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Meshless Lattice Boltzmann Method for pore-scale porous media flow

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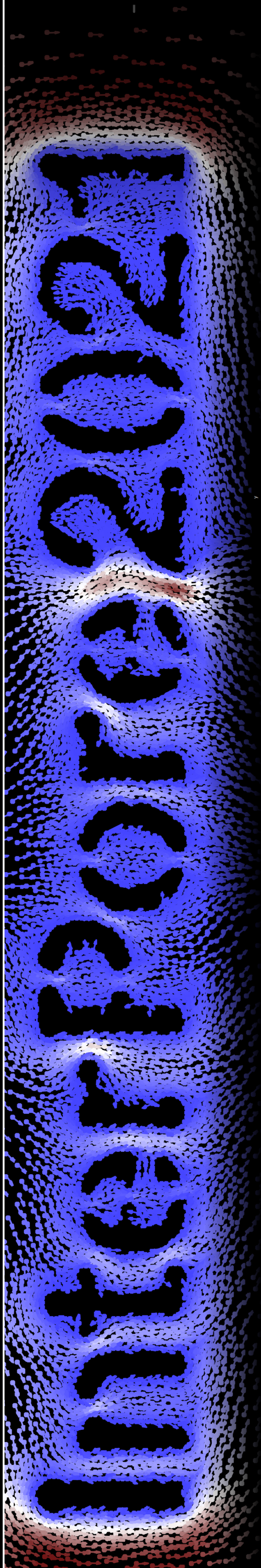
Motivation - how can porous flow solver benefit from being meshless?

Tortuosity (T) and permeability (k) are the two main hydrodynamic properties of porous media. To determine their values one has to know the velocity field of a fluid flowing through a porous sample. Due to the problems with direct measurements as well as with obtaining analytical solutions to hydrodynamics equations in such complex geometries the usage of numerical methods is mandatory in this field of science. However, although a wide range of the latter is available - from finite differences, through finite volume to for example Lattice Boltzmann method, the process of obtaining wanted solutions is still a challenging task. Here we present an application of a fusion of well-established Lattice Boltzmann Method with meshless radial basis functions (RBF) interpolation scheme presented in [4] to calculating tortuosity and permeability of random porous geometries. The algorithm is operating on domains discretized with irregular, non-connected sets of nodes, in contrast to [4] where discretization were rectangular or O-type grids.



RBF interpolation

As early as in 1970s radial functions have been first used as a basis for interpolation of scattered data sets [2]. Since then this method has been both widely studied and utilized to problems varying from pricing financial instruments to biomed flows (see, e.g. [1]). This was due to its **meshless nature** - giving up the necessity of connectivity between nodes discretizing the space, **multidimensionality** as well as **ease of implementation** and subsequent code changes.

A radial function ϕ maps d-dimensional space into a set of scalars. Its value depends only on Euclidean distance between the function's center $\mathbf{x}$ and its argument $\mathbf{x}_0$:

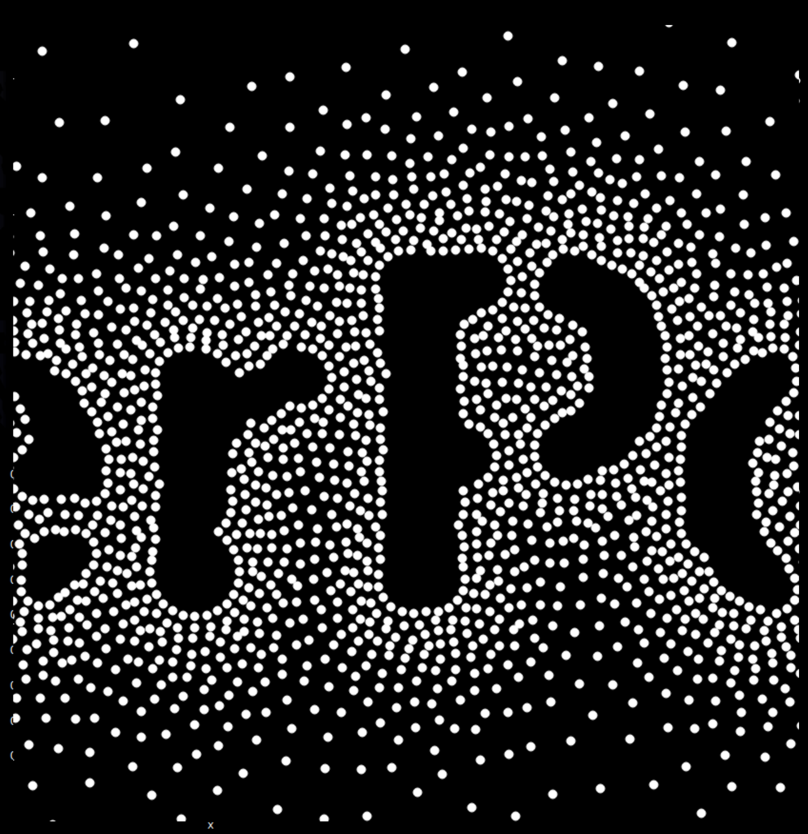


Fig. 1. Discretization used for calculations shown to the left. "InterPore2021" text was approximated using 347 circles distributed with a genetic algorithm. Node set was obtained with Medusa library [5].

$$\phi : \mathbb{R}^d \mapsto \mathbb{R}$$

$$\phi_0(\mathbf{x}) = \phi(|\mathbf{x} - \mathbf{x}_0|) = \phi(r)$$

Real-life examples of radial functions may be point gravitational or electric fields. Many choices for RBFs are available, starting at cubic functions (1), through multiquadratics (2) to Gaussian bells (3):

$$\phi(r) = r^3 \quad (1), \quad \phi(r) = \sqrt{\epsilon^2 + r^2} \quad (2), \quad \phi(r) = e^{-\epsilon^2 r^2} \quad (3); \quad \epsilon - \text{a constant}$$

Giving up mesh in favor of point clouds to discretize the problem makes room for using irregular, locally refined sets of points easily (see Figure 1). To conveniently handle such sets and to account for "non-smoothness" of the interpolated function, interpolation is usually performed in a local manner - interpolation stencils are composed of subsets of all the nodes used for discretization (see Fig. 2). This way, the whole domain is covered by a set of interpolants - one is free to choose which of them will be used in interpolation at an arbitrary point.

Meshless, semi-Lagrangian LBM

The idea of meshless LBM presented in [4] is to interpolate post-collision distribution function values from each of the irregularily placed nodes to Lagrangian departure points from which it is transported to appropriate Eulerian nodes (usually stencil center nodes) in streaming steps (see Fig. 2).

Introducing RBF interpolation to classic Lattice Boltzmann Method on such a basis allows for overcoming the latter's limitations inherited from Lattice Gas Automata - the need for structured, square-grid discretization in physical space and coupling of physical space and velocity discretization.

What follows is among others the approach ability to oprate on boundary-fitted discretizations with variable nodes density and to change the set of discretized velocities independently of physical space discretization.

Below are shown the results of tortuosity and permeability calculations obtained on irregular node sets, compared to one of earlier works in this matter [3].

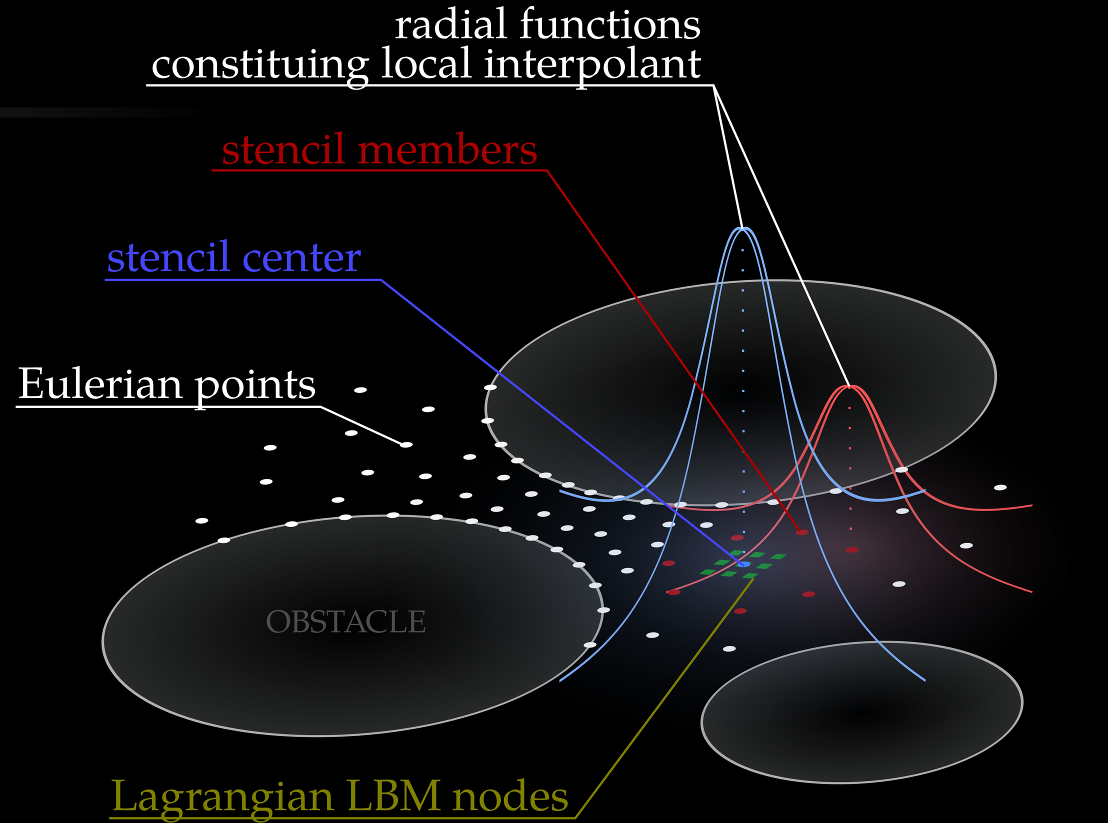


Fig 2. Main ingredients of the discussed interpolation procedure

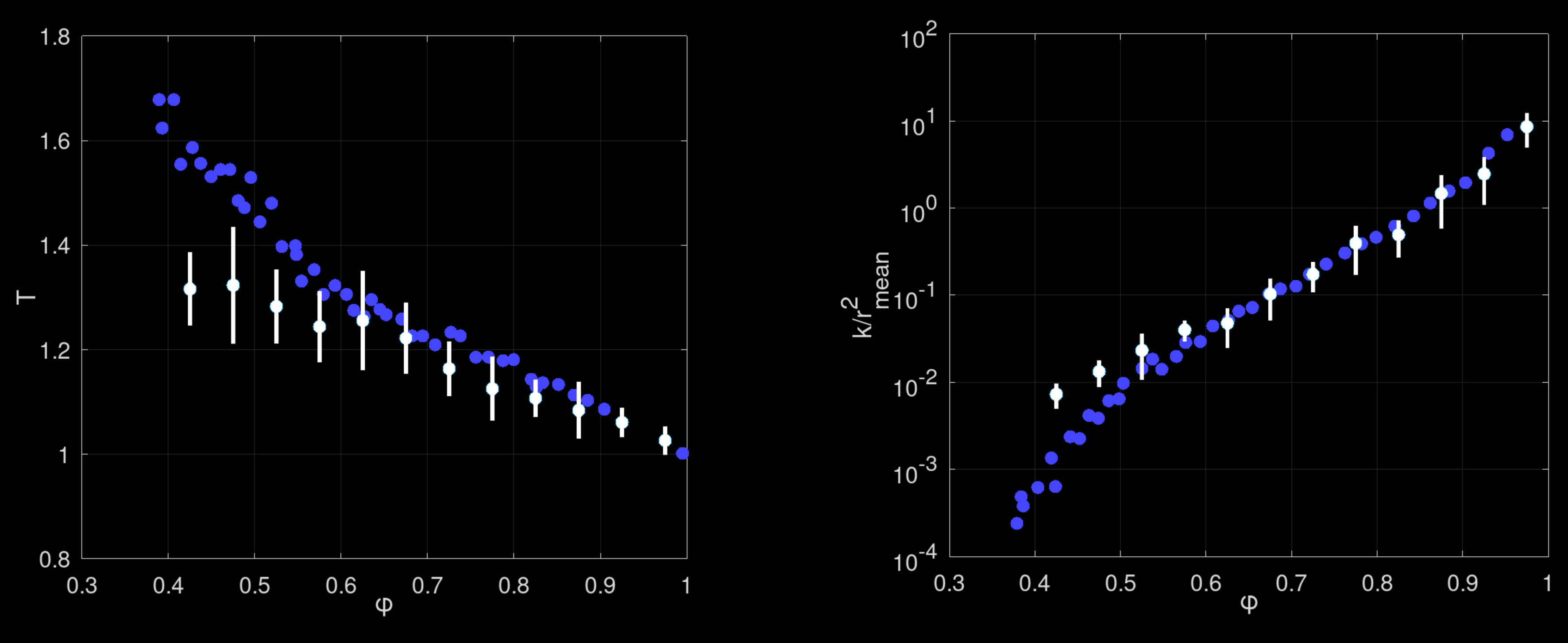


Fig 3. Plots of tortuosity (left) and dimensionless permeability (right) obtained with the discussed algorithm (white markers) and presented in [3] (blue markers) versus porosity.

References

[1] Fornberg, Bengt & Flyer, Natasha (2015). *Solving PDEs with radial basis functions.* Acta Numerica, 24, pp 215-258 doi:10.1017/S0962492914000130

[2] Hardy, R. L. (1971). *Multiquadric equations of topography and other irregular surfaces.* Journal of Geophysical Research, 76(8), 1905–1915. doi:10.1029/jb076i008p01905

[3] Koponen, Antti & Kataja, Markku & Timonen, J. (1997). *Permeability and effective porosity of porous media.* Phys. Rev. E. 56. 10.1103/PhysRevE.56.3319.

[4] Lin, Xingjian & Wu, Jie & Zhang, Tongwei. (2019). A Mesh-Free Radial Basis Function-based Semi-Lagrangian Lattice Boltzmann Method for Incompressible Flows. International Journal for Numerical Methods in Fluids. 91. 10.1002/fld.4749.

[5] Slak, J., & Kosec, G. (2019). *Medusa: A C++ Library for solving PDEs using Strong Form Mesh-Free methods.* arXiv preprint arXiv:1912.13282.

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