Thermodynamics of two-phase flow in porous media using the Lattice Boltzmann model

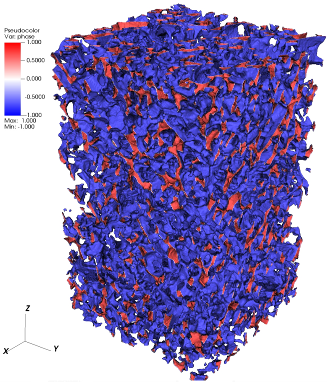
InterPore 2022

Håkon Pedersen, Santanu Sinha, Subhadeep Roy, Alex Hansen and more

May 21, 2022

PoreLab
NTNU-UiO Porous Media Laboratory

What is the problem?

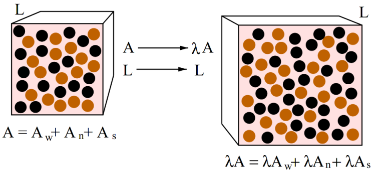


Steady state CLBM simulation, strong wetting affinity at walls.

- **Thermodynamic limit of incompressible, immiscible two-phase flow in porous media: a upscaling problem**
- Macroscopic scale: a single effective fluid with velocity v_p, at a single effective pressure and viscosity
- Continuum, Euler homogeneity, notion of equilibrium $\overset{?}{\Leftrightarrow}$ Thermodynamic theory
- Good correspondence with experimental data[1]
- Can this be obtained in a mesoscopic model like Lattice-Boltzmann?

[1] Roy, Pedersen, et al. 2022.

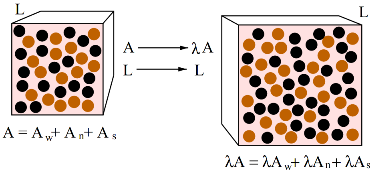
Fundamentals of thermodynamic theory



Deformation of REV by factor λ (*Hansen et al.*)

- Assumption: volumetric flow rate Q extensive in wetting/nonwetting areas $A_w, A_n \leftrightarrow$ *Euler homogeneous function of order one*

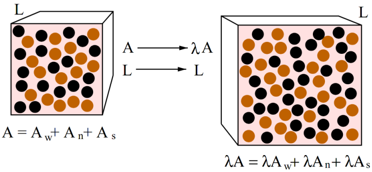
Fundamentals of thermodynamic theory



Deformation of REV by factor λ (*Hansen et al.*)

- Assumption: volumetric flow rate Q extensive in wetting/nonwetting areas A_w, A_n $\leftrightarrow$ *Euler homogeneous function of order one*
- $Q(\lambda A_w, \lambda A_n) = \lambda Q(A_w, A_n)$

Fundamentals of thermodynamic theory



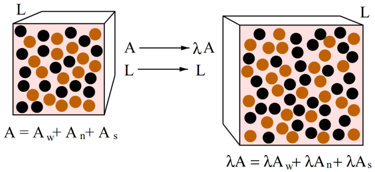
Deformation of REV by factor λ (*Hansen et al.*)

- Assumption: volumetric flow rate Q extensive in wetting/nonwetting areas $A_w, A_n \leftrightarrow$ *Euler homogeneous function of order one*
- $Q(\lambda A_w, \lambda A_n) = \lambda Q(A_w, A_n)$
- Euler theorem for hom. functions of order one:

$$Q(A_w, A_n) =$$

$$A_w \left(\frac{\partial Q}{\partial A_w} \right)_{A_n} + A_n \left(\frac{\partial Q}{\partial A_n} \right)_{A_w}$$

Fundamentals of thermodynamic theory



Deformation of REV by factor λ (*Hansen et al.*)

- Assumption: volumetric flow rate Q extensive in wetting/nonwetting areas A_w, A_n $\leftrightarrow$ *Euler homogeneous function of order one*
- $Q(\lambda A_w, \lambda A_n) = \lambda Q(A_w, A_n)$
- Euler theorem for hom. functions of order one:

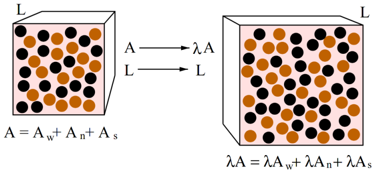
$$Q(A_w, A_n) =$$

$$A_w \left(\frac{\partial Q}{\partial A_w} \right)_{A_n} + A_n \left(\frac{\partial Q}{\partial A_n} \right)_{A_w}$$

Velocity transformation

- Idea: perform a transformation $(v_w, v_n) \leftrightarrow (v_p, v_m)$

Fundamentals of thermodynamic theory



Deformation of REV by factor λ (*Hansen et al.*)

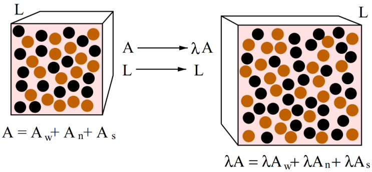
- Assumption: volumetric flow rate Q extensive in wetting/nonwetting areas A_w, A_n $\leftrightarrow$ *Euler homogeneous function of order one*
- $Q(\lambda A_w, \lambda A_n) = \lambda Q(A_w, A_n)$
- Euler theorem for hom. functions of order one:

$$Q(A_w, A_n) = A_w \left(\frac{\partial Q}{\partial A_w} \right)_{A_n} + A_n \left(\frac{\partial Q}{\partial A_n} \right)_{A_w}$$

Velocity transformation

- Idea: perform a transformation $(v_w, v_n) \leftrightarrow (v_p, v_m)$
- Additional field, the *co-moving velocity* v_m

Fundamentals of thermodynamic theory



Deformation of REV by factor λ (*Hansen et al.*)

- Assumption: volumetric flow rate Q extensive in wetting/nonwetting areas A_w, A_n $\leftrightarrow$ *Euler homogeneous function of order one*
- $Q(\lambda A_w, \lambda A_n) = \lambda Q(A_w, A_n)$
- Euler theorem for hom. functions of order one:

$$Q(A_w, A_n) = A_w \left(\frac{\partial Q}{\partial A_w}\right)_{A_n} + A_n \left(\frac{\partial Q}{\partial A_n}\right)_{A_w}$$

Velocity transformation

- Idea: perform a transformation $(v_w, v_n) \leftrightarrow (v_p, v_m)$
- Additional field, the *co-moving velocity* v_m
- Most general relation:

$$\left(\frac{\partial Q}{\partial A_w}\right)_{A_n} = \hat{v}_w = v_w + S_n v_m$$

$$\left(\frac{\partial Q}{\partial A_n}\right)_{A_w} = \hat{v}_n = v_n - S_w v_m$$

Why introduce v_m?

Simple, general, unifying

Why introduce v_m?

Simple, general, unifying

- Linear law for v_m:

$$v_m = a + b\frac{\mathrm{d}v_p}{\mathrm{d}S_w}$$

Why introduce v_m?

Simple, general, unifying

- Linear law for v_m:

$$v_m = a + b\frac{\mathrm{d}v_p}{\mathrm{d}S_w}$$

  - a, b dependent on capillary number, external driving

Why introduce v_m?

Simple, general, unifying

- Linear law for v_m:

$$v_m = a + b\frac{\mathrm{d}v_p}{\mathrm{d}S_w}$$

  - a, b dependent on capillary number, external driving

- Can show that

$$v_m = \frac{\mathrm{d}v_p}{\mathrm{d}S_w} - v_w + v_n$$

Why introduce v_m?

Simple, general, unifying

- Linear law for v_m:

$$v_m = a + b\frac{\mathrm{d}v_p}{\mathrm{d}S_w}$$

  - a, b dependent on capillary number, external driving
- Can show that

$$v_m = \frac{\mathrm{d}v_p}{\mathrm{d}S_w} - v_w + v_n$$

- Constitutive equations:

$$\begin{aligned} v_p &= v_p(S_w, \nabla S_w, \nabla P) \\ v_m &= v_m(S_w, \nabla S_w, \nabla P) \end{aligned}$$

Statistical mechanics with areas

- Connection between pore- and thermodynamic theory: measure velocity distribution in terms of *transversal pore areas*[2]
  - **Idea**: consider small areas in plane perpendicular to flow, create distributions which integrate to total pore areas A_p, A_w, A_n

[2] Roy, Sinha, and Hansen 2020.

Statistical mechanics with areas

- Connection between pore- and thermodynamic theory: measure velocity distribution in terms of *transversal pore areas*[2]
  - **Idea**: consider small areas in plane perpendicular to flow, create distributions which integrate to total pore areas A_p, A_w, A_n
- $a_p(S_w, v):\ v \in [v_i, v_i + dv]$ means that $a_p dv$ is the pore area covered by fluid with velocity in the given range
  - Similar for wetting-, nonwetting- and co-moving velocity $\to a_w, a_n, (a_m)$

[2] Roy, Sinha, and Hansen 2020.

Mesoscopic $\rightarrow$ Macroscopic

Mesoscopic $\rightarrow$ Macroscopic

- A_i: integrate corresponding a_i over all v ($i = p, m, w, n$)

Mesoscopic $\rightarrow$ Macroscopic

- A_i: integrate corresponding a_i over all v ($i = p, m, w, n$)
- v_i: —— 〃 ——, weighted by v

Mesoscopic $\to$ Macroscopic

- A_i: integrate corresponding a_i over all v ($i = p, m, w, n$)
- v_i: ——"——, weighted by v
- Can show: $A_m = \int_{-\infty}^{\infty} \mathrm{d}v \, a_m = 0 \quad \Longrightarrow \; Q_m = A_m v_m = 0$

Mesoscopic $\rightarrow$ Macroscopic

- A_i: integrate corresponding a_i over all v ($i = p, m, w, n$)
- v_i: ——〃——, weighted by v
- Can show: $A_m = \int_{-\infty}^{\infty} \mathrm{d}v \, a_m = 0 \quad \Longrightarrow \; Q_m = A_m v_m = 0$
  - ▶ v_m does not transport anything
  - ▶ Results: $A_m \sim$ one voxel area

Mesoscopic $\to$ Macroscopic

- A_i: integrate corresponding a_i over all v ($i = p, m, w, n$)
- v_i: ——″——, weighted by v
- Can show: $A_m = \int_{-\infty}^{\infty} \mathrm{d}v \, a_m = 0 \quad \Longrightarrow \; Q_m = A_m v_m = 0$
  - ▶ v_m does <u>not</u> transport anything
  - ▶ Results: $A_m \sim$ one voxel area
- Special case: relative permeability. We have

$$k_{rw} \sim \int a_w v \, dv$$

$$k_{rn} \sim \int a_n v \, dv$$

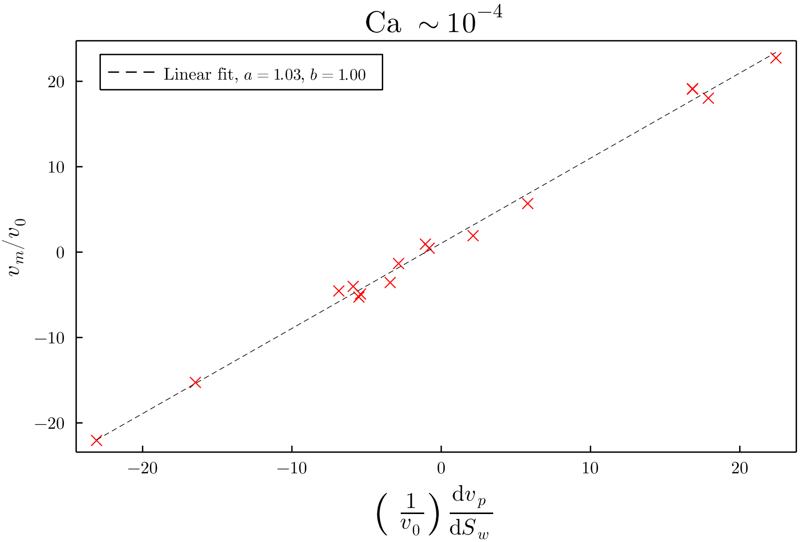
Mesoscopic $\rightarrow$ Macroscopic

- A_i: integrate corresponding a_i over all v ($i = p, m, w, n$)
- v_i: —— 〃 ——, weighted by v
- Can show: $A_m = \int_{-\infty}^{\infty} \mathrm{d}v \, a_m = 0 \quad \Longrightarrow \; Q_m = A_m v_m = 0$
  - ▶ v_m does <u>not</u> transport anything
  - ▶ Results: $A_m \sim$ one voxel area
- Special case: relative permeability. We have

$$k_{rw} \; \sim \; \int a_w v \, dv$$

$$k_{rn} \; \sim \; \int a_n v \, dv$$

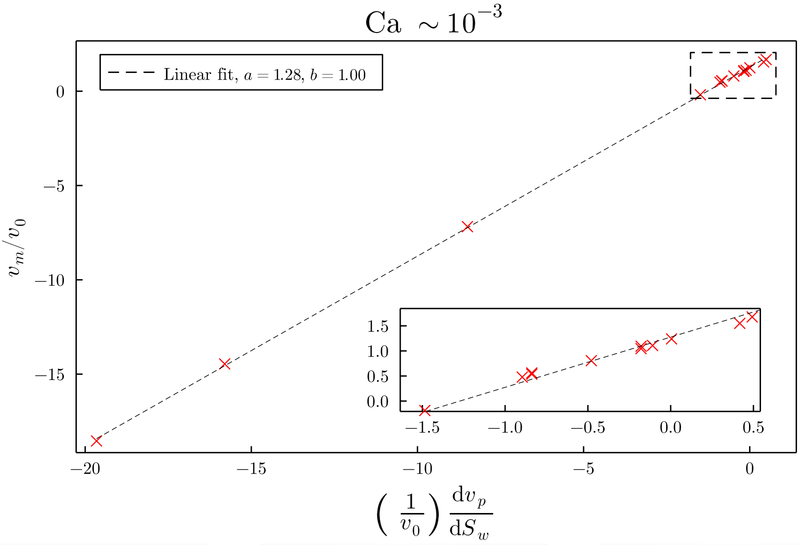
Some results for v_m



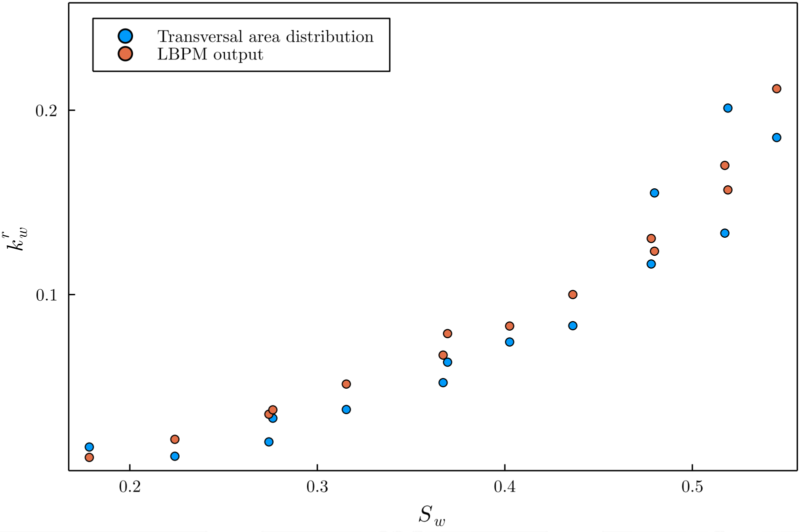
All figures: Lattice-Boltzmann data obtained using open source simulator (LBPM)[3]

[3] McClure et al. 2021.

Some results for v_m

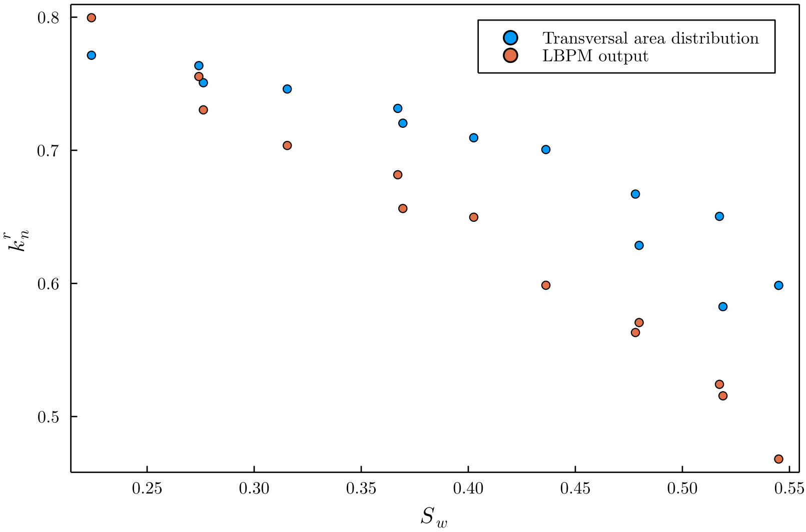


Special case: relative permeability curves



Relative perm. of wetting fluid, scaled with average pore velocity

Special case: relative permeability curves



Relative perm. of nonwetting fluid, scaled with average pore velocity

Summary and future work

- A possible thermodynamic description of immiscible two-phase flow in porous media

PoreLab
NTNU-UiO Porous Media Laboratory

Summary and future work

- A possible thermodynamic description of immiscible two-phase flow in porous media
- $(v_w, v_n) \leftrightarrows (v_p, v_m)$

Summary and future work

- A possible thermodynamic description of immiscible two-phase flow in porous media
- $(v_w, v_n) \leftrightarrows (v_p, v_m)$
- Simple law for v_m

Summary and future work

- A possible thermodynamic description of immiscible two-phase flow in porous media
- $(v_w, v_n) \leftrightarrows (v_p, v_m)$
- Simple law for v_m
- In the future: statistical mechanics, (differential) geometrical description of flow

Thank you for the attention