

A new coupled approach for solving convection-diffusion problems with discontinuous capillary pressure

HMFEM+Operator Splitting+FV

- Hybridized Mixed FEM + FV scheme via operator splitting (Abreu 2014)
- Three-phase flow (systems) in two dimensions
- Multiplicative discontinuous capillary pressure model $P_c(S, K(x)) = p_c(S) \sqrt{\frac{K(x)}{\phi(x)}}$

FVM+Interface Coupling

- Finite volume method + interface coupling (Andreianov et. al 2013)
- Two-phase (scalar) one-dimensional
- Additive discontinuous capillary pressure model $P_c(S, K(x)) = P_{L,R} - \log(1 - S)$

New Coupled Approach

- • Unified fully-coupled hybrid mixed finite element and finite volume formalism
- • Novel reinterpretation of Robin coupling conditions
- • More general capillary pressure models

Two-phase flow problem, with discontinuous capillary pressure:

$$\frac{\partial}{\partial t} (\phi S) + \frac{\partial \mathbf{F}}{\partial x} = \varepsilon \frac{\partial \mathbf{w}}{\partial x},$$

with \mathbf{F} and \mathbf{w} being the convective and diffusive fluxes defined by

$$\mathbf{F}(S, x) = \mathbf{v}f + \mathbf{G},$$

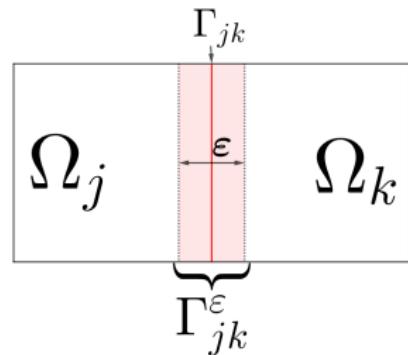
$$\mathbf{w}(S, x) = -K(x) [\lambda(S)(1-f)] \frac{\partial p_c}{\partial x}.$$

The hybrid mixed finite element method is finding $\mathbf{w}_j \in W_j$, $S \in V_j$ and $\ell_{ij} \in \mathbb{R}$ such that

$$\begin{cases} \left(H^{-1} \mathbf{w}_j, \psi_j \right)_{\Omega_j} - \left(S_j, \frac{\partial \psi_j}{\partial x} \right)_{\Omega_j} + \ell_{k,j} \psi_j^R - \ell_{j,k} \psi_j^L = 0, \\ \left(\frac{\partial \phi S_j}{\partial t}, \varphi_j \right)_{\Omega_j} + \left(\frac{\partial \mathbf{F}_j}{\partial x}, \varphi_j \right)_{\Omega_j} - \left(\varepsilon \frac{\partial \mathbf{w}_j}{\partial x}, \varphi_j \right)_{\Omega_j} = 0. \end{cases}$$

The classical Robin Coupling Condition

$$-\chi_{k,j} \mathbf{w}_j \cdot \nu_j + \ell_{k,j} = -\chi_{k,j} \mathbf{w}_k \cdot \nu_k + \ell_{k,j}$$



The weakening of the Robin Coupling Condition

$$-\chi^L \int_{\Gamma_\varepsilon} \mathbf{w}_j^L dx + \ell_j^L |\Gamma_\varepsilon| = -\chi^R \int_{\Gamma_\varepsilon} \mathbf{w}_k^R dx + \ell_k^R |\Gamma_\varepsilon|.$$

