



Contribution ID: 917

Type: Oral 20 Minutes

A continuum model of gravity fingering endowed with an entropy function and bounded saturation overshoot

Thursday 17 May 2018 11:41 (15 minutes)

Gravity-driven fingering is commonly observed during water infiltration in soil. An important feature of gravity fingering is the presence of saturation overshoot, which is understood to be a necessary prerequisite for gravity fingering in unsaturated flows. The Richards model in its basic form, along with the standard (monotonic) pressure-saturation relation, is incompatible with saturation overshoot for infiltration into unsaturated homogeneous media and cannot reproduce the fingering phenomenon.

Most model extensions of the Richards equation do not guarantee that the saturation overshoot be bounded. To remedy this unphysical situation, additional techniques have been invoked and incorporated into multiphase models, such as the use of a relaxation term (Nieber et al., 2005) or adding a compressibility term in the capillary potential (Cueto-Felgueroso and Juanes, 2009). Here we provide the first continuum mathematical model of unstable infiltration in porous media which respects the bounded saturation by construction, therefore overcoming a serious limitation of previous models. The model is developed based on the phase-field methodology, where liquid saturation is the phase field (Cueto-Felgueroso and Juanes, 2008). We design a new formulation of the free energy of the system and develop a special function of saturation, which we term the capillary pinning function, multiplying the square-gradient term. This function is constructed based on equilibrium considerations in the direction orthogonal to gravity. Our approach leads to a positive pinning function whose structure and properties under fully saturated conditions allow the proposed model, by construction, to honor that water saturations remain bounded.

The proposed continuum model is an extension of the Richards equation with a nonlinear high-order term which introduces a macroscopic surface tension at the wetting front and exhibits a strong link with the capillary pressure. We prove that the proposed model leads to an entropy-increasing evolution for an isolated system. The model reproduces the saturation overshoot at the wetting front and the formation of gravity fingers. Our theoretical analysis shows that the proposed model satisfies the bounded saturation by construction. Our numerical simulations show that the proposed model resolves the pinning behavior at the base of the infiltration front, and confirm the bounded saturation of the proposed continuum model.

References

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Authors: Dr BELJADID, Abdelaziz (MIT); Dr CUETO-FELGUEROSO, Luis (MIT and Technical University of Madrid); Prof. JUANES, Ruben (MIT)

Presenter: Dr BELJADID, Abdelaziz (MIT)

Session Classification: Parallel 10-E

Track Classification: GS 4: Porous media applications (renamed)