

# The numerical solution of the micro-scale phase-field equation and its role in a two-scale two-phase flow model.

Manuela Bastidas<sup>1,2</sup>, Sohely Sharmin<sup>3</sup>, Carina Bringedal<sup>4</sup> and Sorin Pop<sup>3</sup>

1. Inria, 2 rue Simone Iff, 75589 Paris, France.
2. CERMICS, Ecole des Ponts, 77455 Marne-la-Vallée, France.
3. Faculty of Sciences, Hasselt University, Diepenbeek, Belgium.
4. Stuttgart Center for Simulation Science, Institute for Modelling Hydraulic and Environmental Systems, University of Stuttgart, Stuttgart, Germany.

Porous media are complex domains involving hierarchically organized structures, where various processes take place at different scales. An example in this sense is the fluid flow through the pores of a porous medium and, in particular, the two-phase flow. Prominent real-life applications in this sense are geological CO<sub>2</sub> sequestration or oil recovery.

In [1], a two-scale model for two-phase porous-media flow is proposed. This model includes variable surface-tension effects, depending on the concentration of a surfactant dissolved in one of the fluid phases. A phase field is employed to approximate the freely moving interfaces separating the different fluid phases. By formal homogenization techniques, a fully coupled two-scale model is derived, where the macro-scale parameters are determined by solving micro-scale cell problems, which, on their turn, depend on the macro-scale variables.

Our main challenge is to design a robust numerical scheme for the model, accounting for the coupling between the two scales. We study the macro-scale impact of the micro-scale evolution of the phase-field. Here we center the attention on the phase-field cell problems, which are Cahn-Hilliard type equations. To solve it, we propose a mixed formulation and a robustly convergent linear iterative scheme that is combined with a mesh refinement strategy, improving the efficiency of the algorithm.

## References

[1] S. Sharmin, M. Bastidas, C. Bringedal, I.S. Pop, Upscaling a Navier-Stokes-Cahn-Hilliard model for two-phase porous-media flow with solute-dependent surface tension effects. (SUBMITTED), 2021.

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