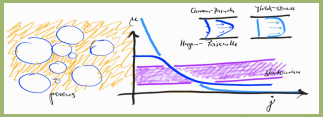

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On the inverse problem of identifying the effective pore size distribution using non-Newtonian fluids

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Pore size distribution in capillary bundle framework

Flow through the porous media is represented as a flow through a bundle of capillaries of various (but uniform) radii:

$$v(\ldots) = \int_0^{r_{\max}} q(\ldots, r) w(r)\, \mathrm{d}r \approx \sum_{j=1}^{N} q_{r_j}(\ldots) w_{r_j}$$

where $q(\ldots, r)$ is given, for given fluid and hydraulic gradient, by Hagen–Poiseulle flow.

The inverse problem

Using non-Newtonian fluids in the flow experiments, say from M fluxes $v_1, \ldots, v_M$ observed for varying hydraulic gradients or fluid rheologies, one hopes to solve the inverse problem

$$v_i \approx \sum_{j=1}^{N_{\text{appx}}} q_{r_j}(\nabla P_i, c_i)\, w_{r_j} = \sum_{j=1}^{N_{\text{appx}}} a_{ij} w_{r_j},$$

obtaining the representative pore size distribution weights $w_{r_1}, \ldots, w_{r_{N_{\text{appx}}}}$.

Refer to

a number of recent works by Abou Najm M.R., Atallah N.M., Hauswirth S.C. and Rodriguez de Castro A., Oukhlef A., Champmartin S., Ambari A., and many others.

General open problems

- Given the experimental data, what is the optimal representative PSD and the best numerical algorithm to reach it?
- Given the data, the algorithm and the results, what is the reliability of the solution (the error estimate).
- Given a rough expectation about the pore size distribution, how to plan the optimal set of experiments (polymer concentrations, pressure drops)?

Sensitivity of inversion

- How much is the computed PSD affected by data error?
- How that depends on the chosen data set (fluid rheologies and hydraulic gradients).
- (And how to estimate that numerically?)

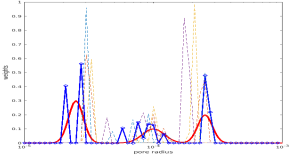
Numerical experiments with artificial data

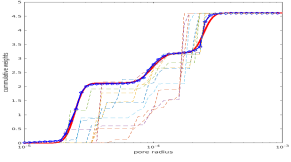
can be used to separate different aspects of the problem:

- the error due to the simplified model (capillary bundle representation of the media);
- the capability of the inversion for given data set;
- the sensitivity of the inversion with respect to data error.

Sensitivity example

See this example of exact pore size distribution, the exact data inversion, and the inversions from data with 1% noise. In this case, some lower hydraulic gradients are missing in the data set.





In this example, the missing experiments in the data set still allow for a very good exact inversion, but completely destroy its rebustness with respect to small random data error.