Pore Pressure Sensitivity-Permeability Decay Evaluation for Nonlinear Oil Flow in Porous Media through Green's Functions

Fernando Bastos Fernandes[1,2]
[1]Pontifical Catholic University of Rio de Janeiro (PUC-Rio), [2]Petróleo Brasileiro S.A. (Petrobras)

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Abstract

This work proposes a permeability decay evaluation through a new integro-differential analytical model using Green's functions (GF's). The model is used to solve the nonlinear hydraulic diffusivity equation (NHDE) for radial oil flow with source term for two case studies. The results showed close convergence, when compared to a numerical simulator and presented errors less than 0.5%.

Introduction

GF's-based analytical models have been proposed to solve the NHDE for isothermal flow through porous media and has shown close agreement, when compared to numerical flow simulator, (Barreto Jr. et al., 2011 and 2012), (Sousa et al., 2015) and (Fernandes et al., 2021a). This work proposes evaluating the permeability decay as a function of pore pressure for two reservoir layers, named as *case study A* and *B*, using a new coupled-pseudopressure model with variable permeability and GF's to solve NHDE with source in an infinite-acting-radial oil flow (**Fig.1**).

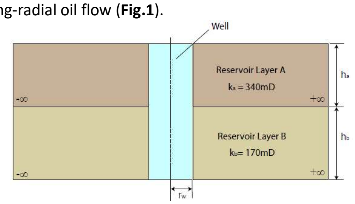

Fig.1: Well sketch in an infinite Reservoir.

Model Assumptions

The proposed model is based on combined asymptotic series expansion and GF for the sealing fault well-reservoir setting. The GF related to this problem can be found in Carslaw and Jaegger (1959). The solution is based on the following premises:

(1) Permeability pore pressure-dependent
(2) Mechanical hysteresis of porous media is negligible
(3) Small pressure gradient
(4) Newtonian fluid inside porous media
(5) Uniform net pay reservoir
(6) Skin and storage effects are not considered
(7) Well fully penetrates reservoir rock
(8) Deformable, homogeneous, linear elastic and isotropic reservoir
(9) Isothermal, single-phase flow in the porous media
(10) Homogeneous initial and boundary conditions

Model Calibration

The model is calibrated through a numerical oil flow simulator. The calibration was performed by replacing the pressure and permeability values in a computational table in the simulator. The values of the analytical model and the numerical simulator were presented in a *semi-log* and *log-log plot* and the results matched.

Analytical Model

The NHDE for oil flow in a homogeneous and isotropic porous media is expressed by:

$$\nabla^2 p - \frac{1}{\eta(p)}\frac{\partial p}{\partial t} = f(\boldsymbol{r}, t) \qquad (1)$$

The new proposed pseudopressure function is:

$$m(p) = \int_{p}^{p_b} k(p')dp' \qquad (2)$$

Where: $\eta(p)$ is hydraulic diffusivity function [*sec/m²*]; $f(\boldsymbol{r}, t)$ is the oil source [*kgf/cm²*], $\boldsymbol{r}$ is the position vector in Cartesian coordinates (x, y, z) [*m*], t is the time [sec], p_b is a reference pressure [*kgf/cm²*], $k(p')$ is the permeability pore pressure dependent function [*mD*] and $m(p)$ is the pseudopressure function [*mD kgf/cm²*].

The dimensionless variables are:

$$r_D = \frac{r}{r_w} \qquad (3)$$

$$t_D = \frac{k_0 t}{\phi \mu c_t r_w^2} \qquad (4)$$

$$k_D = \frac{k(p)}{k_0} \qquad (5)$$

$$\eta_D = \frac{\eta(p)}{\eta_0} \qquad (6)$$

$$m_D = \frac{2\pi h \Delta m(p)}{q\mu} \qquad (7)$$

Where: r_D, t_D, k_D, η_D and m_D are the dimensionless radius, time, permeability, diffusivity function, hydraulic diffusivity factor and pseudo-pressure, respectively. k_0 is permeability in initial pressure [m²]; $\eta(p)$ is hydraulic diffusivity function [sec/m²]; η_0 is hydraulic diffusivity in initial pressure, [sec/m²]; t is time [sec]; c_t is total compressibility, [(kgf/cm²)⁻¹]; μ is fluid dynamic viscosity [cp]; r_w is wellbore radius [m]; r is radial component of cylindrical system of coordinates [m] and h is reservoir thickness [m].

For radial oil flow in porous media, the dimensionless NDHE

$$\frac{1}{r_D}\frac{\partial}{\partial r_D}\left(r_D\frac{\partial m_D}{\partial r_D}\right) - \left(\frac{1}{k_D(p)} + 1\right)\frac{\partial m_D}{\partial t_D} = f_D(r_D, t_D) \qquad (8)$$

The general solution is expressed by the implicit Volterra's second kind integro-differential equation below:

$$m_{wD}(t_D) = -\frac{1}{2}\boldsymbol{Ei}\left(-\frac{1}{4t_D}\right) + \int_0^{\infty}\int_0^{t_D}\left\{\frac{1}{k_D(p_{wD}(t_D))} - 1\right\}\frac{\partial p_{wD}}{\partial t_D} G_D(r_D = 1, r'_D, t_D, t_D')dt'_D dr'_D \qquad (9)$$

Results and Discussions

Fig.2 shows the *semi-log plot* of the dimensionless pseudo-pressure as function of the dimensionless time. The smooth displacement between the curves shows the effect of the permeability drop, in comparison to the linear solution p_{wD}. **Fig.3** shows the *log-log plot* of the dimensionless pseudo-pressure and its Bourdet derivative as function of the dimensionless time.

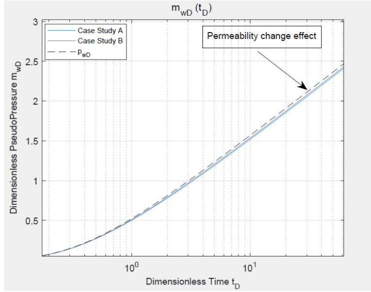

Fig.2: *Semi-log plot* of the dimensionless pseudo-pressure as function of the dimensionless time.

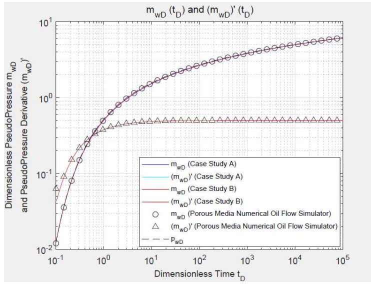

Fig.3: *Log-log plot* of the dimensionless pseudo-pressure and its derivative as function of the dimensionless time.

Conclusions

This work presented an analytical solution of the NHDE for permeability decay evaluation. The results were compared to a porous media oil flow simulator and has shown close accuracy , therefore it may be a useful mathematical tool to calibrate new numerical models that may arise in porous media literature. The *semi-log plot* of the pseudo-pressure can be used to compute the instantaneous economic impairment caused by permeability loss.

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Fernando Bastos Fernandes
PUC-Rio and Petrobras
Email: fernandobastos@petrobras.com.br

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