Positive DDFV scheme for degenerate parabolic equations arising from infiltration problem

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Abstract:

Nonlinear degenerate parabolic equations are the main core to study some complex problems arising from petroleum engineering and hydrology. In our study, the problem describes the infiltration of a single fluid through a porous medium with no gravity effects.

We carry out the convergence analysis of a positive DDFV (Dual Discrete Finite Volume) method for approximating solutions of degenerate parabolic equations. The basic idea rests upon different approximations of the fluxes on the same interface of the control volume. Precisely, the approximated flux is split into two terms corresponding to the primal and dual normal components. Then the first term is discretized using a centred scheme whereas the second one is approximated in a non-evident way by an upstream scheme. The novelty of our approach is twofold: on the one hand, we prove that the resulting scheme preserves the positivity and on the other hand we establish energy estimates. Some numerical tests are presented and they show that the scheme in question turns out to be robust and efficient with an accuracy of second order on quadrilateral grids.