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# A posteriori estimates for the Richards equation (nonlinearity, degeneracy & heterogeneity)

Wednesday, 2 June 2021 19:50 (15 minutes)

Richards equation is commonly used to model the flow of water and air through soil, and it serves as a gateway equation for multiphase flow through porous domains. With pressure p being the primary variable, it equates  $bein{align}$ 

where  $S: [-\infty, \infty] \to [0, 1]$  is the increasing saturation function,  $\kappa : [0, 1] \to [0, 1]$  is the relative permeability function, **K** the absolute permeability tensor, **g** the gravitational acceleration, and *f* the reaction/absorption term. Apart from having nonlinear advection/reaction/diffusion components, Richards equation also exhibits both parabolic-hyperbolic (at S(p) = 0 since  $\kappa(S(p)) = 0$ ) and parabolic-elliptic (at S(p) = 1 since S'(p) = 0) type of degeneracies. Further challenges in its numerical treatment comes from the heterogeneity in **K**.

In this study, we provide fully computable, locally space-time efficient, and reliable a posteriori error bounds [1] for numerical solutions of the fully degenerate Richards equation: if p is the exact solution of (1) and  $p_{h\tau}$  is a known approximate solution, then for a composite distance metric dist $(\cdot, \cdot)$  it holds that \begin{align}

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where both  $\underline{\eta}(\cdot)$  and  $\overline{\eta}(\cdot)$  are fully computable, and a version of the lower bound holds in any space-time subdomain. The bounds are proven in a variation of the  $H^1(H^{-1}) \cap L^2(L^2) \cap L^2(H^1)$  norm which corresponds to the minimum regularity inherited by the exact solutions, thus avoiding further smoothness assumptions like in [2]. For showing the upper bound, error estimates are derived individually for the  $H^1(H^{-1}), L^2(L^2)$ and the  $L^2(H^1)$  error components with a maximum principle and a novel degeneracy estimator being used for the last one. Local and global space-time efficient error bounds are obtained following [3]. Error contributors such as flux and time non-conformity, quadrature, linearisation, data oscillation are identified and separated. The estimates also work in a fully adaptive space-time discretization and linearisation setting.

To investigate the effectiveness of the estimators, numerical tests are conducted for non-degenerate and degenerate cases having exact solutions. It is shown that the estimators correctly identify the errors, both spatially and temporally, up to a factor in the order of unity. Finally, to demonstrate the prowess of the estimators, a degenerate problem is analyzed in a heterogeneous, anisotropic domain with discontinuous initial condition and mixed boundary conditions.

## **Time Block Preference**

Time Block B (14:00-17:00 CET)

#### References

 M. Ainsworth and J.T. Oden. A posteriori error estimation in finite element analysis. Pure and Applied Mathematics (New York). Wiley-Interscience [John Wiley & Sons], New York, 2000 [2] V.Dolejší, A. Ern, and M. Vohralík. A framework for robust a posteriori error control in unsteady nonlinear advection-diffusion problems. SIAM Journal on Numerical Analysis, 51(2): 773-793, 2013.

[3] A. Ern, I. Smears, and M. Vohralík. Guaranteed, locally space-time efficient, and polynomial-degree robust a posteriori error estimates for high-order discretizations of parabolic problems. SIAM Journal of Numerical Analysis, 55(6): 2811-2834, 2017.

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