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A posteriori estimates for the Richards equation (nonlinearity, degeneracy & heterogeneity)

Wednesday, 2 June 2021 19:50 (15 minutes)

Richards equation is commonly used to model the flow of water and air through soil, and it serves as a gateway equation for multiphase flow through porous domains. With pressure p being the primary variable, it equates
\begin{align}
\partial_t S(p) -\nabla\cdot[\mathbf{\bar{K}} \, \kappa(S(p))\, (\nabla p - \mathbf{g})]= f(s,\mathbf{x},t), \tag{1}
\end{align}
where $S : [-\infty, \infty] \to [0, 1]$ is the increasing saturation function, $\kappa : [0, 1] \to [0, 1]$ is the relative permeability function, $\mathbf{K}$ the absolute permeability tensor, $\mathbf{g}$ the gravitational acceleration, and f the reaction/absorption term. Apart from having nonlinear advection/reaction/diffusion components, Richards equation also exhibits both parabolic-hyperbolic (at $S(p) = 0$ since $\kappa(S(p)) = 0$) and parabolic-elliptic (at $S(p) = 1$ since $S'(p) = 0$) type of degeneracies. Further challenges in its numerical treatment comes from the heterogeneity in $\mathbf{K}$.

In this study, we provide fully computable, locally space-time efficient, and reliable a posteriori error bounds [1] for numerical solutions of the fully degenerate Richards equation: if p is the exact solution of (1) and $p_{h\tau}$ is a known approximate solution, then for a composite distance metric $\mathrm{dist}(\cdot, \cdot)$ it holds that
\begin{align}
\underline{\eta}(p_{h\tau})\leq \mathrm{dist}(p,p_{h\tau})\leq \bar{\eta}(p_{h\tau}), \tag{2}
\end{align}
where both $\underline{\eta}(\cdot)$ and $\bar{\eta}(\cdot)$ are fully computable, and a version of the lower bound holds in any space-time subdomain. The bounds are proven in a variation of the $H^1(H^{-1}) \cap L^2(L^2) \cap L^2(H^1)$ norm which corresponds to the minimum regularity inherited by the exact solutions, thus avoiding further smoothness assumptions like in [2]. For showing the upper bound, error estimates are derived individually for the $H^1(H^{-1})$, $L^2(L^2)$ and the $L^2(H^1)$ error components with a maximum principle and a novel degeneracy estimator being used for the last one. Local and global space-time efficient error bounds are obtained following [3]. Error contributors such as flux and time non-conformity, quadrature, linearisation, data oscillation are identified and separated. The estimates also work in a fully adaptive space-time discretization and linearisation setting.

To investigate the effectiveness of the estimators, numerical tests are conducted for non-degenerate and degenerate cases having exact solutions. It is shown that the estimators correctly identify the errors, both spatially and temporally, up to a factor in the order of unity. Finally, to demonstrate the prowess of the estimators, a degenerate problem is analyzed in a heterogeneous, anisotropic domain with discontinuous initial condition and mixed boundary conditions.

Time Block Preference

Time Block B (14:00-17:00 CET)

References

[1] M. Ainsworth and J.T. Oden. A posteriori error estimation in finite element analysis. Pure and Applied Mathematics (New York). Wiley-Interscience [John Wiley & Sons],
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[2] V.Dolejší, A. Ern, and M. Vohralík. A framework for robust a posteriori error control in unsteady nonlinear advection-diffusion problems. SIAM Journal on Numerical Analysis, 51(2): 773-793, 2013.

[3] A. Ern, I. Smears, and M. Vohralík. Guaranteed, locally space-time efficient, and polynomial-degree robust a posteriori error estimates for high-order discretizations of parabolic problems. SIAM Journal of Numerical Analysis, 55(6): 2811-2834, 2017.

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