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Coupled flow in porous media with thin inclusions: preconditioning based on rational approximations of the fractional interface operators

Thursday, 2 June 2022 14:00 (15 minutes)

Flow in porous media with inclusions is a determining processes within natural and artificial materials [6]. One important feature of thin inclusions is that they represent fractures. In fractured formations, the equations are strongly coupled, so the accurate and robust numerical methods are of great importance.

Flow in fractured porous media is a typical example of non-local process, thus naturally connected with fractional diffusion. Here we use some recent advances in numerical methods for fractional diffusion problems [2]. It may be surprising that the matrix-vector multiplication is a more difficult task than solving systems with $\mathbb{A}^{\alpha}$.

The presented results are in the spirit of multiphysics or multiscale problems which involve coupling at interfaces that are manifolds of lower dimensions, thus giving rise to interface conditions formulated in fractional Sobolev spaces [1]. Aimed at large-scale applications we focus on the iterative solution of the arising linear systems. Two examples are considered.

E1: Let $\Omega = \Omega_S \cup \Omega_D \subset \mathbb{R}^2, \Omega_S \cap \Omega_D = \emptyset$. The exchange trough the interface Γ between a Stokes flow in Ω_S and a Darcy flow in Ω_D is modeled. The preconditioner $C_{D-S} = Diag\left(-\Delta + D\mathcal{T}_t^T\mathcal{T}_t,\ K^{-1}(-\nabla\nabla\cdot+\mathcal{I}),\ \mathcal{I},\ K\mathcal{I},\ (-\Delta+\mathcal{I})^{1/2}\right)$, $\mathcal{T}_t$ is a tangential trace operator, provides an optimal condition number estimate.

E2: Let $\Omega \subset \mathbb{R}^3$ be a bounded domain, while Γ represents a 1D manifold inside Ω. The trace coupled problem is studied. We consider the preconditioner $C_{3-1} = Diag\left(-\Delta + I, -\Delta + I, -\Delta^{-0.14}\right)$. The choice of fractional power is analyzed in \cite{KLMZ-21}, observing that $\alpha \in [-0.145, -0.1]$ yields bounded condition numbers. The additional stability analysis has led to the preferred value $\alpha = -0.14$.

The question is how to implement the nonlocal blocks of C_{D-S} and C_{3-1}. Until recently, this was an open problem in the case of general geometry of Γ. In 2019, multigrid methods for discrete fractional Sobolev spaces were proposed in [1]. The constructed smoother involves $(\nu+1)\times(\nu+1)$ blocks $\mathbb{A}_{\ell,\nu}^{-\alpha}$, $\alpha \in (-1,1)$, where ν is the graph-degree of the mesh node ℓ.

As an alternative, we propose to use the best uniform rational approximation (BURA) of $\mathbb{A}^{-\alpha}$, $\alpha \in (0,1)$. As correctly noted in \cite[1], a disadvantage of the method introduced in 2018 in [4] is that its accuracy depends on the condition number of $\mathbb{A}$. This drawback was completely overcome in [3]. Thus, the currently used BURA method is defined as $\mathbb{A}^{-\alpha} \approx \lambda_{1,h}^{-\alpha} r_{\alpha,k}(\lambda_{1,h}\mathbb{A}^{-1})$, $r_{\alpha,k}$ is the best uniform rational approximation of degree k of z^{α} on $[0,1]$. In the case of negative degree $\alpha \in (-1,0)$, the approximation $\mathbb{A}^{-\alpha} \approx \mathbb{A}\lambda_{1,h}^{-\beta} r_{\beta,k}(\lambda_{1,h}\mathbb{A}^{-1})$, $\beta = 1 + \alpha \in (0,1)$, is applied.

The obtained results show that presumably small degrees of BURA (say, $k = 2,3,4$) are enough to get a relative condition number which is competitive to the multigrid preconditioner. Among the advantages of the BURA based preconditioners are that: (i) the condition number estimates are independent of the geometry of the interface Γ; (ii) the new approach is applicable to coupled PDEs defined on domains and interfaces with different dimensionality; (iii) no regularity assumptions are required.

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Time Block Preference

Time Block B (14:00-17:00 CET)

Participation

In person

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