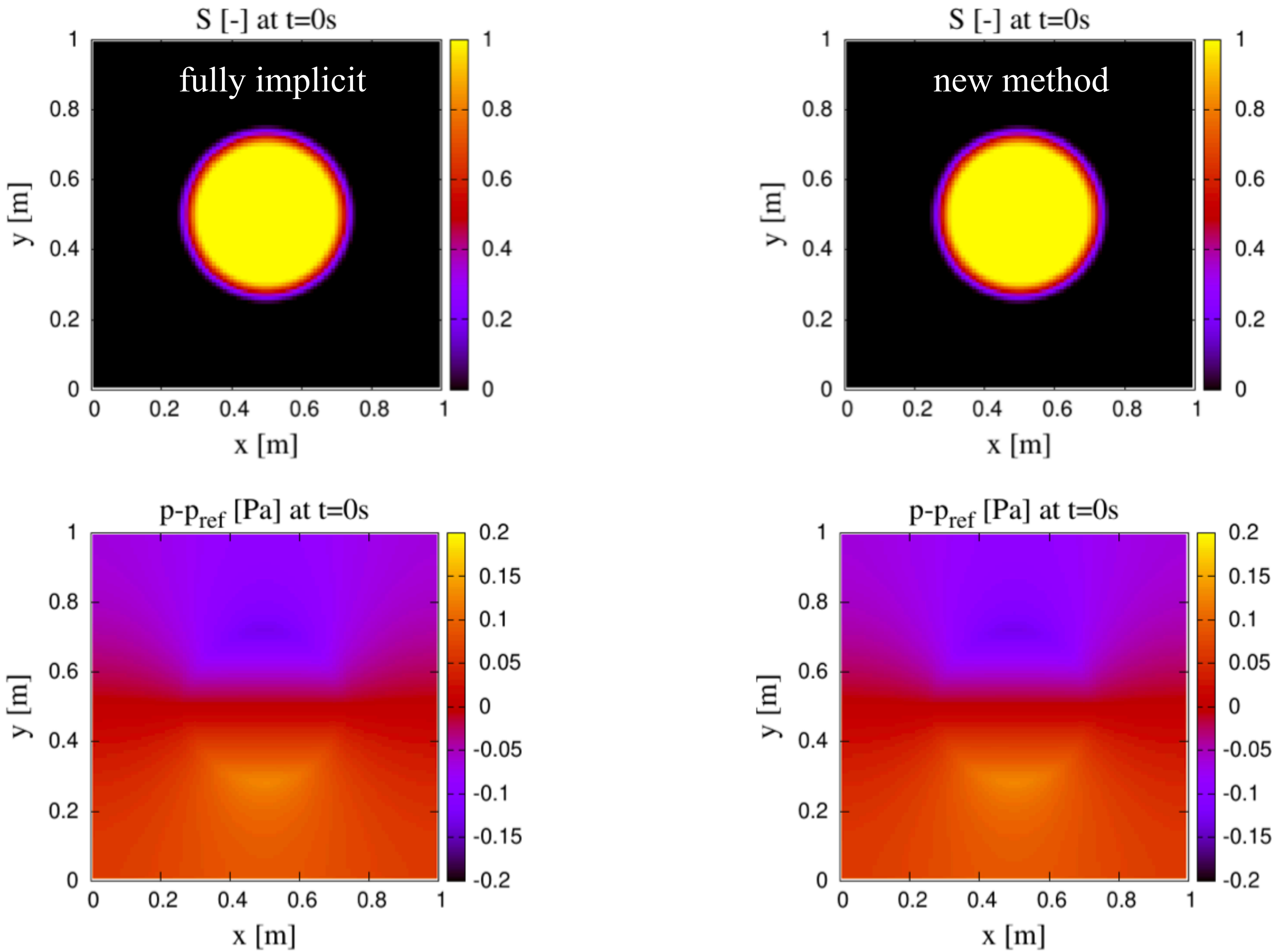
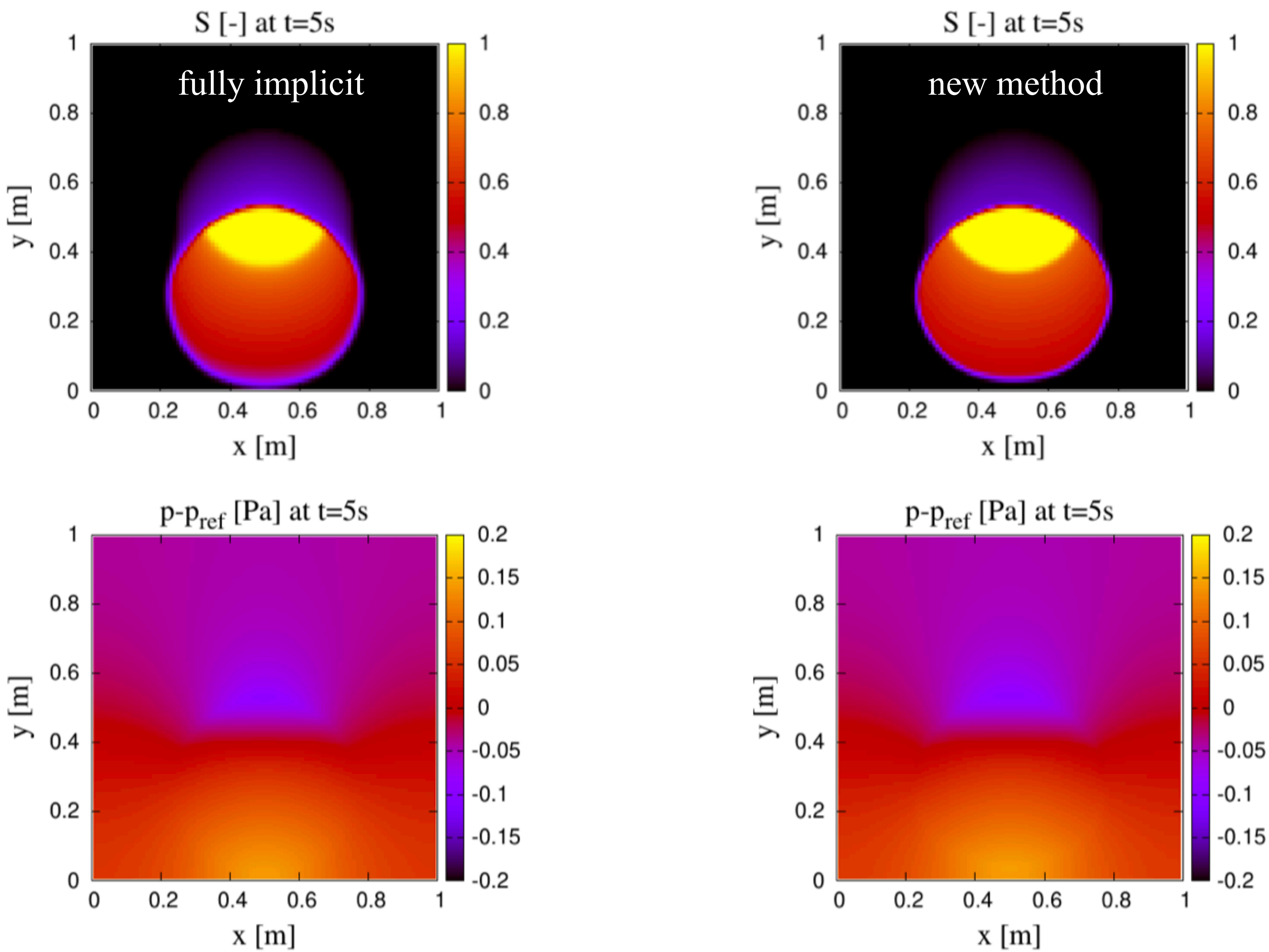
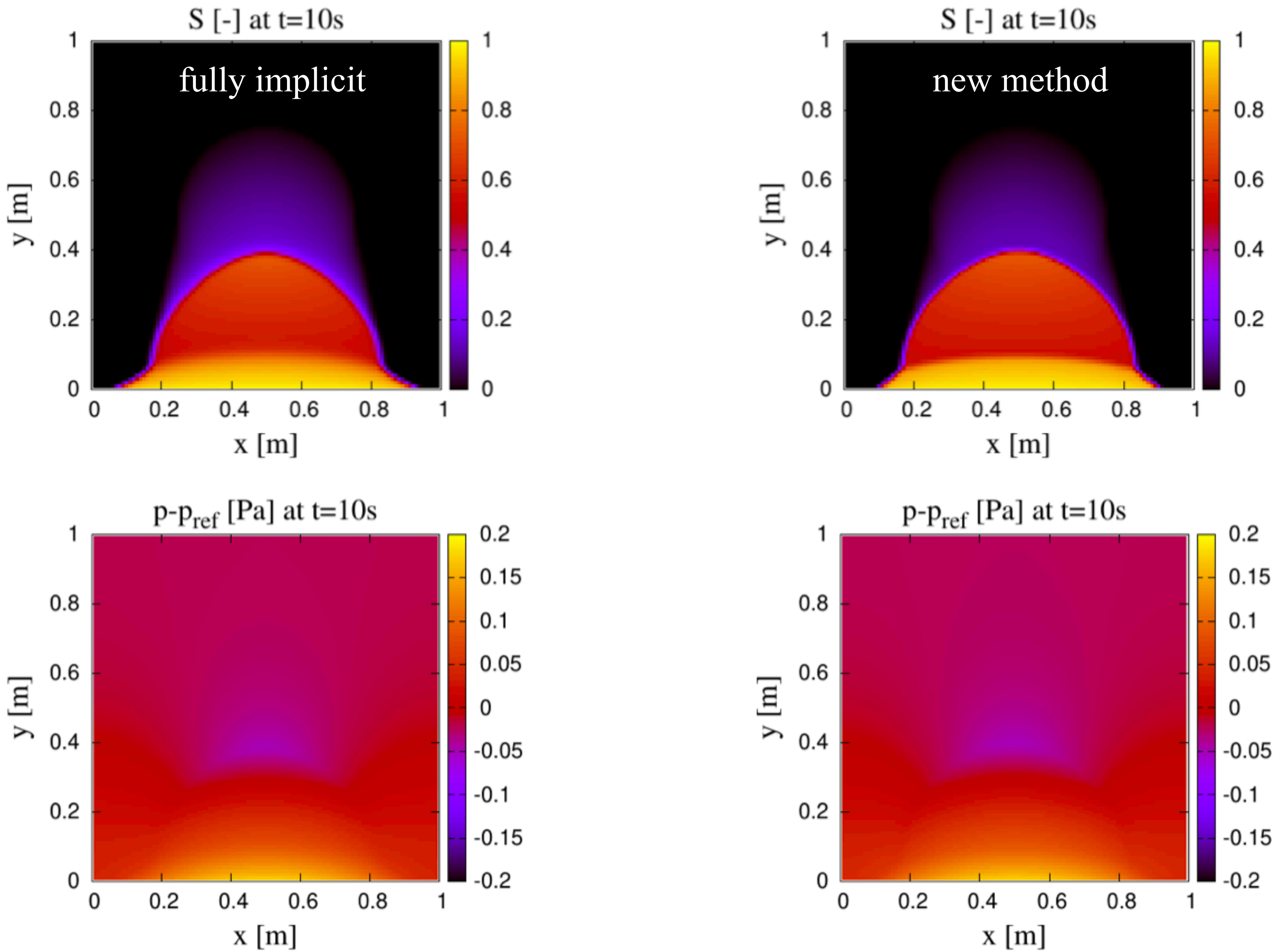
Case 2: 2D Plume



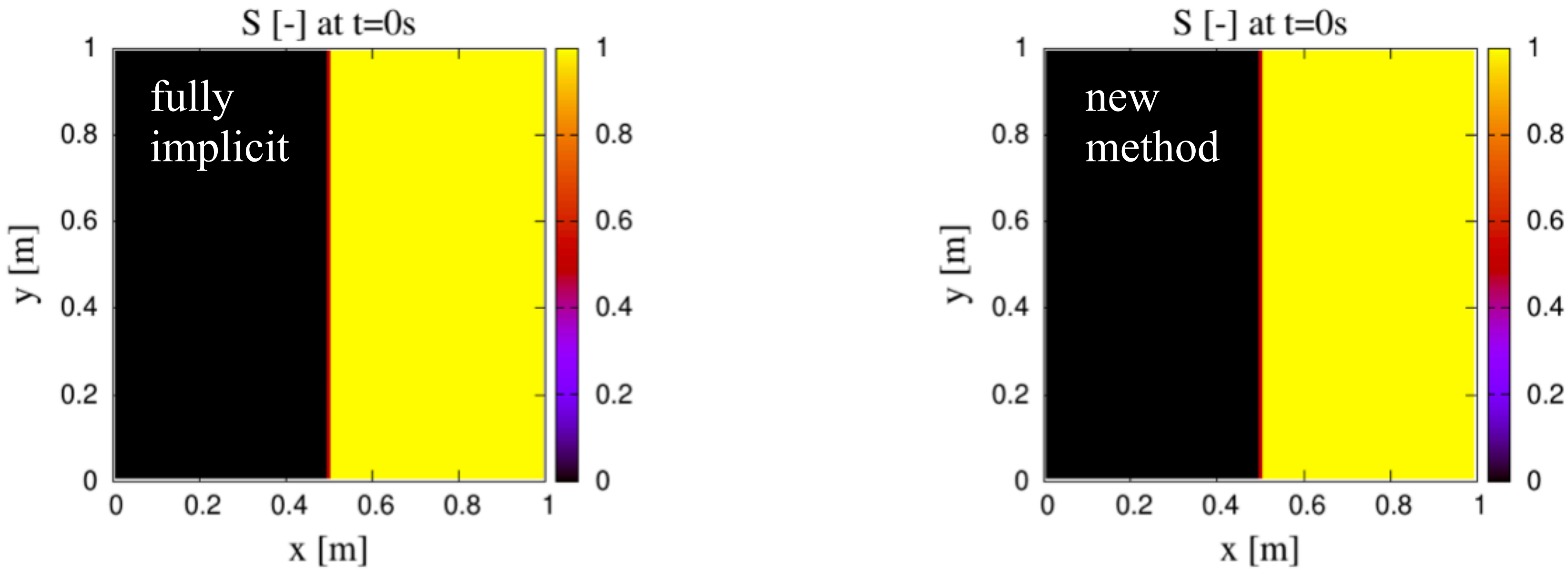
Case 2: 2D Plume



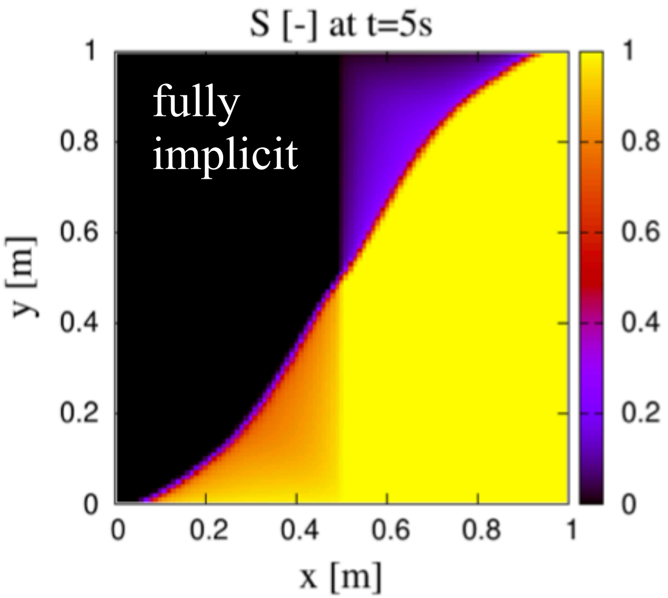
Case 2: 2D Plume

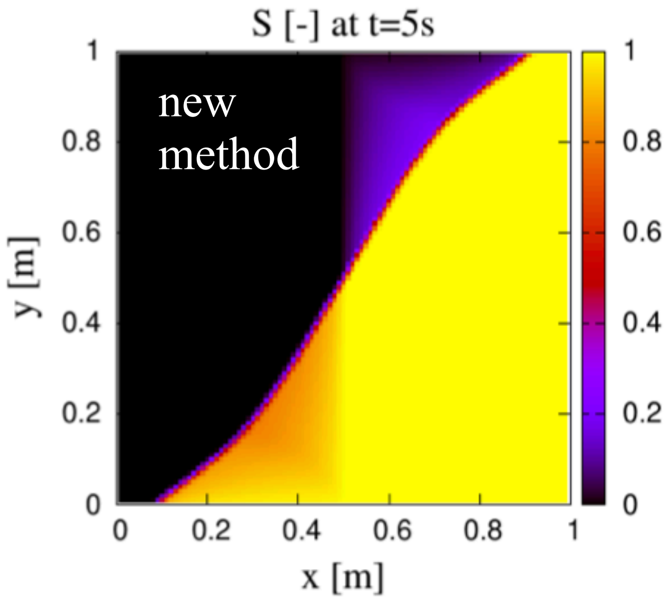


Case 3: Lock Exchange



Case 3: Lock Exchange





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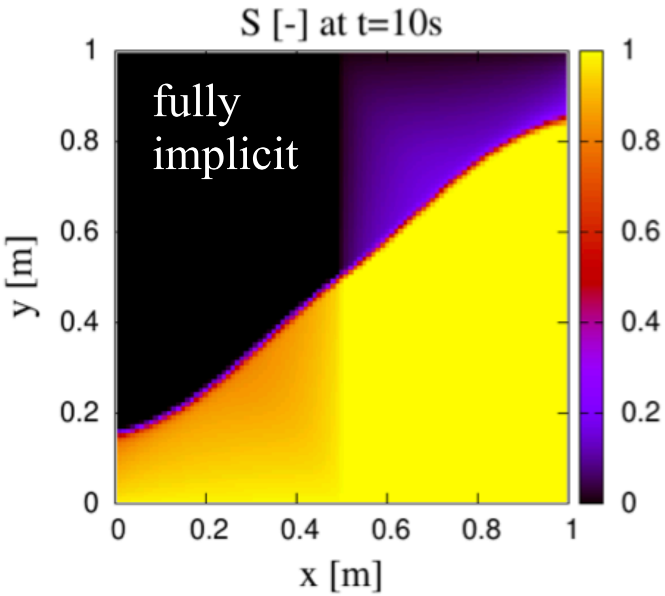
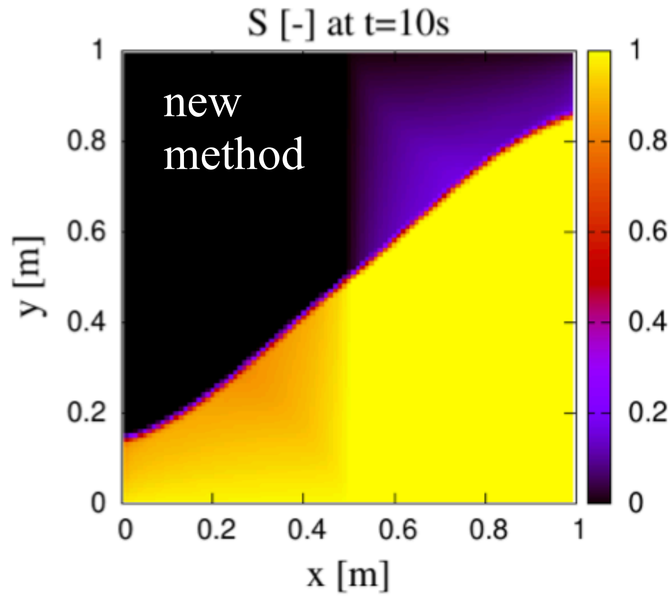
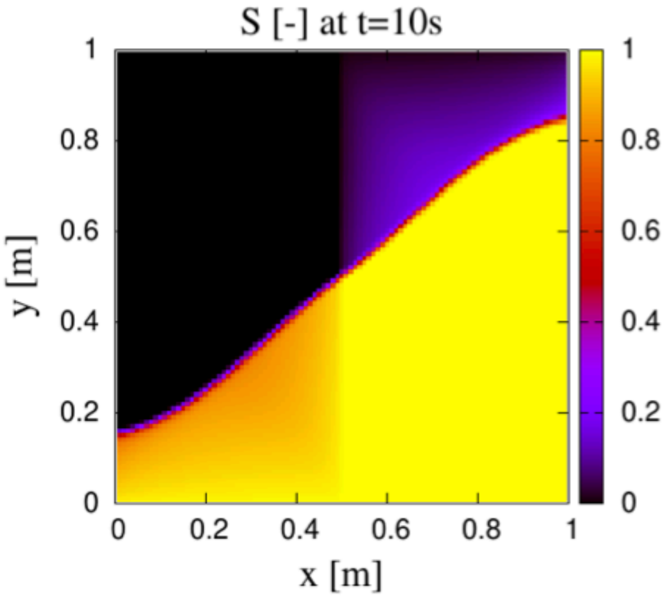
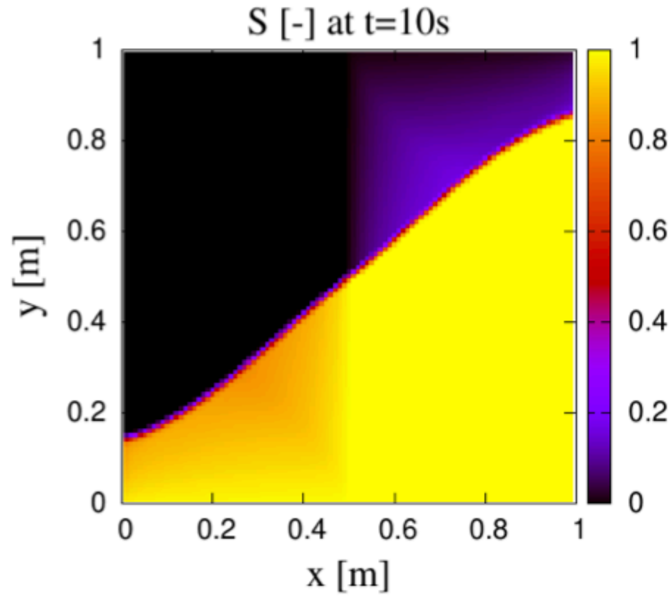




Efficiency





		case 1 (time steps)	case 2 (time steps)	case 3 (time steps)
fully implicit	1st order	100	1000	1000
new method with $\{c = 1\text{m/s}, \rho_{mean} = 4\text{kg/m}^3\}$	1st order		2075	2124
	2nd order	1001	5213	5328
new method with $\{c = 5\text{m/s}, \rho_{mean} = 1\text{kg/m}^3\}$	1st order		10073	10118
	2nd order	5001	40351	40517

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More details in

Patrick Jenny, Rasim Hasanzade, Hamdi Tchelepi.
Tightly Coupled Hyperbolic Treatment of Buoyant Two-Phase Flow and Transport in Porous Media.
Submitted to Journal of Computational Physics.

thank you for your attention