How simplifying capillary effects can affect the traveling wave solution profiles of the foam flow in porous media

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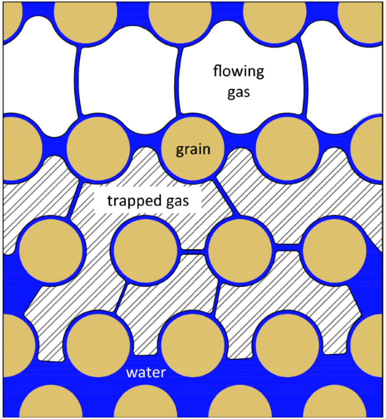
Foam in porous media

What is foam in porous media

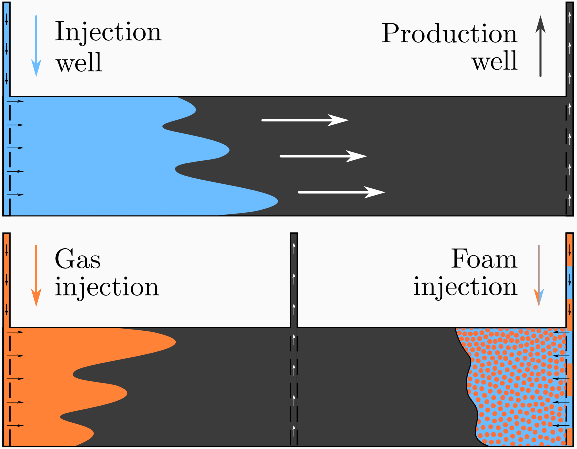
- Lamellae (liquid films) separate gas bubbles.
- Foam reduces the gas mobility.
- Foam texture is modeled as a tracer in the gas phase.
- Foam does not affect water phase relative mobility.



Almajid M.M. and Kovscek A. R., TiPM, 2020 131(1):289-313.

Applications of foam in porous media

- Soil/aquifer remediation.
- EOR.
- CO_2 sequestration.
- Others.



Two-dimensional simulation in heterogeneous porous media with gravity.

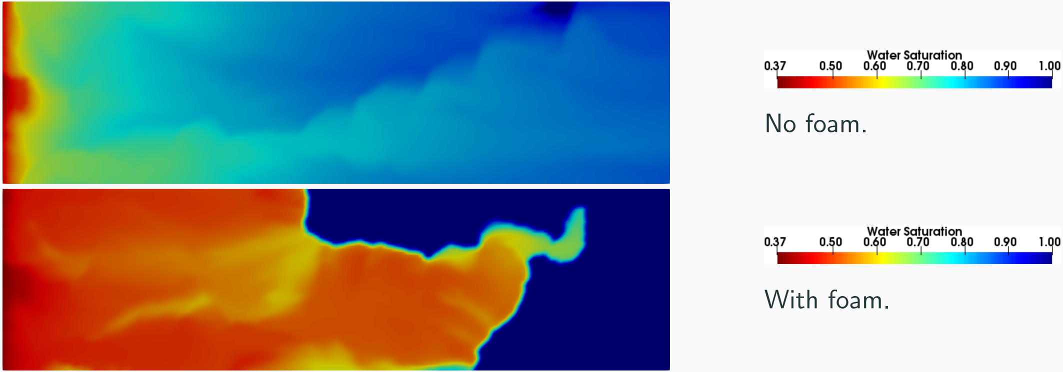


Figure 1: Water saturation in SPE10(36) por. medium at time $t = 10000$s.

F. F. de Paula, T. Quinelato, I. Igreja, G. C., LNCS, 2020.

How to model foam displacement in porous media

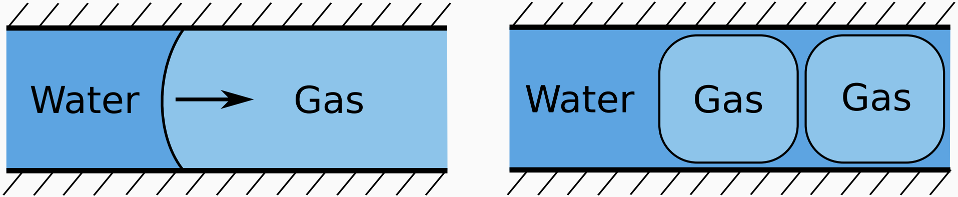


Figure 2: Schematic representation of the gas-water flow in a pore throat.

Typical 2-phase flow foam model

$$\begin{cases} \dfrac{\partial}{\partial t}(\rho_w \phi S_\mathrm{w}) + \nabla \cdot (\rho_w \mathbf{u}_\mathrm{w}) = 0\,, \\ \dfrac{\partial}{\partial t}(\rho_g \phi S_\mathrm{g} n_\mathrm{D}) + \nabla \cdot (\rho_g \mathbf{u}_g n_\mathrm{D}) = \phi S_\mathrm{g} \Phi\,, \\ \mathbf{u}_w = \mathbf{u} f_\mathrm{w} + \mathbf{k} \lambda_\mathrm{g} f_\mathrm{w} \nabla P_\mathrm{c}\,. \end{cases}$$

Gas phase $\neq$ Foamed gas

$$\begin{cases} \nabla \cdot (\rho_w \mathbf{u}_\mathrm{w}) = \nabla \cdot (\rho_w \mathbf{u} f_\mathrm{w} + \rho_w \mathbf{k} \lambda_\mathrm{g} f_\mathrm{w} \nabla P_\mathrm{c})\,, \\ \nabla \cdot (\rho_g \mathbf{u}_g n_\mathrm{D}) = \nabla \cdot (\rho_g \mathbf{u}_g f_\mathrm{g} n_\mathrm{D} - \rho_g \mathbf{k} \lambda_\mathrm{g} f_\mathrm{w} n_\mathrm{D} \nabla P_\mathrm{c})\,. \end{cases}$$

Mathematical model

First-order kinetic model

Model is composed by conservations laws:

$$\begin{cases} \dfrac{\partial}{\partial t}\left(\rho_w \phi S_\mathrm{w}\right) + \nabla \cdot \left(\rho_w \mathbf{u}_\mathrm{w}\right) = 0\,, \\ \dfrac{\partial}{\partial t}\left(\rho_g \phi S_\mathrm{g} n_\mathrm{D}\right) + \nabla \cdot \left(\rho_g \mathbf{u}_g n_\mathrm{D}\right) = \phi S_\mathrm{g} \Phi\,, \\ \mathbf{u}_w = \mathbf{u} f_\mathrm{w} + \mathbf{k} \lambda_\mathrm{g} f_\mathrm{w} \nabla P_\mathrm{c}\,. \end{cases}$$

Assumptions:

- Rigid & homogeneous PM (ϕ, $\mathbf{k}$ constants);
- Incompressible fluids (ρ_w, ρ_g - constants);
- Constant injection rate ($\mathbf{u}$ is constant);
- One-dimensional case;
- Surfactant amount above CMC.

- Foamed gas relative permeability is reduced by a linear factor (Ashoori et al., 2011): $\mathrm{MRF}(n_\mathrm{D}) = 18500 n_\mathrm{D} + 1$; $k_\mathrm{rg}(S_\mathrm{w}, n_\mathrm{D}) = \frac{k^0_\mathrm{rg}(S_\mathrm{w})}{\mathrm{MRF}(n_\mathrm{D})}$.
- Linear generation and coalescence rate: $\Phi = (r_\mathrm{g} - r_\mathrm{c}) = K_c(n^\mathrm{LE}_\mathrm{D}(S_\mathrm{w}) - n_\mathrm{D})\,.$
- Local equilibrium foam texture:

$$n^\mathrm{LE}_\mathrm{D}(S_\mathrm{w}) = \begin{cases} \tanh(A(S_\mathrm{w} - S^*_\mathrm{w}))\,, & S_\mathrm{w} > S^*_\mathrm{w} \\ 0\,, & S_\mathrm{w} \leq S^*_\mathrm{w} \end{cases},$$

where S^*_w is the water saturation at the limiting capilary pressure $P^*_\mathrm{c} = P_\mathrm{c}(S^*_\mathrm{w})$.

Model's simplifications

Original model from Ashoori et al. (2011):

$$\begin{cases} \dfrac{\partial S_{\mathrm{w}}}{\partial t} + \dfrac{\partial f_{\mathrm{w}}}{\partial x} = -\dfrac{\partial}{\partial x}\left(\alpha\,\lambda_{\mathrm{g}}\,f_{\mathrm{w}}\dfrac{\mathrm{d}P_{\mathrm{c}}}{\mathrm{d}S_{\mathrm{w}}}\dfrac{\partial S_{\mathrm{w}}}{\partial x}\right), \\ \dfrac{\partial (S_{\mathrm{g}}n_{\mathrm{D}})}{\partial t} + \dfrac{\partial (f_{\mathrm{g}}n_{\mathrm{D}})}{\partial x} = \dfrac{\partial}{\partial x}\left(\alpha\,n_{\mathrm{D}}\,\lambda_{\mathrm{g}}\,f_{\mathrm{w}}\dfrac{\mathrm{d}P_{\mathrm{c}}}{\mathrm{d}S_{\mathrm{w}}}\dfrac{\partial S_{\mathrm{w}}}{\partial x}\right) + \mathcal{K}_c\,S_{\mathrm{g}}(n_{\mathrm{D}}^{\mathrm{LE}} - n_{\mathrm{D}}). \end{cases}$$

Define:

$$\epsilon_{\mathrm{w}} = -\alpha\,\lambda_{\mathrm{g}}\,f_{\mathrm{w}}\,\mathrm{d}P_{\mathrm{c}}/\mathrm{d}S_{\mathrm{w}}, \qquad \epsilon_{\mathrm{g}} = -\alpha\,n_{\mathrm{D}}\lambda_{\mathrm{g}}\,f_{\mathrm{w}}\,\mathrm{d}P_{\mathrm{c}}/\mathrm{d}S_{\mathrm{w}}.$$

We consider ϵ_{w} and ϵ_{g} constants satisfying:

- Simplification 1:

$$\epsilon_{\mathrm{g}} = 0.$$

- Simplification 2:

$$\epsilon_{\mathrm{g}} = \epsilon_{\mathrm{w}}.$$

- Simplification 3:

$$\epsilon_{\mathrm{g}} = \epsilon_{\mathrm{w}}\,n_{\mathrm{D}}.$$

Ashoori, E., et al., *Colloids and Surfaces A: Physicochemical and Engineering Aspects*, 377, 228–242, 2011

Estimation of constant ϵ

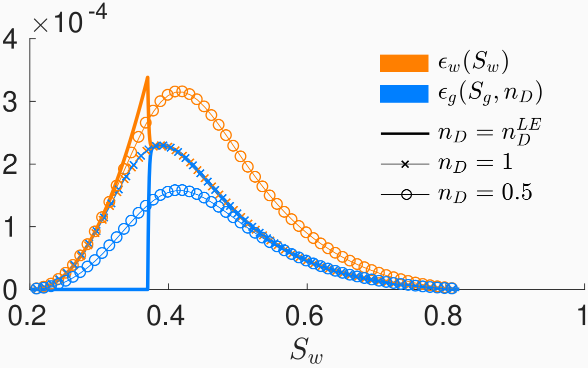


Figure 3: Values of $\epsilon_{\mathrm{w}}(S_{\mathrm{w}})$ and $\epsilon_{\mathrm{g}}(S_{\mathrm{g}}, n_{\mathrm{D}})$ for three choices of n_{D}.

These functions guide us on choosing a constant value for ϵ_{w} and ϵ_{g}:

- ϵ_{w} is up to $3.387 \cdot 10^{-4}$;
- ϵ_{g} is up to $2.299 \cdot 10^{-4}$;
- When adopted we assume this constants are equal to $\epsilon = 10^{-4}$.

What is a traveling wave solution?

Consider the PDE:

$$\frac{\partial u}{\partial t} - \frac{\partial u}{\partial x} + \frac{\partial F(u)}{\partial x} = \epsilon \Delta_{xx} u + G(u), \qquad u, F(u) \in \mathbb{R}^n.$$

Two (three?) steps:

(1) Change of variables $(x, t) \to (\xi, t)$, where $\xi = x - vt$ with v - constant traveling wave velocity, ξ - traveling variable (Euler–Lagrange coordinates)

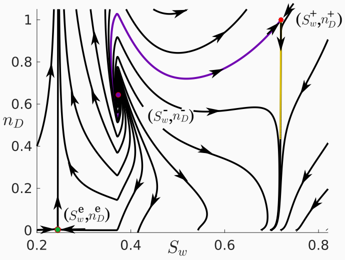
(2) Search for the stationary solution of the system

$$\frac{\partial u}{\partial t} - V \frac{\partial u}{\partial \xi} + \frac{\partial F(u)}{\partial \xi} = \epsilon \Delta_{\xi\xi} u + G(u), \qquad u, F(u) \in \mathbb{R}^n.$$

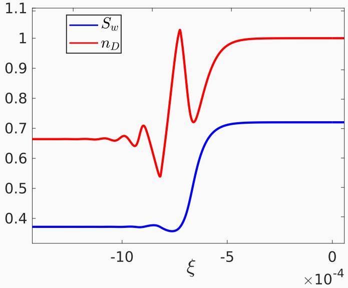
(3) If we are dealing with the Riemann problem, the solution must satisfy the corresponding asymptotic boundary conditions.

A. I. Volpert et al., *AMS*, V. 140, 2000.

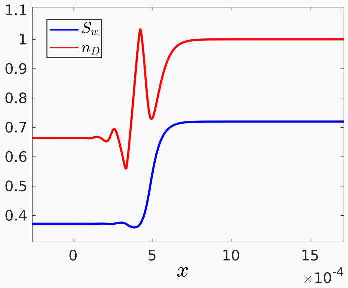
Results



(a) Traveling wave phase space.



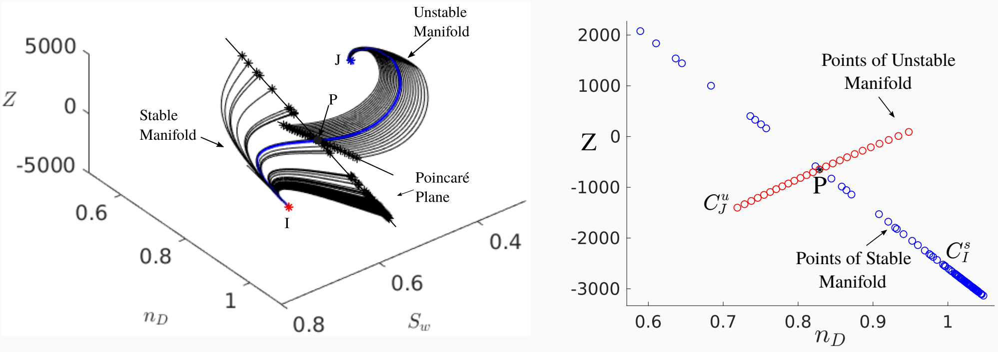
(b) Analytical solution.



(c) Numerical solution.

Figure 4: Solution of the Riemann problem for simplification 1 with $(S_\mathrm{w}^-, n_\mathrm{D}^-) = (0.372, 0.664)$, $(S_\mathrm{w}^+, n_\mathrm{D}^+) = (0.72, 1.0)$, $\epsilon = 10^{-4}$, and $K_\mathrm{c} = 1.0$.

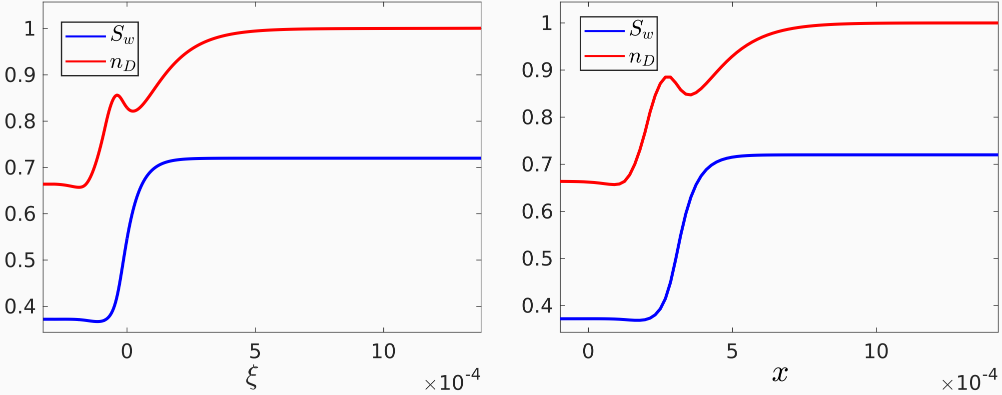
Simplification 2



(a) Traveling wave phase space. **(b)** Poincaré plane.

Figure 5: Solution of the Riemann problem for simplification 2 with $(S_{\mathrm{w}}^-, n_{\mathrm{D}}^-) = (0.372, 0.664)$, $(S_{\mathrm{w}}^+, n_{\mathrm{D}}^+) = (0.72, 1.0)$, $\epsilon = 10^{-4}$, and $K_c = 1.0$.

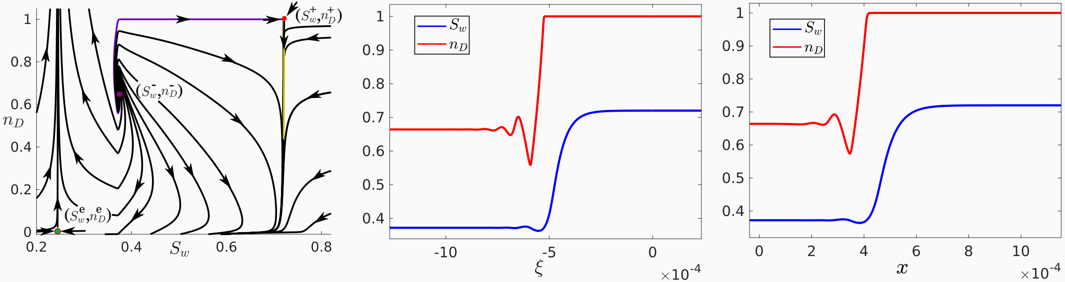
Simplification 2



(c) Analytical solution.

(d) Numerical solution.

Figure 5: Solution of the Riemann problem for simplification 2 with $(S_{\mathrm{w}}^-, n_{\mathrm{D}}^-) = (0.372, 0.664)$, $(S_{\mathrm{w}}^+, n_{\mathrm{D}}^+) = (0.72, 1.0)$, $\epsilon = 10^{-4}$, and $K_{\mathrm{c}} = 1.0$.



(a) Traveling wave phase space. **(b)** Analytical solution. **(c)** Numerical solution.

Figure 6: Solution of the Riemann problem for simplification 3 with $(S_{\mathrm{w}}^-, n_{\mathrm{D}}^-) = (0.372, 0.664)$, $(S_{\mathrm{w}}^+, n_{\mathrm{D}}^+) = (0.72, 1.0)$, $\epsilon = 10^{-4}$, and $K_{\mathrm{c}} = 1.0$.

Traveling wave solutions for all simplified capillary pressure models

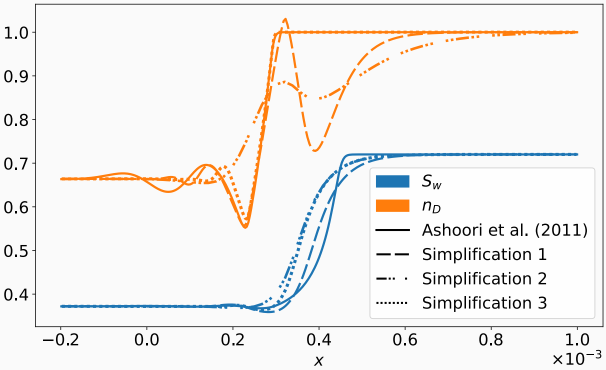


Figure 7: Numerical solution for different models.

- Numerical and analytical solutions match.
- All profiles present oscillations before the wavefront.
- All models present the same wave velocity.
- Simplification 3 is the closest to the original kinetic model.

L. F. Lozano, R. V. Q. Zavala and G. C., *Computational Geosciences*, 25, 515–527, 2021

Discussions & Conclusions

- There should be a difference in modeling two-phase gas-water flow in the presence / absence of foam.
- We analyzed different mathematical simplifications pointing out that the traveling wave profile presents different oscillations.
- We stress that physically acceptable simplification procedures can result in qualitatively inaccurate solutions describing foam texture.
- We wonder whether this phenomenon can be observed in laboratory experiments.

L. F. Lozano, J. B. Cedro, R. V. Q. Zavala and G. C., *International Journal of Non-Linear Mechanics*, 139, p. 103867, 2022

Thank you for attention!

Questions?

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