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CENTER OF
APPLIED SPACE TECHNOLOGY
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ZARM

A MACROSCOPIC TWO-LENGTH-SCALE MODEL FOR NATURAL CONVECTION IN POROUS MEDIA DRIVEN BY A SPECIES-CONCENTRATION GRADIENT

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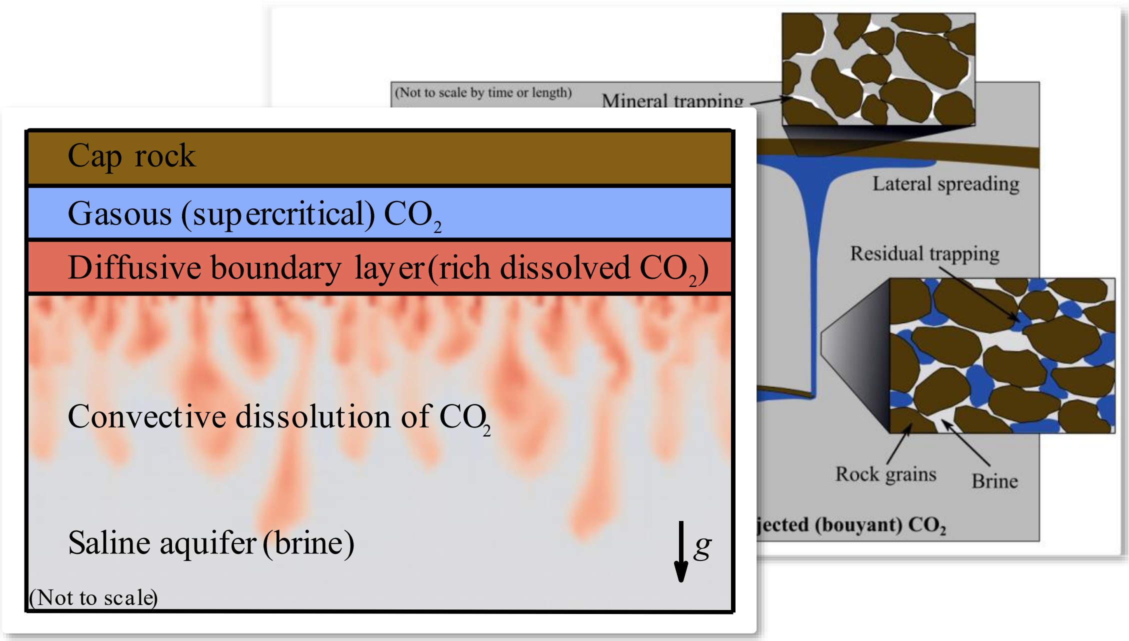
Motivation

- Reduction of anthropogenic CO_2 emissions by CO_2 sequestration in deep saline aquifers
  - Dissolution driven natural convection with high Rayleigh-Darcy numbers ($\mathrm{Ra} \sim 10^4$)



Objective

- Better understanding of the physics of “macroscopic turbulent” natural convection in porous media.
- Can “turbulent” natural convection in porous media be precisely modeled by the state of the art Darcy-Oberbeck-Boussinesq (DOB) equations?

Proposal of a new macroscopic model:
Two Length Scale Diffusion model (TLSD)

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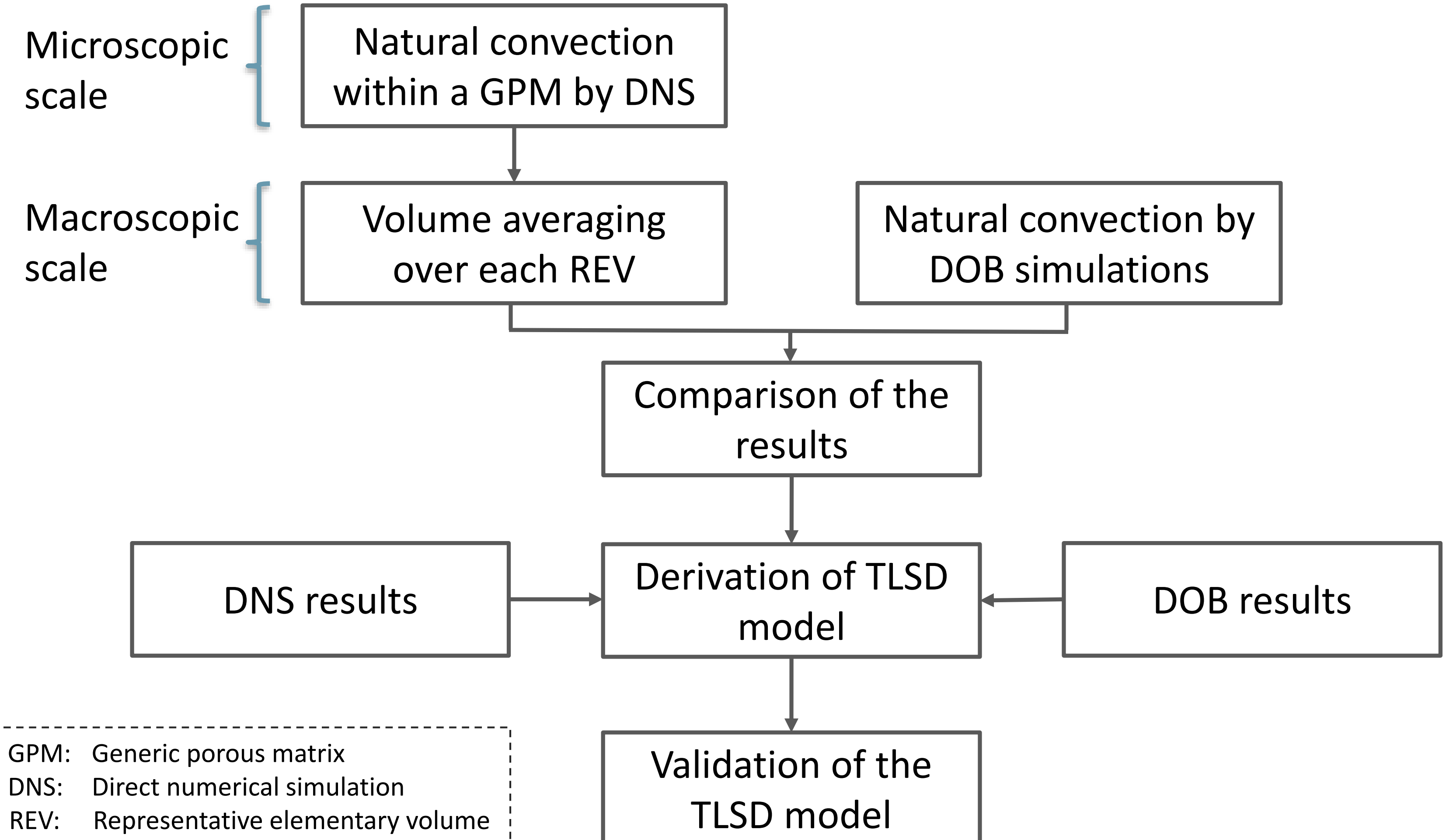
General procedure



GPM: Generic porous matrix
DNS: Direct numerical simulation
REV: Representative elementary volume
DOB: Darcy-Oberbeck-Boussinesq

Governing microscopic equations (DNS)

$$\frac{\partial \tilde{u}_i}{\partial \tilde{x}_i} = 0$$

$$\frac{\partial \tilde{u}_i}{\partial \tilde{t}} + \frac{\partial\left(\tilde{u}_i \tilde{u}_j\right)}{\partial \tilde{x}_j} = -\frac{\partial \tilde{p}}{\partial \tilde{x}_i} + \frac{\mathrm{Sc}}{\mathrm{Ra}_f \mathrm{Da}} \frac{\partial^2 \tilde{u}_i}{\partial \tilde{x}_j^2} - \boxed{\frac{\mathrm{Sc}}{\mathrm{Ra}_f \mathrm{Da}^2} z_i \tilde{c}}$$

$$\frac{\partial \tilde{c}}{\partial \tilde{t}} + \frac{\partial(\tilde{u}_i \tilde{c})}{\partial \tilde{x}_i} = \frac{1}{\mathrm{Ra}_f \mathrm{Da}} \frac{\partial^2 \tilde{c}}{\partial \tilde{x}_j^2}$$

$\sim$: Dimensionless variable

$\mathrm{Sc} = \nu / D_f$ $\quad$ $\mathrm{Ra}_f = H^3 \beta \Delta c g / \nu D_f$ $\quad$ $\tilde{t} = t\, u_m / H$

$\mathrm{Da} = K / H^2$ $\quad$ $\tilde{u} = u / u_m$ $\quad$ $\tilde{x} = x / H$

K: Permeability $\quad$ $u_m = \beta \Delta c g K / \nu$ $\quad$ $\tilde{c} = c - c_0 / \Delta c$

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Governing macroscopic equations (DOB)

$$\frac{\partial \hat{u}_i}{\partial \hat{x}_i} = 0$$

$$\frac{\partial \hat{p}}{\partial \hat{x}_i} + \hat{c} z_i + \boxed{\hat{u}_i} = 0$$

$$\frac{\partial \hat{c}}{\partial \hat{t}} + \frac{\partial (\hat{u}_i \hat{c})}{\partial \hat{x}_i} = \frac{1}{\boxed{\mathrm{Ra}}} \frac{\partial^2 \hat{c}}{\partial \hat{x}_j^2}$$

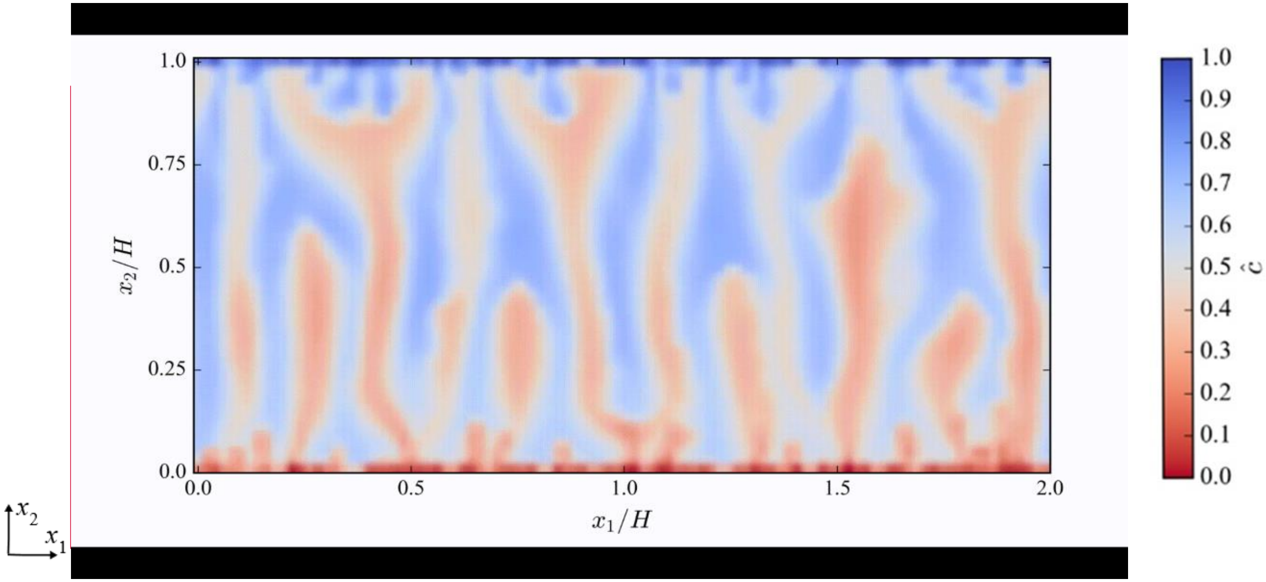
^: Dimensionless and volume averaged variable

$$\mathrm{Ra} = \frac{\mathrm{Ra}_f \mathrm{Da}}{\gamma_m} = \frac{H \beta \Delta c g K}{D_m \nu} \qquad \hat{p} = \frac{\mathrm{Ra}\mathrm{Da} \langle \tilde{p} \rangle^i}{\gamma_m \mathrm{Sc}} \qquad \hat{c} = \frac{\langle c \rangle^i - c_0}{c_1 - c_0}$$

$$\gamma_m = \frac{D_m}{D_f} \qquad \hat{u}_i = \phi \langle \tilde{u}_i \rangle^i$$

ϕ: Porosity

Case study



s/d -> $\phi[0.33 - 0.96]$, $K/d^2[1 \times 10^{-6} - 1.7]$ and γ_m

H/s -> $\mathrm{Da}[7 \times 10^{-8} - 1.7 \times 10^{-4}]$

Ra: [500-20,000]

Derivation of TLSD model

- The DNS revealed significant effects which can not be captured by the state of the art model the DOB:
  - Increase of the domain size H=> increase of the mega-plumes
  - Species boundary layer thickness /characteristic length scale ~ pore size $\sqrt{K}$
  - Increase of the porosity => change of the Sherwood(Ra) scaling
  - The Darcy term $\frac{\gamma_m \mathrm{Sc}}{\mathrm{RaDa}} \hat{u}_i$ can not account for all losses at the boundary layer

Derivation of TLSD model

- Macroscopic momentum diffusion:

$$\boxed{\breve{R}_i} + \boxed{\nu \frac{\partial^2 \breve{u}_i}{\partial \breve{x}_j^2}} = \boxed{\frac{\nu}{K} \breve{u}_i} + \boxed{\nu_m \frac{\partial^2 \breve{u}_i}{\partial \breve{x}_j^2}}$$

$\breve{}$: Volume averaged variable

  - TLSD hypothesis: macroscopic diffusion is determined by the pore size $\sqrt{K}$, and the domain size H => combined in Da:

$$\nu_m = \nu \frac{a_\nu^*}{Da}$$

  - a_ν^*: a constant assumed to be solely determined by the pore-scale geometry

Derivation of TLSD model

- Governing equations of the TLSD model

$$\frac{\partial \hat{u}_i}{\partial \hat{x}_i} = 0$$

$$\frac{\partial \hat{u}_i}{\partial \hat{t}} + \frac{\partial(\hat{u}_i \hat{u}_i/\phi)}{\partial \hat{x}_j} = -\frac{\partial\left(\phi\langle\tilde{p}\rangle^i\right)}{\partial \hat{x}_i} + \boxed{\frac{a_v^* \mathrm{Sc}}{\gamma_m \mathrm{RaDa}} \frac{\partial^2 \hat{u}_i}{\partial \hat{x}_j^2}} - \boxed{\frac{\phi \mathrm{Sc}}{\gamma_m \mathrm{RaDa}} z_i \hat{c}} - \boxed{\frac{\phi \mathrm{Sc}}{\gamma_m \mathrm{RaDa}} \hat{u}_i}$$

$$\frac{\partial(\phi \hat{c})}{\partial \hat{t}} + \frac{\partial(\hat{u}_i \hat{c})}{\partial \hat{x}_i} = \frac{1}{\mathrm{Ra}} \frac{\partial^2 \hat{c}}{\partial \hat{x}_j^2}$$

^: Dimensionless and volume averaged variable

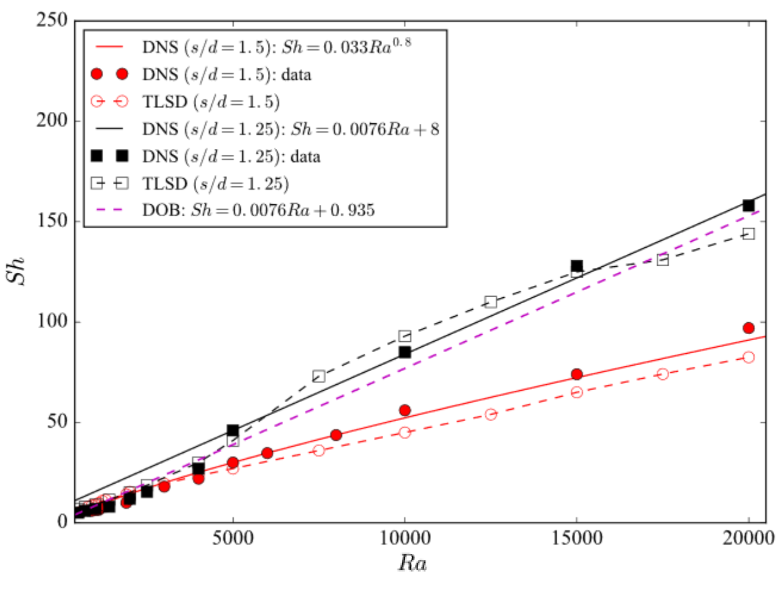
~: Dimensionless variable

$\langle\ \rangle^i$: Intrinsic volume averaged variable

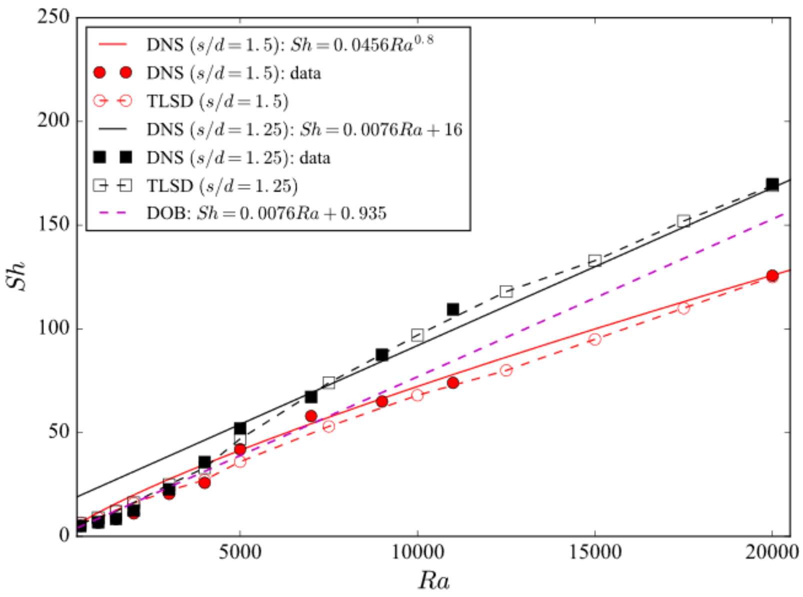
Results

- Sh correlations in dependence on Ra:

Sc=1

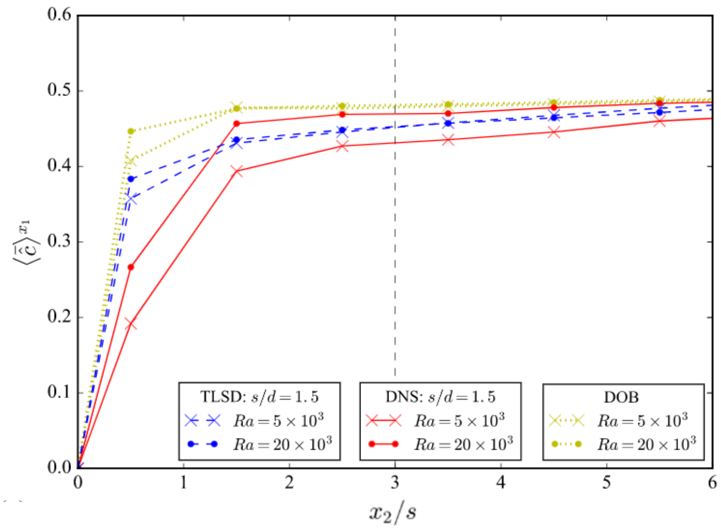


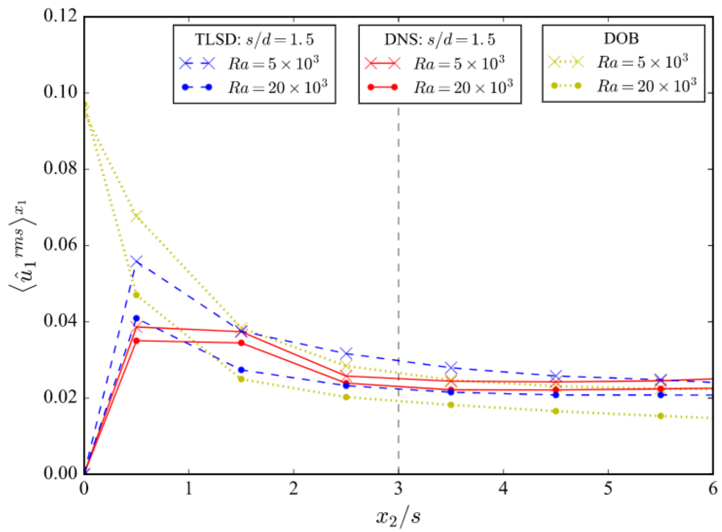
Sc=250



Results

- Vertical profiles of $\langle\bar{\hat{c}}\rangle^{x1}$ and root mean square velocity fluctuations $\langle\hat{u}_i{}^{rms}\rangle^{x1}$ (Sc=250; Ra=20,000):





Conclusion

- A new macroscopic model for natural convection in porous media has been proposed the TLSD model:
  - Accounts for macroscopic diffusion
  - Macroscopic diffusion is determined by two length scales the pore size $\sqrt{K}$ and the domain size H
  - The TLSD model predicts more accurate Sherwood numbers, mass concentration, and r.m.s. velocity than the state of the art DOB equations

Thank you for your attention!

If you are interested in more results or would like to discuss the topic, I am looking forward to meet you at my poster session.

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