How does the power law dependency of flow rate on pressure gradient when viscous and capillary forces compete, scale with system size?

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A. Introduction

Immiscible 2-phase Flow: When two immiscible fluids flow together through a porous media and both of them are fighting for the same pore space.

Controlling Parameters

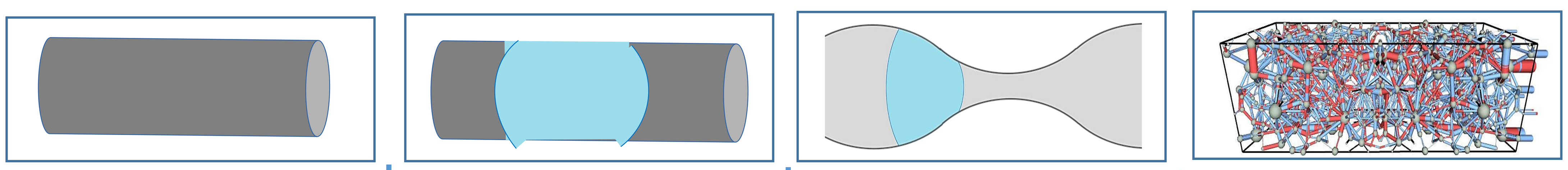
- Pressure gradient
- Geometry of the system
- Saturation of the fluids
- Capillary number → $Ca = \frac{Q\mu}{\gamma A}$
- Viscosity ratio

Area of Application

Oil recovery, CO_2 sequestration, transport in fuel cells, ground-water management, catalyst support in automotive industry, blood flow in capillary vessels.

Rheological Behavior

p_a [q →] p_b $\quad \Delta p = p_a - p_b$



$q = -\frac{\kappa A(p_b - p_a)}{\mu l}$

Henry Darcy, Les Fontaines Publiques de la Ville de Dijon, Dalmont, Paris, 647 (1856)

$q = -\frac{\pi r^4}{8 l \mu}\left(p_b - p_a - \sum p_c\right)$

E. W. Washburn, *The dynamics of capillary flow*, Phys. Rev. 17, 273 (1921)

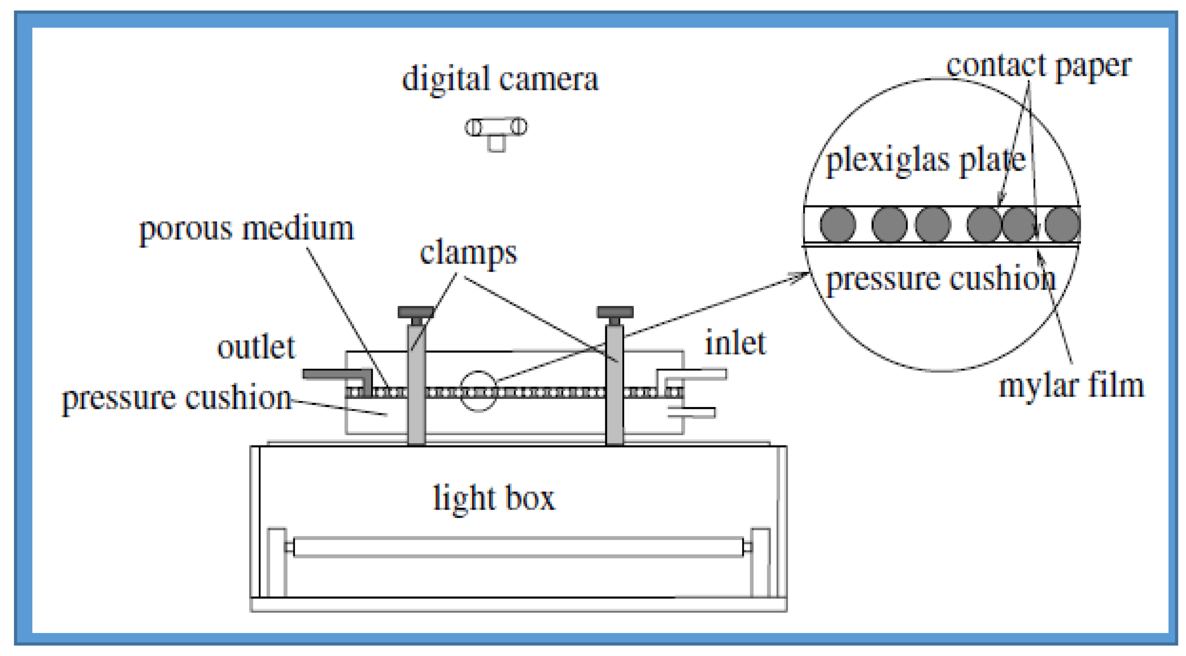
$< q > \sim \sqrt{\Delta p^2 - p_t^2}$

S. Sinha, A. Hansen, D. Bedeaux and S. Kjelstrup, Phys. Rev. E 87, 025001 (2013)

$Q \sim \begin{cases} (|\Delta P| - P_t)^2 & , |\Delta P| > \Delta P_c \\ 0 & , |\Delta P| \le \Delta P_c \end{cases}$

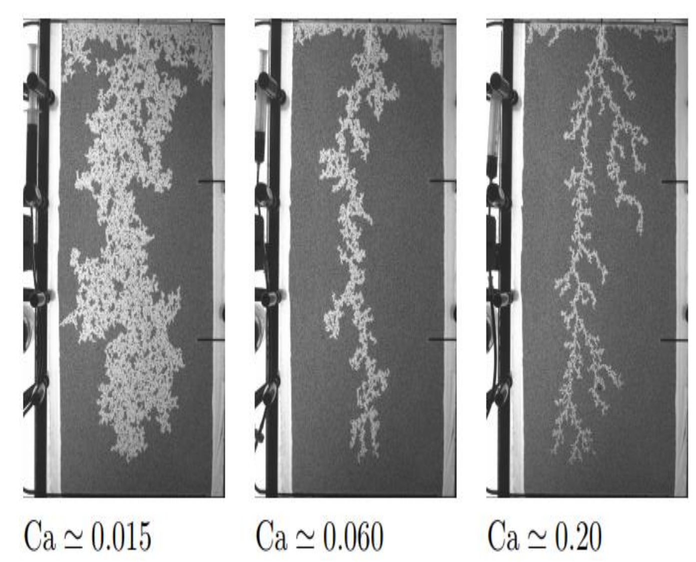
Europhys. Lett. 99, 44004 (2012), Transp. Porous Med. 119, 77 (2017)

B. Experiments [1,2,3]

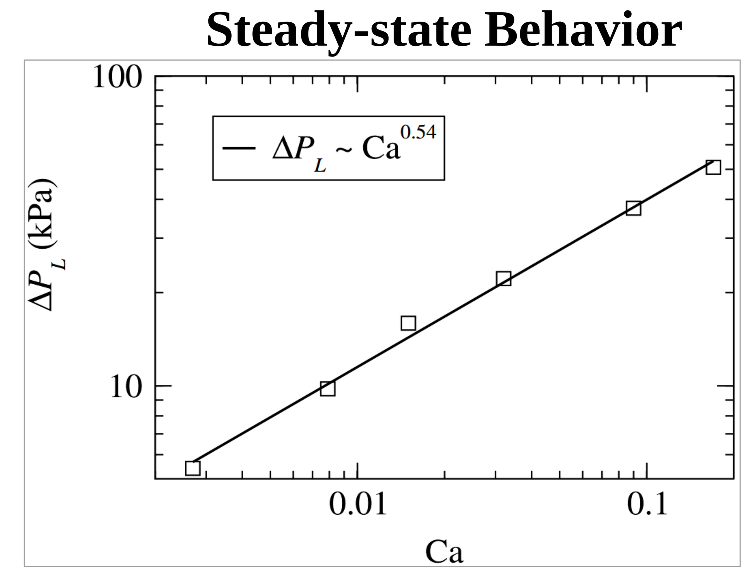


$\Delta P \sim Ca^{0.54}$ ↓ $\Delta P \sim Q^{0.54}$ ↓ $Q \sim \Delta P^{0.54}$

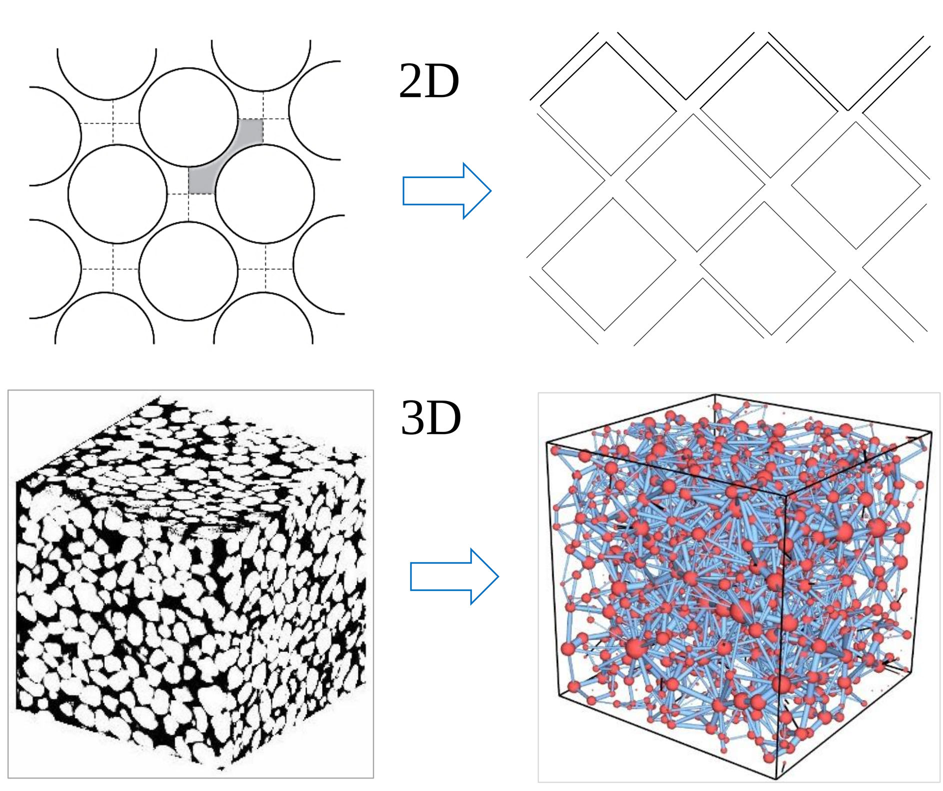
Transient Behavior



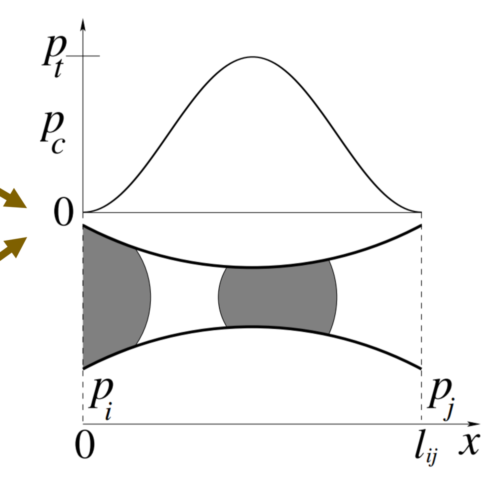
Steady-state Behavior



C. Dynamic Pore Network Model [4]



❑ Capillary pressure at an interface: **Young-Laplace equation [5]**



$$p_c(x) = \frac{2\gamma \cos\vartheta_{ij}}{r_{ij}}\left[1 - \cos\left(\frac{2\pi x}{l_{ij}}\right)\right]$$

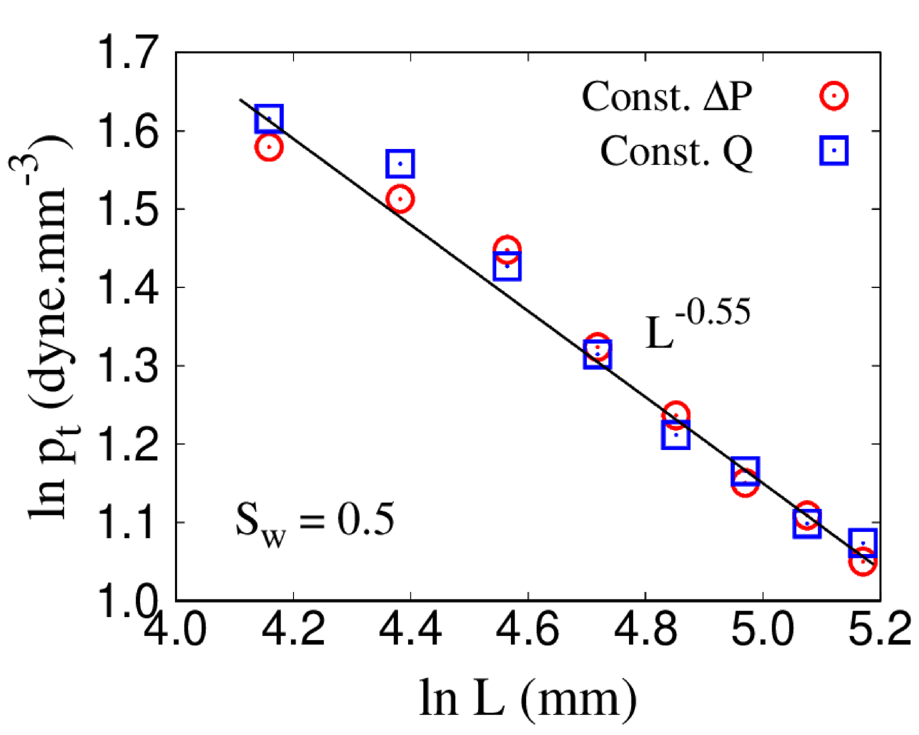
❑ Flow through each pore: **Washburn equation [6]**

l_{ij} : link length
g_{ij} : link mobility

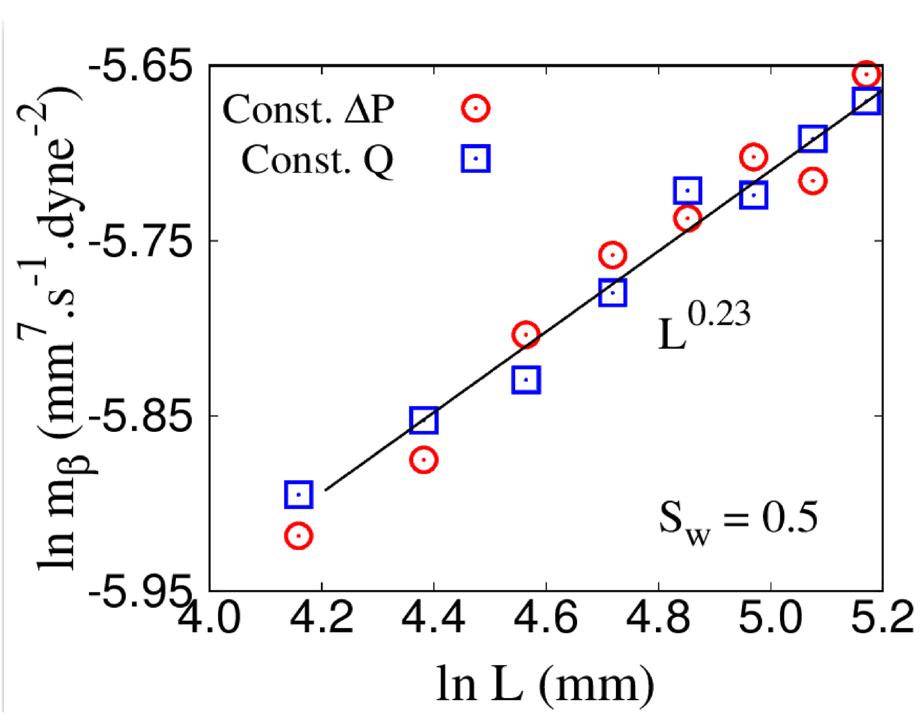
$$q_{ij}(t) = -\frac{g_{ij}}{l_{ij}}\left(\Delta p_{ij} - \sum p_c\right)$$

Assumptions

- ✓ The fluids are incompressible
- ✓ There is no velocity gradient inside a link. Each link comes with a single velocity/flowrate.



$$p_c \sim L^{-\alpha(S_w)}$$



$$m_\beta \sim L^{\gamma(S_w)}$$

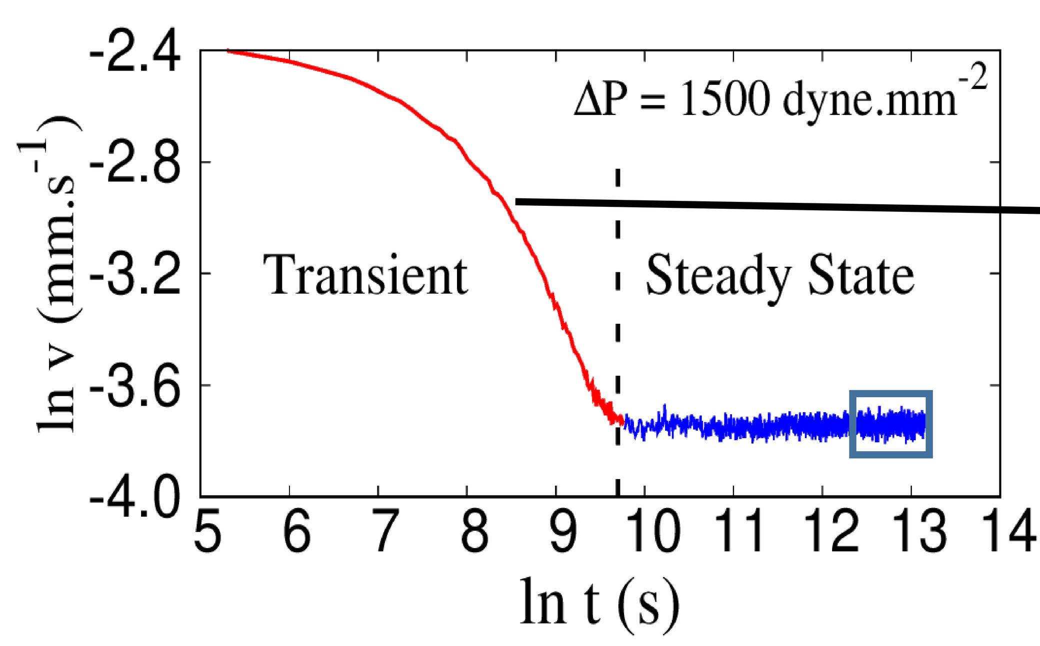
❖ Our numerical results suggest the following observations:

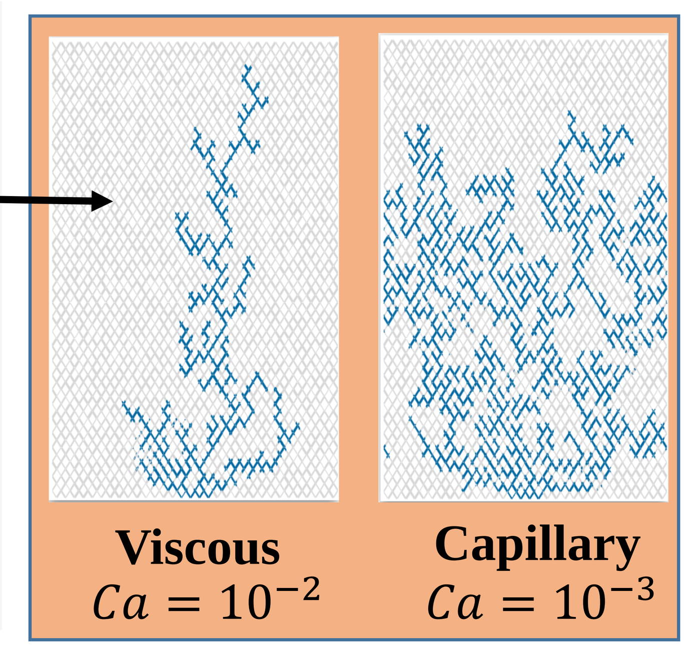
I. The threshold pressure p_t decreases in a scale-free manner with system size L. In the thermodynamic limit we observe zero resistance even if it is a two-phase flow.

II. The mobility m_β increases in a scale-free manner with L and reaches infinite in the thermodynamic limit.

III. Since the 'Darcy line' in case of velocity does not depend on system size, above two factor makes sense only if the non-linear to linear transition point p_m decreases with increasing L. This suggest more linear region as size of the system is increased.

D. Results





❑ Due to the capillary forces there exists a threshold pressure p_t below which there is no flow.

❑ Just above p_t the relation between fluid velocity $\boldsymbol{v}$ (= $\boldsymbol{Q/A}$) and pressure gradient p is non-linear due to path opening dynamics **[1]**.

$$Q = -M_\beta\, sign(\Delta P)\theta(|\Delta P| - P_t).(|\Delta P| - P_t)^\beta$$

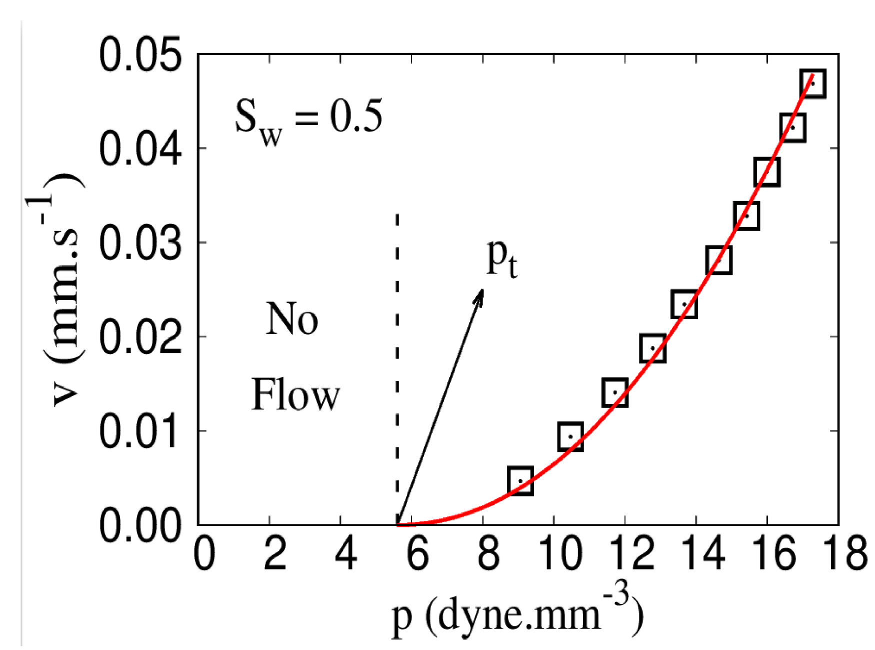
$$v = -m_\beta\, sign(p)\theta(|p| - p_t).(|p| - p_t)^\beta$$

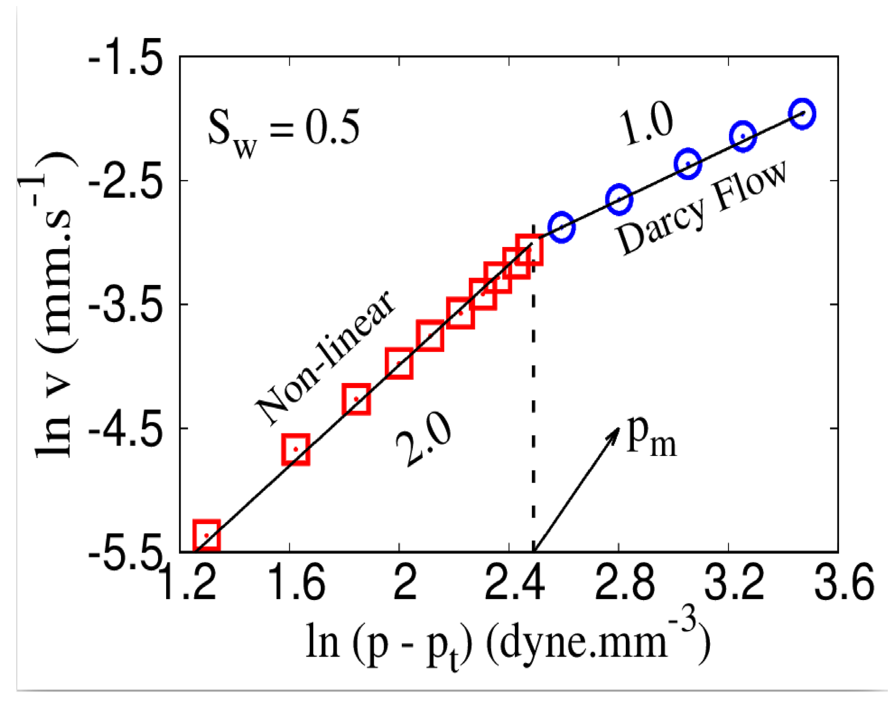
Here, $p_t = P_t/L$, $p = |\Delta P|/L$ and $m_\beta = M_\beta L^\beta/A$.

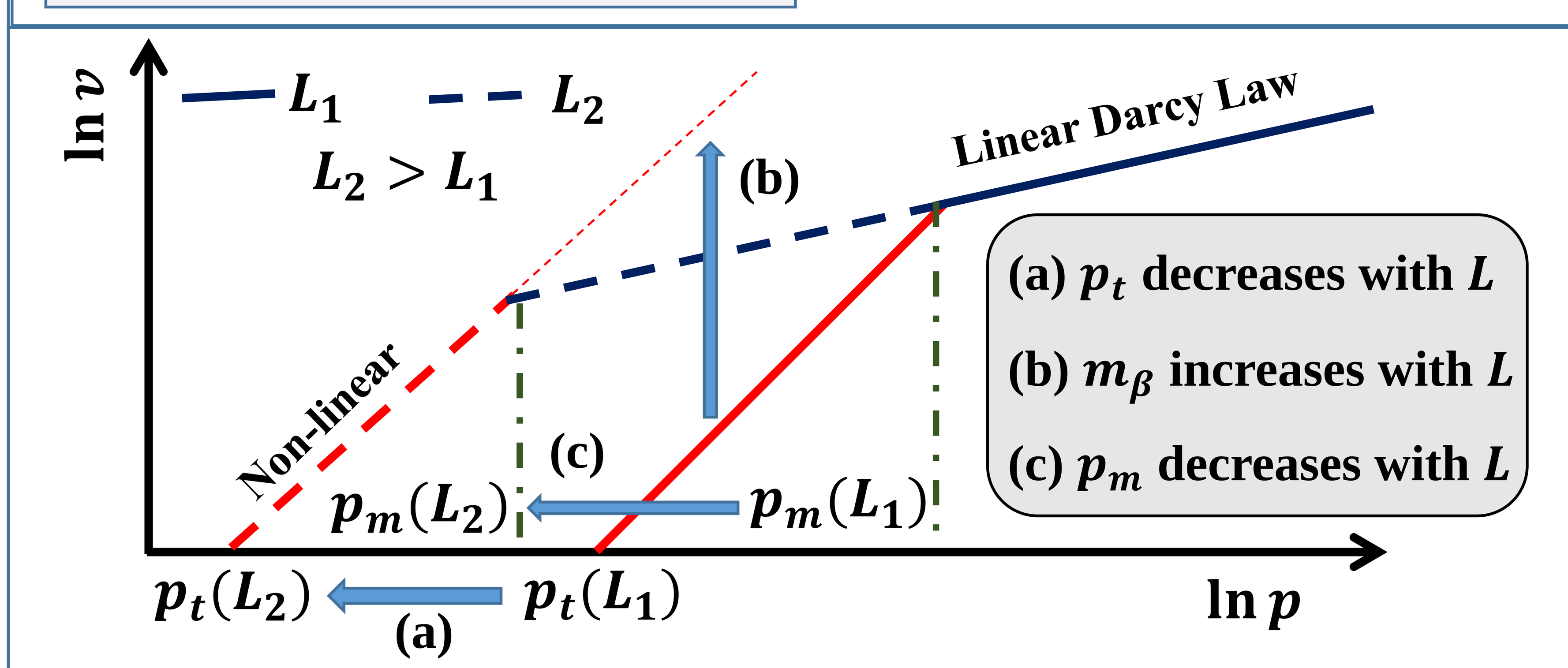
❑ When p is sufficiently high and all possible paths are open, we enter the Darcy limit **[7]**.

$Q = -M_d \Delta P$ and $v = -m_d . p$

Here, $p_t = P_t/L$, $p = |\Delta P|/L$ and $m_\beta = M_\beta L/A$.







(a) p_t decreases with L
(b) m_β increases with L
(c) p_m decreases with L

References: [1] Tallakstad et. al, Phys. Rev. Lett. 102, 074505 (2009); [2] Rassi et. al, New. J. Phys. 13, 015007 (2011); [3] Zhang et. al, Geophys. Res. Lett. e2020GL090477 (2021); [4] Sinha et. al, Front. Phys. 8:548497 (2021); [5] Sinha et. al, Phys. Rev. E. 87, 025001 (2013); [6] Washburn, Phys. Rev. 17, 273 (1921); [7] Darcy, H. (1856). Les Fontaines publiques de la ville de Dijon. Paris: Victor Dalamont, 647.

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